



**QUANTUM AND STATISTICAL PROPERTIES OF  
THREE-LEVEL LASER DRIVEN BY COHERENT LIGHT**

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OPTICS)

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*This Work is Dedicated  
to my 'mom'*

# Contents

<b>Acknowledgment</b>	<b>iii</b>
<b>Abstract</b>	<b>iv</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Statement of Problem . . . . .	4
1.2 Research Questions . . . . .	5
1.3 Objectives . . . . .	6
1.3.1 General Objective . . . . .	6
1.3.2 Specific Objectives . . . . .	6
1.4 Significance of Study . . . . .	7
<b>2 Operator Dynamics</b>	<b>8</b>
2.1 Master Equation . . . . .	10
2.2 Quantum Langevin Equation . . . . .	11
2.3 Equations of Evolution for Atomic and Cavity Mode Operators . . . . .	12
<b>3 Photon Statistics</b>	<b>24</b>
3.1 Mean Photon Number . . . . .	24
3.2 Photon Number Variance . . . . .	31
<b>4 Quadrature Fluctuations</b>	<b>36</b>
4.1 Quadrature Variance . . . . .	36
4.2 The quadrature Squeezing . . . . .	40

<b>5 Conclusion</b>	<b>42</b>
<b>Bibliography</b>	<b>43</b>

# List of Figures

1.1	Degenerate three-level atom . . . . .	2
2.1	Schematic representation of a degenerate three-level laser with a degenerate parametric amplifier (DPA) coupled to a squeezed vacuum reservoir. . . . .	8
3.1	Plots of the steady-state mean photon number [Eq.(3.24)] versus $\Omega$ , for $\kappa=0.8$ , $\gamma_c=0.4$ , $N=50$ , $r=0.5$ , and for different values of $\varepsilon$ . . . . .	29
3.2	Plots of the steady-state mean photon number [Eq.(3.24)] versus $\Omega$ , for $\kappa=0.8$ , $\varepsilon=0.02$ , $N=50$ , $r=0.5$ , and for different values of $\gamma_c$ . . . . .	30
3.3	Plots of the steady-state mean photon number [Eq.(3.24)] versus $\Omega$ , for $\kappa=0.8$ , $\varepsilon=0.02$ , $N=50$ , $\gamma_c=0.4$ , and for different values of $r$ . . . . .	31
3.4	Plots of the photon number variance [Eq.(3.32)] versus $\Omega$ , for $\kappa=0.8$ , $r=0.5$ , $N=50$ , $\gamma_c=0.4$ and for different value of $\varepsilon$ . . . . .	33
3.5	Plots of the steady-state photon number variance [Eq.(3.32)] versus $\Omega$ , for $\kappa=0.8$ , $\varepsilon=0.02$ , $N=50$ , $\gamma_c=0.4$ and for different value of $r$ . . . . .	34
3.6	Plots of the photon number variance [Eq.(3.32)] versus $\Omega$ , for $\kappa=0.8$ , $\varepsilon=0.02$ , $N=50$ , $r=0.5$ and for different value of $\gamma_c$ . . . . .	35
4.1	Plots of $(\Delta a_-)^2$ [Eq.(4.11)] versus $\Omega$ , for $\kappa=0.8$ , $\gamma_c=0.4$ , and $N=50$ , and for different values of the amplitude of the pump mode that drives the NLC, $\varepsilon$ . . . . .	39
4.2	Plots of quadrature squeezing vs $\Omega$ [Eq.(4.13)], for $\kappa=0.8$ , $\gamma_c=0.4$ , and for different value of the amplitude of pump mode that drives the NLC, $\varepsilon$ . . . . .	40

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# Abstract

We have analyzed the quantum and statistical properties of the cavity light beam produced by a coherently driven degenerate three-level laser with a degenerate parametric amplifier (DPA) in a closed cavity and coupled to a squeezed vacuum reservoir via a single-port mirror. Applying the large-time approximation scheme, we have obtained the steady-state solutions of the equations of evolution for the expectation values of the atomic operators and the quantum Langevin equations for the cavity mode operators. We calculated the mean photon number, photon number variance, quadrature variance, and quadrature squeezing for the cavity light using the steady-state solutions. We have seen that the light generated by the three-level laser is in a squeezed state and the squeezing occurs in the minus quadrature. The maximum quadrature squeezing of light generated by three level atom is 60% below vacuum-state level. The parametric amplifier and stimulated emission rate increase the mean and photon number variance, whereas the squeezing parameter decreases mean and photon number variance. It so turns out that the parametric amplifier increases the squeezing significantly. It is also found that one effect of the coupling of the top and bottom levels is to increase the mean and photon number variance.

# Chapter 1

## Introduction

Three-level cascade lasers have received considerable interest in connection with its potential as a source of light with interesting non-classical features [1–4]. A three-level laser is a quantum optical system in which light is generated by three-level atoms inside a cavity usually coupled to a vacuum reservoir via a single-port mirror. The quantum properties of the light is attributed to atomic coherence that can be induced either by preparing the atoms initially in a coherent superposition of the top and bottom levels [5] or pumping its top and bottom energy levels with a strong classical driving field [6,7] or using these mechanisms together [8]. Such mechanism imposes a constraint on the populations of atoms in the bottom and top levels in which transitions to and from could not be made in the electric dipole approximation. The classical pumping radiation, therefore, contributes to the observed non-classical properties by facilitating atomic population transfer pathway in which the induced correlation is transferred to the emitted photons.

In squeezed light the noise in one quadrature is below the vacuum or coherent level at the expense of enhanced fluctuation in the other quadrature, with product of the uncertainties in the two quadrature's satisfying the uncertainty relation [9,10]. This feature is a direct consequence of quantum nature of light and cannot be explained within the classical framework. Due to the quantum noise reduction achievable below the vacuum state level, squeezed light has potential applications in the detection of weak signals, in low-noise communications and high precision measurements in atomic fountain clocks

and interferometers [11–14]. Squeezed light can be generated by various quantum optical processes such as subharmonic generations [15], four-wave mixing [9, 16], resonance fluorescence [15, 17] and three-level laser under certain conditions [15, 18]. When a three-level atom in a cascade configuration makes a transition from top to the bottom level via the intermediate level, two photons are generated. If the two photons have the same frequency, then the three-level atom is called a degenerate three-level atom; otherwise, it is called non-degenerate. The two photons are highly correlated and this correlation is responsible for the production of squeezed light. Degenerate three-level atom is an atom, which is its valence electron has three possible energy levels. The energy gap between the first level and the second level is the same with the energy gap between the second level and the third level as shown in Fig(1.1).

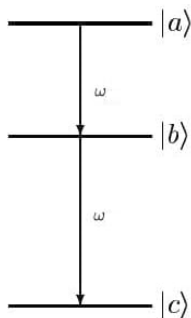


Figure 1.1: Degenerate three-level atom

Moreover, one of the various possible mechanisms employed in enhancing the non-classical properties is introducing a non-linear crystal inside the laser cavity [19–21]. Usually, these properties get amplified in a certain condition in response to the non-linearity introduced into the laser cavity. The non-linear crystals used in such cases are often degenerate or non-degenerate parametric amplifiers, which are defined in terms of the frequencies produced after the down conversion process. A parametric oscillator is one of the most interesting and well characterized optical device in quantum optics. In this device a pump photon interacts with a non-linear crystal inside a cavity and is down converted in two highly correlated photons. If these photons have the same frequency the device is called a degenerate parametric oscillator, otherwise it is called a non-degenerate parametric oscillator. The correlation between these frequencies leads to the production

of squeezing up to 50% below the standard quantum limit [22]. Degenerate parametric amplifier is a phenomenon in the non-linear optics, where a pump mode of frequency  $\omega$  down converted into a pair of signal photons each of frequency  $2\omega$  [9].

In our case we seek to analyze the quantum and statistical properties of degenerate three-level laser whose cavity contain degenerate parametric amplifier (DPA) driven by coherent light with a closed cavity coupled to a squeezed vacuum reservoir via a single-port mirror. Strong coherent light is used to couple the top and bottom levels of a degenerate three-level atom, and a non-linear crystal called a degenerate parametric amplifier is placed inside the laser cavity, with the pump mode emerging from the parametric amplifier not coupling the top and bottom levels of the atoms.

## 1.1 Statement of Problem

Many prior work focused on the quantum and statistical properties of a degenerate three-level laser with a parametric amplifier that is driven by coherent light coupled to a vacuum reservoir in which the effect of an external noise on the quantum properties and photon statistics is completely neglected. Apart from the quantum and statistical properties of degenerate three-level lasers with parametric amplifiers driven by coherent light coupled to squeezed vacuum reservoir, the calculation of the squeezing and statistical properties of the cavity mode, as well as the effects of vacuum reservoir on photon statistics and quadrature squeezing of the cavity light, has received special attention. The quantum and statistical properties of degenerate three-level lasers with a degenerate parametric amplifier would be the main focus of this thesis. This thesis also focuses on the impact of squeezed vacuum reservoir noise on photon statistics, as well as quadrature squeezing of cavity light. Numerous scientists and engineers have been working to understand the quantum and statistical properties of three-level atoms, which has resulted in many fascinating and helpful theoretical and experimental findings. The quantum optical system under consideration is expected to produce a squeezed light which has potential applications in precision measurements and in noiseless communications.

## 1.2 Research Questions

1. What effect does the squeezing parameter have on the quantum and statistical properties of a degenerate three-level laser that is coherently driven?
2. What effect does the parametric amplifier, coupling coherent light amplitude, and photon stimulated emission rate have on the quantum and statistical features of a coherently driven degenerate three level laser?
3. For cavity light beams, how can we calculate and describe the photon statistics, quadrature variance, and quadrature squeezing?

## 1.3 Objectives

### 1.3.1 General Objective

The primary goal of this thesis is to investigate the quantum and statistical properties of a degenerate three-level laser with a degenerate parametric amplifier in side a cavity.

### 1.3.2 Specific Objectives

The specific objectives of this thesis are:-

- ✓ To calculate the photon statistics, and the quadrature squeezing of the cavity light.
- ✓ To study the effect of squeezing parameter on the quantum and statistical properties of coherently driven degenerate three-level laser.
- ✓ To study significance of the parametric amplifier, amplitude of the coupling coherent light, and the stimulated emission rate of photon on the quantum, and statistical properties of degenerate three-level laser.

## 1.4 Significance of Study

The out come of our study can be:

- Used to go from squeezing production to application, as well as to demonstrate experiments and well-understood squeezing sources.
- Useful to find some convenient means of generating a bright squeezed light.
- Other researchers interested in the quantum and statistical features of light created by a coherently driven degenerate three-level laser with parametric amplifier can use this document.

## Chapter 2

# Operator Dynamics

In this chapter, we consider a three-level laser that consists of a cavity with degenerate three-level atoms in a cascade configuration. As shown in Fig (2.1) (Adapted from [23] with some modifications), the top, middle, and bottom levels of a three-level atom are represented by  $|a\rangle_k, |b\rangle_k$ , and  $|c\rangle_k$  respectively, where as  $k=1, 2, \dots, N$  are the number of atoms inside the cavity. In addition, we assume the transitions between levels  $|a\rangle_k$  to  $|b\rangle_k$  and  $|b\rangle_k$  to  $|c\rangle_k$  dipole allowed, and with direct transitions between levels  $|a\rangle_k$  to  $|c\rangle_k$  dipole forbidden.

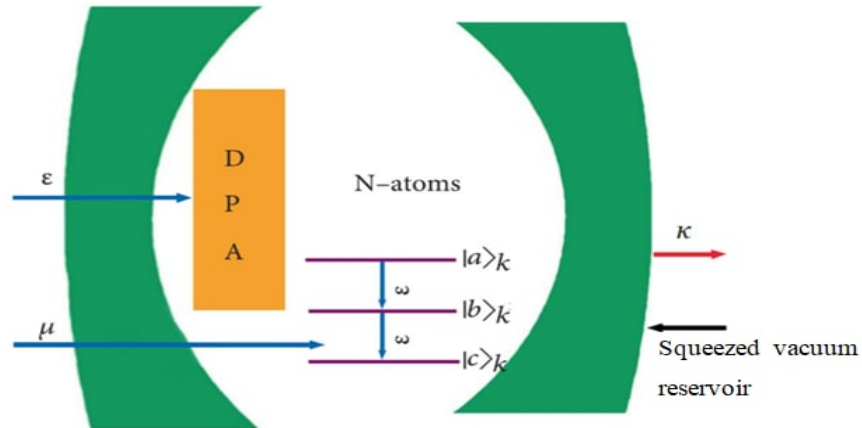


Figure 2.1: Schematic representation of a degenerate three-level laser with a degenerate parametric amplifier (DPA) coupled to a squeezed vacuum reservoir.

Before obtaining the master equation for our system, we first write the Hamiltonian describing various interactions. In this system, there are three cavity interactions. One of

this is the driving coherent light interacts with the three-level atom. In this interaction, the coherent light couples the top and bottom levels of the atom. The Hamiltonian describing the coupling of the top and bottom levels of the three-level atom by driving coherent light is given by [7]

$$\hat{H}_1 = i\lambda (\hat{\sigma}_c^{\dagger k} \hat{c} - \hat{c}^\dagger \hat{\sigma}_c^k), \quad (2.1)$$

where

$$\sigma_c^k = |c\rangle_{kk}\langle a| \quad (2.2)$$

is lowering atomic operator,  $\lambda$  is the coupling constant,  $\hat{c}$  is annihilation operator of driving coherent light. We can write  $\hat{c} = \hat{c}^\dagger = \mu$  for classically treated driving coherent light and rewrite Eq. (2.1) as

$$\hat{H}_1 = i\frac{\Omega}{2} (\hat{\sigma}_c^{\dagger k} - \hat{\sigma}_c^k), \quad (2.3)$$

where  $\Omega = 2\lambda\mu$  is a positive real c-number proportional to the amplitude of the coupling coherent light.

The Hamiltonian describing the interaction between a degenerate three level atom and the cavity mode  $\hat{a}$  in the dipole and rotating wave approximations is expressible as [10]

$$\hat{H}_2 = ig \left( (\hat{\sigma}_a^{\dagger k} + \hat{\sigma}_b^{\dagger k}) \hat{a} - \hat{a}^\dagger (\hat{\sigma}_a^k + \hat{\sigma}_b^k) \right), \quad (2.4)$$

where

$$\hat{\sigma}_a^k = |b\rangle_{kk}\langle a|, \quad (2.5)$$

and

$$\hat{\sigma}_b^k = |c\rangle_{kk}\langle b| \quad (2.6)$$

are lowering atomic operators,  $\hat{a}$  and  $\hat{a}^\dagger$  are the annihilation and creation operators for the cavity modes,  $g$  is the coupling constant between the atom and the cavity modes. The light emerging from the non-linear crystal does not couple the top and bottom levels.

The parametric interaction is described by [24, 25]

$$\hat{H}_3 = \frac{i\varepsilon}{2} (\hat{a}^2 - \hat{a}^{\dagger 2}), \quad (2.7)$$

in which  $\varepsilon = 2\beta\lambda_0$  with  $\lambda_0$  is the coupling constant between the pump mode and nonlinear crystal, and  $\beta$  is proportional to the amplitude of the driving pump mode.

Hence, on the basis of Eqs. (2.3), (2.4), and (2.7) the interaction of a degenerate three level laser with the cavity modes and the coupling of the top and bottom levels of the three-level atom by driving coherent with DPA can be described by the Hamiltonian

$$\hat{H} = i\frac{\Omega}{2} (\hat{\sigma}_c^{\dagger k} - \hat{\sigma}_c^k) + \frac{i\varepsilon}{2} (\hat{a}^2 - \hat{a}^{\dagger 2}) + ig \left( (\hat{\sigma}_a^{\dagger k} + \hat{\sigma}_b^{\dagger k})\hat{a} - \hat{a}^{\dagger}(\hat{\sigma}_a^k + \hat{\sigma}_b^k) \right). \quad (2.8)$$

## 2.1 Master Equation

The quantum analysis of the interaction of a system such as a cavity mode or a three-level atom with the external environment is a relatively complex problem. In order to study the dynamics of the cavity radiation of the combined system, it is necessary to obtain the relevant equations of evolution. The external environment, usually referred to as a reservoir, can be thermal light, ordinary, or squeezed vacuum. The dynamics of the system is describable by the master equation. The master equation for a degenerate three-level atom interacting with a squeezed vacuum reservoir is given by [26]

$$\begin{aligned} \frac{d\hat{\rho}}{dt} = & -i[\hat{H}, \hat{\rho}] + \frac{\kappa}{2}(N_0 + 1) \left[ 2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a} \right] + \frac{\kappa}{2}N_0 \left[ 2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{a}\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^{\dagger} \right] \\ & + \frac{\kappa}{2}M_0 \left[ 2\hat{a}\hat{\rho}\hat{a} - \hat{a}^2\hat{\rho} - \hat{\rho}\hat{a}^2 + 2\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2} \right], \end{aligned} \quad (2.9)$$

where

$$N_0 = \sinh^2(r) \quad (2.10)$$

$$M_0 = \sinh(r) \cosh(r) \quad (2.11)$$

with  $r$  being squeezed parameter.

By substituting Eq. (2.8) into Eq. (2.9), the master equation for coherently driven degenerate three level atom with parametric amplifier coupled to squeezed vacuum reservoir is given by

$$\begin{aligned}
\frac{d\hat{\rho}}{dt} = & \frac{\Omega}{2} [\hat{\sigma}_c^{\dagger k} \hat{\rho} - \hat{\rho} \hat{\sigma}_c^{\dagger k} + \hat{\rho} \hat{\sigma}_c^k - \hat{\sigma}_c^k \hat{\rho}] + \frac{\varepsilon}{2} [\hat{a}^2 \hat{\rho} - \hat{\rho} \hat{a}^2 + \hat{a}^{\dagger 2} \hat{\rho} - \hat{\rho} \hat{a}^{\dagger 2}] \\
& + g \left[ \hat{\sigma}_a^{\dagger k} \hat{a} \hat{\rho} - \hat{\rho} \hat{\sigma}_a^{\dagger k} \hat{a} + \hat{\sigma}_b^{\dagger k} \hat{a} \hat{\rho} - \hat{\rho} \hat{\sigma}_b^{\dagger k} \hat{a} - \hat{a}^{\dagger} \hat{\sigma}_a^k \hat{\rho} - \hat{a}^{\dagger} \hat{\sigma}_b^k \hat{\rho} + \hat{\rho} \hat{a}^{\dagger} \hat{\sigma}_a^k + \hat{\rho} \hat{a}^{\dagger} \hat{\sigma}_b^k \right] \\
& + \frac{\kappa}{2} (N_0 + 1) \left[ 2\hat{a} \hat{\rho} \hat{a}^{\dagger} - \hat{a}^{\dagger} \hat{a} \hat{\rho} - \hat{\rho} \hat{a}^{\dagger} \hat{a} \right] + \frac{\kappa}{2} N_0 \left[ 2\hat{a}^{\dagger} \hat{\rho} \hat{a} - \hat{a} \hat{a}^{\dagger} \hat{\rho} - \hat{\rho} \hat{a} \hat{a}^{\dagger} \right] \\
& + \frac{\kappa}{2} M_0 \left[ 2\hat{a} \hat{\rho} \hat{a} - \hat{a}^2 \hat{\rho} - \hat{\rho} \hat{a}^2 + 2\hat{a}^{\dagger} \hat{\rho} \hat{a}^{\dagger} - \hat{a}^{\dagger 2} \hat{\rho} - \hat{\rho} \hat{a}^{\dagger 2} \right].
\end{aligned} \tag{2.12}$$

## 2.2 Quantum Langevin Equation

We recall that the laser cavity is coupled to a squeezed vacuum reservoir via a single-port mirror. The quantum Langevin equation is helpful to obtain the correlation properties, while they are not helpful to study the analysis of the system. This equation tells the time evolution of the annihilation operator of the system.

The quantum Langevin equation for the cavity mode operator  $\hat{a}$  is expressible as [5, 10]

$$\frac{d\hat{a}}{dt} = -\frac{\kappa}{2} \hat{a} - i[\hat{a}, \hat{H}] + \hat{F}_a(t), \tag{2.13}$$

where  $\kappa$  is the cavity damping constant and  $\hat{F}_a(t)$  is noise operator associated with the squeezed vacuum reservoir.

This noise operator has the following correlation properties, when the cavity modes is coupled to a squeezed vacuum reservoir [18, 27]

$$\langle \hat{F}_a(t) \rangle = 0, \tag{2.14}$$

$$\langle \hat{F}_a^\dagger(t') F_a(t) \rangle = \langle \hat{F}_a^\dagger(t) F_a(t') \rangle = \kappa N_0 \delta(t - t'), \quad (2.15)$$

$$\langle F_a(t) \hat{F}_a^\dagger(t') \rangle = \langle F_a(t') \hat{F}_a^\dagger(t) \rangle = \kappa(N_0 + 1) \delta(t - t'), \quad (2.16)$$

$$\langle F_a(t) \hat{F}_a(t') \rangle = \langle \hat{F}_a(t') F_a(t) \rangle = -\kappa M_0. \quad (2.17)$$

Then, in view of Eq. (2.8), the quantum Langevin equation for cavity mode operator  $\hat{a}$  turn out to be

$$\frac{d\hat{a}}{dt} = -\frac{\kappa}{2}\hat{a} - g(\hat{\sigma}_a^k + \hat{\sigma}_b^k) - \varepsilon\hat{a}^\dagger + \hat{F}_a(t). \quad (2.18)$$

## 2.3 Equations of Evolution for Atomic and Cavity Mode Operators

Here we seek to derive the equations of evolution of the expectation values of the atomic operators by applying the master equation and the large-time approximation scheme. Moreover, we find the steady-state solutions of the equations of evolution of the atomic operators. To this end, employing the relation

$$\frac{d}{dt}\langle \hat{A} \rangle = Tr\left(\frac{d\hat{\rho}}{dt}\hat{A}\right), \quad (2.19)$$

along with the master equation given by Eq.(2.12), one can readily establish that

$$\frac{d}{dt}\langle \hat{\sigma}_a^k \rangle = \frac{\Omega}{2}\langle \hat{\sigma}_b^{\dagger k} \rangle + g(\langle \hat{\eta}_b^k \hat{a} \rangle - \langle \hat{\eta}_a^k \hat{a} \rangle + \langle \hat{a}^\dagger \hat{\sigma}_c^k \rangle), \quad (2.20)$$

$$\frac{d}{dt}\langle \hat{\sigma}_b^k \rangle = -\frac{\Omega}{2}\langle \hat{\sigma}_a^{\dagger k} \rangle + g(\langle \hat{\eta}_c^k \hat{a} \rangle - \langle \hat{\eta}_b^k \hat{a} \rangle - \langle \hat{a}^\dagger \hat{\sigma}_c^k \rangle), \quad (2.21)$$

$$\frac{d}{dt}\langle\hat{\sigma}_c^k\rangle = \frac{\Omega}{2}(\langle\hat{\eta}_c^k\rangle - \langle\hat{\eta}_a^k\rangle) + g(\langle\hat{\sigma}_b^k\hat{a}\rangle - \langle\hat{\sigma}_a^k\hat{a}\rangle), \quad (2.22)$$

$$\frac{d}{dt}\langle\hat{\eta}_a^k\rangle = \frac{\Omega}{2}(\langle\hat{\sigma}_c^{\dagger k}\rangle + \langle\hat{\sigma}_c^k\rangle) + g(\langle\hat{\sigma}_a^{\dagger k}\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_a^k\rangle), \quad (2.23)$$

$$\frac{d}{dt}\langle\hat{\eta}_b^k\rangle = g(\langle\hat{\sigma}_b^{\dagger k}\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_b^k\rangle - \langle\hat{\sigma}_a^{\dagger k}\hat{a}\rangle - \langle\hat{a}^\dagger\hat{\sigma}_a^k\rangle), \quad (2.24)$$

$$\frac{d}{dt}\langle\hat{\eta}_c^k\rangle = -\frac{\Omega}{2}(\langle\hat{\sigma}_c^{\dagger k}\rangle + \langle\hat{\sigma}_c^k\rangle) - g(\langle\hat{\sigma}_b^{\dagger k}\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_b^k\rangle). \quad (2.25)$$

Where  $\langle\hat{\eta}_a^k\rangle$ ,  $\langle\hat{\eta}_b^k\rangle$  and  $\langle\hat{\eta}_c^k\rangle$  are the probability of the atom in the top, middle and bottom level of three level atom, and Eqs.(2.20)-(2.25) are nonlinear differential equations and hence it is not possible to find exact time-dependent solutions of these equations. We intend to overcome this problem by applying the large-time approximation scheme [7,17]. Employing this approximation scheme, we get from Eq.(2.18) the approximately valid relation

$$\hat{a} = -\frac{2g}{\kappa}(\hat{\sigma}_a^k + \hat{\sigma}_b^k) - \frac{2\varepsilon}{\kappa}\hat{a}^\dagger + \frac{2}{\kappa}\hat{F}_a(t). \quad (2.26)$$

Evidently, this turns out to be exact relation at steady state. Employing the adjoint of Eq.(2.26) into Eq.(2.18), one easily verify that

$$\frac{d\hat{a}}{dt} = -\left(\frac{\kappa^2 - 4\varepsilon^2}{2\kappa}\right)\hat{a} - g(\hat{\sigma}_a^k + \hat{\sigma}_b^k) + \frac{2g\varepsilon}{\kappa}(\hat{\sigma}_a^{\dagger k} + \hat{\sigma}_b^{\dagger k}) - \frac{2\varepsilon}{\kappa}\hat{F}_a^\dagger(t) + \hat{F}_a(t). \quad (2.27)$$

The solution of this equation is given by

$$\hat{a} = -\frac{2g\kappa}{\kappa^2 - 4\varepsilon^2}(\hat{\sigma}_a^k + \hat{\sigma}_b^k) + \frac{4g\varepsilon}{\kappa^2 - 4\varepsilon^2}(\hat{\sigma}_a^{\dagger k} + \hat{\sigma}_b^{\dagger k}) - \frac{4\varepsilon}{\kappa^2 - 4\varepsilon^2}\hat{F}_a^\dagger(t) + \frac{2\kappa}{\kappa^2 - 4\varepsilon^2}\hat{F}_a(t). \quad (2.28)$$

Now introducing Eq.(2.28) and its adjoint into Eqs.(2.20)-(2.25), the equations of evolution of the atomic operators take the form

$$\begin{aligned} \frac{d}{dt}\langle\hat{\sigma}_a^k\rangle &= \frac{\Omega}{2}\langle\hat{\sigma}_b^{\dagger k}\rangle + \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_b^{\dagger k}\rangle - \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_a^k\rangle - \frac{\gamma_c\kappa\varepsilon}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_a^{\dagger k}\rangle \\ &\quad - \frac{4\varepsilon g}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\eta}_b^k\hat{F}_a^\dagger(t)\rangle - \langle\hat{\eta}_a^k\hat{F}_a^\dagger(t)\rangle + \langle\hat{F}_a(t)\hat{\sigma}_c^k\rangle] \\ &\quad + \frac{2\kappa g}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\eta}_b^k\hat{F}_a(t)\rangle - \langle\hat{\eta}_a^k\hat{F}_a(t)\rangle + \langle\hat{F}_a^\dagger(t)\hat{\sigma}_c^k\rangle], \end{aligned} \quad (2.29)$$

$$\begin{aligned} \frac{d}{dt}\langle\hat{\sigma}_b^k\rangle &= -\frac{\Omega}{2}\langle\hat{\sigma}_a^{\dagger k}\rangle - \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_b^{\dagger k}\rangle + \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_a^k\rangle - \frac{\gamma_c\kappa^2}{2(\kappa^2 - 4\varepsilon^2)}\langle\hat{\sigma}_b^k\rangle \\ &\quad - \frac{4\varepsilon g}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\eta}_c^k\hat{F}_a^\dagger(t)\rangle - \langle\hat{\eta}_b^k\hat{F}_a^\dagger(t)\rangle - \langle\hat{F}_a(t)\hat{\sigma}_c^k\rangle] \\ &\quad + \frac{2\kappa g}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\eta}_c^k\hat{F}_a(t)\rangle - \langle\hat{\eta}_b^k\hat{F}_a(t)\rangle - \langle\hat{F}_a^\dagger(t)\hat{\sigma}_c^k\rangle], \end{aligned} \quad (2.30)$$

$$\begin{aligned} \frac{d}{dt}\langle\hat{\sigma}_c^k\rangle &= \frac{\Omega}{2}(\langle\hat{\eta}_c^k\rangle - \langle\hat{\eta}_a^k\rangle) - \frac{\gamma_c\kappa^2}{2(\kappa^2 - 4\varepsilon^2)}\langle\hat{\sigma}_c^k\rangle + \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}(\langle\hat{\eta}_c^k\rangle - \langle\hat{\eta}_b^k\rangle) \\ &\quad - \frac{4\varepsilon}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\sigma}_b^k\hat{F}_a^\dagger(t)\rangle - \langle\hat{\sigma}_a^k\hat{F}_a^\dagger(t)\rangle] + \frac{2\kappa}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\sigma}_b^k\hat{F}_a(t)\rangle - \langle\hat{\sigma}_a^k\hat{F}_a(t)\rangle], \end{aligned} \quad (2.31)$$

$$\begin{aligned} \frac{d}{dt}\langle\hat{\eta}_a^k\rangle &= \left(\frac{\Omega}{2} + \frac{2\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}\right)(\langle\hat{\sigma}_c^{\dagger k}\rangle + \langle\hat{\sigma}_c^k\rangle) - \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}\langle\hat{\eta}_a^k\rangle \\ &\quad - \frac{4\varepsilon}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\sigma}_a^{\dagger k}\hat{F}_a^\dagger(t)\rangle + \langle\hat{F}_a(t)\hat{\sigma}_a^k\rangle] + \frac{2\kappa}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\sigma}_a^{\dagger k}\hat{F}_a(t)\rangle + \langle\hat{F}_a^\dagger(t)\hat{\sigma}_a^k\rangle], \end{aligned} \quad (2.32)$$

$$\begin{aligned} \frac{d}{dt}\langle\hat{\eta}_b^k\rangle &= \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}(\langle\hat{\eta}_a^k\rangle - \langle\hat{\eta}_b^k\rangle) - \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}(\langle\hat{\sigma}_c^{\dagger k}\rangle + \langle\hat{\sigma}_c^k\rangle) \\ &\quad - \frac{4\varepsilon}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\sigma}_b^{\dagger k}\hat{F}_a^\dagger(t)\rangle + \langle\hat{F}_a(t)\hat{\sigma}_b^k\rangle - \langle\hat{\sigma}_a^{\dagger k}\hat{F}_a^\dagger(t)\rangle - \langle\hat{F}_a(t)\hat{\sigma}_a^k\rangle] \\ &\quad + \frac{2\kappa}{\kappa^2 - 4\varepsilon^2}[\langle\hat{\sigma}_b^{\dagger k}(t)\hat{F}_a(t)\rangle + \hat{F}_a^\dagger(t)\langle\hat{\sigma}_b^k(t)\rangle - \langle\hat{\sigma}_a^{\dagger k}(t)\hat{F}_a(t)\rangle - \hat{F}_a^\dagger(t)\langle\hat{\sigma}_a^k(t)\rangle], \end{aligned} \quad (2.33)$$

$$\begin{aligned} \frac{d}{dt} \langle \hat{\eta}_c^k \rangle &= -\frac{\Omega}{2} (\langle \hat{\sigma}_c^{\dagger k} \rangle + \langle \hat{\sigma}_c^k \rangle) + \frac{\gamma_c \kappa^2}{\kappa^2 - 4\varepsilon^2} \langle \hat{\eta}_b^k \rangle + \frac{4\varepsilon}{\kappa^2 - 4\varepsilon^2} [\langle \hat{\sigma}_b^{\dagger k} \hat{F}_a^\dagger(t) \rangle + \langle \hat{F}_a(t) \hat{\sigma}_b^k \rangle] \\ &\quad - \frac{2\kappa}{\kappa^2 - 4\varepsilon^2} [\langle \hat{\sigma}_b^{\dagger k} \hat{F}_a(t) \rangle + \langle \hat{F}_a^\dagger(t) \hat{\sigma}_b^k \rangle], \end{aligned} \quad (2.34)$$

where  $\gamma_c = \frac{4g^2}{k}$  is the stimulated emission decay constant.

Ignoring the noncommutativity of the atomic and noise operators as well as neglecting the correlation between  $\hat{F}_a(t)$  and  $\hat{\sigma}_a^k(t')$ , assumed to be considerably small [6], one can write the approximately valid relations

$$\langle \hat{\sigma}_a^k(t) \hat{F}_a(t) \rangle = \langle \hat{\sigma}_a^k(t) \rangle \langle \hat{F}_a(t) \rangle = 0, \quad (2.35)$$

$$\langle \hat{\eta}_a^k(t) \hat{F}_a(t) \rangle = 0, \quad (2.36)$$

$$\langle \hat{F}_a(t) \hat{\sigma}_c^k(t) \rangle = 0, \quad (2.37)$$

$$\langle \hat{\eta}_b^k(t) \hat{F}_a(t) \rangle = 0, \quad (2.38)$$

$$\langle \hat{\eta}_c^k(t) \hat{F}_a(t) \rangle = 0, \quad (2.39)$$

$$\langle \hat{F}_a^\dagger(t) \hat{\sigma}_a^k(t) \rangle = 0, \quad (2.40)$$

$$\langle \hat{F}_a^\dagger(t) \hat{\sigma}_c^k(t) \rangle = 0, \quad (2.41)$$

$$\langle \hat{F}_a^\dagger(t) \hat{\sigma}_b^k(t) \rangle = 0, \quad (2.42)$$

$$\langle \hat{\sigma}_b^k(t) \hat{F}_a^\dagger(t) \rangle = 0, \quad (2.43)$$

$$\langle \hat{\sigma}_b^k(t) \hat{F}_a(t) \rangle = 0, \quad (2.44)$$

$$\langle \hat{\sigma}_b^{\dagger k}(t) \hat{F}_a(t) \rangle = 0. \quad (2.45)$$

With the aid of Eqs. (2.35)-(2.45) we rewrite Eqs. (2.29)-(2.34) as

$$\frac{d}{dt}\langle\hat{\sigma}_a^k\rangle = \frac{1}{2}\left(\Omega + \frac{2\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}\right)\langle\hat{\sigma}_b^{\dagger k}\rangle - \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_a^k\rangle - \frac{\gamma_c\kappa\varepsilon}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_a^{\dagger k}\rangle, \quad (2.46)$$

$$\frac{d}{dt}\langle\hat{\sigma}_b^k\rangle = -\frac{\Omega}{2}\langle\hat{\sigma}_a^{\dagger k}\rangle - \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_b^{\dagger k}\rangle + \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}\langle\hat{\sigma}_a^k\rangle - \frac{\gamma_c\kappa^2}{2(\kappa^2 - 4\varepsilon^2)}\langle\hat{\sigma}_b^k\rangle, \quad (2.47)$$

$$\frac{d}{dt}\langle\hat{\sigma}_c^k\rangle = \frac{\Omega}{2}(\langle\hat{\eta}_c^k\rangle - \langle\hat{\eta}_a^k\rangle) - \frac{\gamma_c\kappa^2}{2(\kappa^2 - 4\varepsilon^2)}\langle\hat{\sigma}_c^k\rangle + \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}(\langle\hat{\eta}_c^k\rangle - \langle\hat{\eta}_b^k\rangle), \quad (2.48)$$

$$\frac{d}{dt}\langle\hat{\eta}_a^k\rangle = \left(\frac{\Omega}{2} + \frac{2\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}\right)(\langle\hat{\sigma}_c^{\dagger k}\rangle + \langle\hat{\sigma}_c^k\rangle) - \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}\langle\hat{\eta}_a^k\rangle, \quad (2.49)$$

$$\frac{d}{dt}\langle\hat{\eta}_b^k\rangle = \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}(-\langle\hat{\eta}_b^k\rangle + \langle\hat{\eta}_a^k\rangle) - \frac{\gamma_c\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}(\langle\hat{\sigma}_c^{\dagger k}\rangle + \langle\hat{\sigma}_c^k\rangle), \quad (2.50)$$

$$\frac{d}{dt}\langle\hat{\eta}_c^k\rangle = -\frac{\Omega}{2}(\langle\hat{\sigma}_c^{\dagger k}\rangle + \langle\hat{\sigma}_c^k\rangle) + \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}\langle\hat{\eta}_b^k\rangle. \quad (2.51)$$

We next sum Eqs. (2.46) - (2.51) over the  $N$  three-level atoms, so that we replace the term exist in above equation by

$$\frac{d}{dt}\langle\hat{m}_a\rangle = -\nu\langle\hat{m}_a\rangle - \frac{1}{2}\Gamma_-\langle\hat{m}_b^\dagger\rangle, \quad (2.52)$$

$$\frac{d}{dt}\langle\hat{m}_b\rangle = -\frac{1}{2}\nu\langle\hat{m}_b\rangle - \frac{1}{2}\Gamma_+\langle\hat{m}_a^\dagger\rangle, \quad (2.53)$$

$$\frac{d}{dt}\langle\hat{m}_c\rangle = \frac{\Omega}{2}(\langle\hat{N}_c\rangle - \langle\hat{N}_a\rangle) - \frac{1}{2}\nu\langle\hat{m}_c\rangle + \Gamma(\langle\hat{N}_c\rangle - \langle\hat{N}_b\rangle), \quad (2.54)$$

$$\frac{d}{dt}\langle\hat{N}_a\rangle = \frac{1}{2}\Gamma_+(\langle\hat{m}_c^\dagger\rangle + \langle\hat{m}_c\rangle) - \nu\langle\hat{N}_a\rangle, \quad (2.55)$$

$$\frac{d}{dt}\langle\hat{N}_b\rangle = \nu(\langle\hat{N}_a\rangle - \langle\hat{N}_b\rangle) - \Gamma(\langle\hat{m}_c^\dagger\rangle + \langle\hat{m}_c\rangle), \quad (2.56)$$

$$\frac{d}{dt}\langle\hat{N}_c\rangle = -\frac{\Omega}{2}(\langle\hat{m}_c^\dagger\rangle + \langle\hat{m}_c\rangle) + \nu\langle\hat{N}_b\rangle, \quad (2.57)$$

where

$$\nu = \frac{\gamma_c\kappa^2}{\kappa^2 - 4\varepsilon^2}, \quad (2.58)$$

$$\Gamma = \frac{\gamma_c\kappa\varepsilon}{\kappa^2 - 4\varepsilon^2}, \quad (2.59)$$

$$\Gamma_\pm = \frac{2\gamma_c\kappa\varepsilon}{\kappa^2 - 4\varepsilon^2} \pm \Omega. \quad (2.60)$$

For  $N$  number of atoms, we see that

$$\hat{m}_j = \sum_{k=1}^N \hat{\sigma}_j^k, \quad (2.61)$$

$$\hat{N}_j = \sum_{k=1}^N \hat{\eta}_j^k, \quad (2.62)$$

where  $j = a, b, c$ . Hence, the operators  $\hat{N}_a$ ,  $\hat{N}_b$ , and  $\hat{N}_c$  represent the number of atoms in the top, middle, and bottom levels, respectively.

Furthermore, using the definition  $\hat{\sigma}_a^k = |b\rangle_{kk}\langle a|$ , we have

$$\hat{m}_a = N|b\rangle_{kk}\langle a|. \quad (2.63)$$

Following the same procedure, one can also easily establish that

$$\hat{m}_b = N|c\rangle_{kk}\langle b|, \quad (2.64)$$

$$\hat{m}_c = N|c\rangle_{kk}\langle a|, \quad (2.65)$$

$$\hat{N}_a = N|a\rangle_{kk}\langle a|, \quad (2.66)$$

$$\hat{N}_b = N|b\rangle_{kk}\langle b|, \quad (2.67)$$

$$\hat{N}_c = N|c\rangle_{kk}\langle c|. \quad (2.68)$$

In addition, using the definition

$$\hat{m} = \hat{m}_a + \hat{m}_b \quad (2.69)$$

and in view of Eqs.(2.64)-(2.68), it can be easily verified that

$$\hat{m}^\dagger \hat{m} = N(\hat{N}_a + \hat{N}_b), \quad (2.70)$$

$$\hat{m} \hat{m}^\dagger = N(\hat{N}_b + \hat{N}_c), \quad (2.71)$$

$$\hat{m}^2 = N\hat{m}_c. \quad (2.72)$$

Employing the completeness relation

$$\hat{\eta}_a^k + \hat{\eta}_b^k + \hat{\eta}_c^k = \hat{I}, \quad (2.73)$$

we easily arrive at

$$\langle \hat{N}_a \rangle + \langle \hat{N}_b \rangle + \langle \hat{N}_c \rangle = N. \quad (2.74)$$

The steady state solution of Eqs.(2.54)-(2.57) are given by

$$\begin{aligned} \frac{\Omega}{2}(\langle \hat{N}_c \rangle - \langle \hat{N}_a \rangle) - \frac{1}{2}\nu\langle \hat{m}_c \rangle + \Gamma(\langle \hat{N}_c \rangle - \langle \hat{N}_b \rangle) &= 0 \\ \frac{1}{2}\Gamma_+(\langle \hat{m}_c^\dagger \rangle + \langle \hat{m}_c \rangle) - \nu\langle \hat{N}_a \rangle &= 0 \\ \nu(\langle \hat{N}_a \rangle - \langle \hat{N}_b \rangle) - \Gamma(\langle \hat{m}_c^\dagger \rangle + \langle \hat{m}_c \rangle) &= 0 \\ -\frac{\Omega}{2}(\langle \hat{m}_c^\dagger \rangle + \langle \hat{m}_c \rangle) + \nu\langle \hat{N}_b \rangle &= 0. \end{aligned} \quad (2.75)$$

There are five equations, which are obtained from the linear combination of only atomic operators. In view of Eq.(2.75), we see that

$$\langle \hat{m}_c \rangle = \frac{\Omega}{\nu}(\langle \hat{N}_c \rangle - \langle \hat{N}_a \rangle) + \frac{2\Gamma}{\nu}(\langle \hat{N}_c \rangle - \langle \hat{N}_b \rangle), \quad (2.76)$$

$$\langle \hat{N}_a \rangle = \frac{\Gamma_+}{2\nu}(\langle \hat{m}_c^\dagger \rangle + \langle \hat{m}_c \rangle), \quad (2.77)$$

$$\langle \hat{N}_b \rangle = \langle \hat{N}_a \rangle - \frac{\Gamma}{\nu}(\langle \hat{m}_c^\dagger \rangle + \langle \hat{m}_c \rangle), \quad (2.78)$$

$$\langle \hat{N}_b \rangle = \frac{\Omega}{2\nu}(\langle \hat{m}_c^\dagger \rangle + \langle \hat{m}_c \rangle). \quad (2.79)$$

We note from Eq.(2.76) that  $\langle \hat{m}_c \rangle = \langle \hat{m}_c^\dagger \rangle$ . We can therefore write

$\langle \hat{m}_c \rangle + \langle \hat{m}_c^\dagger \rangle = 2\langle \hat{m}_c \rangle$ . On the basis of this result the atomic operators can be expressed

interms of  $\langle \hat{m}_c \rangle$  as

$$\langle \hat{N}_a \rangle = \frac{\Gamma_+}{\nu} \langle \hat{m}_c \rangle, \quad (2.80)$$

$$\langle \hat{N}_b \rangle = \frac{\Gamma_+ - 2\Gamma}{\nu} \langle \hat{m}_c \rangle. \quad (2.81)$$

Substituting Eq.(2.74),(2.80) and (2.81) into Eq.(2.76) gives

$$\langle \hat{m}_c \rangle = \frac{\nu(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} N, \quad (2.82)$$

by putting Eq.(2.82),into Eq.(2.80),(2.81), and (2.74) we obtain

$$\langle \hat{N}_a \rangle = \frac{(\Omega + 2\Gamma)\Gamma_+}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} N, \quad (2.83)$$

$$\langle \hat{N}_b \rangle = \frac{\Omega(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} N, \quad (2.84)$$

$$\langle \hat{N}_c \rangle = N - \frac{(\Omega + \Gamma_+)(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} N. \quad (2.85)$$

On the other hand, when the cavity modes are interacting with all  $N$  three-level atoms available in the cavity, we can rewrite Eq.(2.27), as [18]

$$\frac{d\hat{a}}{dt} = -\left(\frac{\kappa^2 - 4\varepsilon^2}{2\kappa}\right)\hat{a} + \lambda_1(\hat{m}_a + \hat{m}_b) + \lambda_2(\hat{m}_a^\dagger + \hat{m}_b^\dagger) + \alpha\hat{F}_a(t) + \beta\hat{F}_a^\dagger(t) \quad (2.86)$$

in which  $\lambda_1$ ,  $\lambda_2$ ,  $\alpha$ , and  $\beta$  are constants whose values remain to be fixed. With aid of

Eq.(2.69) we can write Eq.(2.86) as

$$\frac{d\hat{a}}{dt} = -\left(\frac{\kappa^2 - 4\varepsilon^2}{2\kappa}\right)\hat{a} + \lambda_1\hat{m} + \lambda_2\hat{m}^\dagger + \alpha\hat{F}_a(t) + \beta\hat{F}_a^\dagger(t). \quad (2.87)$$

Moreover, employing Eq.(2.28), the commutation relations of the cavity mode operator is

$$\begin{aligned} [\hat{a}, \hat{a}^\dagger] &= \frac{\gamma_c\kappa}{(\kappa^2 - 4\varepsilon^2)^2} [\kappa^2(\hat{\eta}_c^k - \hat{\eta}_a^k) + 4\varepsilon^2(\hat{\eta}_a^k - \hat{\eta}_c^k)] \\ &+ \frac{4\kappa^2}{(\kappa^2 - 4\varepsilon^2)^2} [\hat{F}_a(t), \hat{F}_a^\dagger(t)] - \frac{16\varepsilon^2}{(\kappa^2 - 4\varepsilon^2)^2} [\hat{F}_a(t), \hat{F}_a^\dagger(t)], \end{aligned} \quad (2.88)$$

and on summing over all atoms, we have

$$\begin{aligned} [\hat{a}, \hat{a}^\dagger] &= \frac{\gamma_c\kappa}{(\kappa^2 - 4\varepsilon^2)^2} [\kappa^2(\hat{N}_c - \hat{N}_a) + 4\varepsilon^2(\hat{N}_a - \hat{N}_c)] \\ &+ \frac{4N\kappa^2}{(\kappa^2 - 4\varepsilon^2)^2} [\hat{F}_a(t), \hat{F}_a^\dagger(t)] - \frac{16N\varepsilon^2}{(\kappa^2 - 4\varepsilon^2)^2} [\hat{F}_a(t), \hat{F}_a^\dagger(t)], \end{aligned} \quad (2.89)$$

where

$$[\hat{a}, \hat{a}^\dagger] = \sum_{k=1}^N [\hat{a}, \hat{a}^\dagger], \quad (2.90)$$

stands for the commutator of  $\hat{a}$  and  $\hat{a}^\dagger$  when light mode  $\hat{a}$  is interacting with all the  $N$  three-level atoms. On the other hand, using the steady-state solution of Eq.(2.87), one can easily verify that

$$\begin{aligned} [\hat{a}, \hat{a}^\dagger] &= \frac{4\kappa^2\lambda_1^2N}{(\kappa^2 - 4\varepsilon^2)^2} (\hat{N}_c - \hat{N}_a) + \frac{4\kappa^2\lambda_2^2N}{(\kappa^2 - 4\varepsilon^2)^2} (\hat{N}_a - \hat{N}_c) \\ &+ \frac{4\kappa^2\alpha^2}{(\kappa^2 - 4\varepsilon^2)^2} [\hat{F}_a(t), \hat{F}_a^\dagger(t)] - \frac{16\beta^2\varepsilon^2}{(\kappa^2 - 4\varepsilon^2)^2} [\hat{F}_a(t), \hat{F}_a^\dagger(t)]. \end{aligned} \quad (2.91)$$

Now once again, comparison of Eq.(2.89) and (2.91) immediately implies that

$$\lambda_1 = \pm \frac{g}{\sqrt{N}}, \quad (2.92)$$

$$\lambda_2 = \pm \frac{2\varepsilon g}{\kappa\sqrt{N}}, \quad (2.93)$$

$$\beta = \pm\sqrt{N}, \quad (2.94)$$

$$\alpha = \pm\sqrt{N}. \quad (2.95)$$

Substituting Eqs. (2.92) - (2.95) into Eq. (2.87) we obtain

$$\frac{d\hat{a}}{dt} = -\left(\frac{\kappa^2 - 4\varepsilon^2}{2\kappa}\right)\hat{a} + \frac{g}{\sqrt{N}}\hat{m} + \frac{2\varepsilon g}{\kappa\sqrt{N}}\hat{m}^\dagger + \sqrt{N}\hat{F}_a(t) + \sqrt{N}\hat{F}_a^\dagger(t). \quad (2.96)$$

As a result of using the large-time approximation approach, the adjoint solution of Eq.(2.53) to be

$$\langle\hat{m}_b^\dagger\rangle = -\frac{\Gamma_+}{\nu}\langle\hat{m}_a(t)\rangle, \quad (2.97)$$

Now, on substituting Eq.(2.97) into Eq.(2.52), it is easy to see that

$$\frac{d}{dt}\langle\hat{m}_a\rangle = -\frac{1}{2}\eta_0\langle\hat{m}_a(t)\rangle, \quad (2.98)$$

where  $\eta_0 = \frac{2\nu^2 - \Gamma_- \Gamma_+}{\nu}$ .

By applying similar procedure on Eq.(2.52), we easily arrive

$$\frac{d}{dt}\langle\hat{m}_b\rangle = -\frac{1}{4}\eta_0\langle\hat{m}_b(t)\rangle. \quad (2.99)$$

With the atoms considered to be initially in the bottom level, the solution of Eq. (2.98) is found to be

$$\langle\hat{m}_a(t)\rangle = 0. \quad (2.100)$$

Similarly the solution of Eq. (2.99) becomes

$$\langle \hat{m}_b(t) \rangle = 0. \quad (2.101)$$

Hence the cavity light to be initially in vacuum state the expectation value of the solution of Eq.(2.96) turns out to be

$$\langle \hat{a} \rangle = 0 \quad (2.102)$$

we assume that the operator  $\langle \hat{a} \rangle$  is a gaussian variable with zero mean.

# Chapter 3

## Photon Statistics

In this chapter, we will look at the statistical features of light produced by a degenerate three-level laser, in which coherent light is used to pump  $N$  degenerate three-level atoms in a closed cavity that is connected to a squeezed vacuum reservoir.

### 3.1 Mean Photon Number

To learn about the brightness of the generated light, it is necessary to study the mean photon number of the cavity radiation. We want to find the mean photon number for light modes  $\hat{a}$  over the entire frequency interval. To this end, using the relation.

$$\frac{d}{dt} \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle = \left\langle \frac{d\hat{a}^\dagger(t)}{dt} \hat{a}(t) \right\rangle + \left\langle \hat{a}^\dagger(t) \frac{d\hat{a}(t)}{dt} \right\rangle. \quad (3.1)$$

By using Eq. (2.98) and the adjoint of this equation, we get

$$\begin{aligned} \frac{d}{dt} \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle &= -\left(\frac{\kappa^2 - 4\varepsilon^2}{\kappa}\right) \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle + \frac{g}{\sqrt{N}} [\langle \hat{a}^\dagger(t) \hat{m}(t) \rangle + \langle \hat{m}^\dagger(t) \hat{a}(t) \rangle] \\ &+ \frac{2\varepsilon g}{\kappa\sqrt{N}} [\langle \hat{a}^\dagger(t) \hat{m}^\dagger(t) \rangle + \langle \hat{m}(t) \hat{a}(t) \rangle] \\ &+ \sqrt{N} [\langle \hat{F}_a^\dagger(t) \hat{a}(t) \rangle + \langle \hat{a}^\dagger(t) \hat{F}_a(t) \rangle + \langle \hat{a}^\dagger(t) \hat{F}_a^\dagger(t) \rangle + \langle \hat{F}_a(t) \hat{a}(t) \rangle]. \end{aligned} \quad (3.2)$$

Next we seek to evaluate  $\langle \hat{a}^\dagger(t)\hat{m}(t) \rangle$ . Applying the large-time approximation, one gets from Eq. (2.98) the approximately valid relation

$$\hat{a}(t) = \frac{2\kappa g}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{m} + \frac{4\varepsilon g}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{m}^\dagger + \frac{2\kappa\sqrt{N}}{\kappa^2 - 4\varepsilon^2} \left[ \hat{F}_a(t) + \hat{F}_a^\dagger(t) \right]. \quad (3.3)$$

Multiplying the adjoint of Eq. (3.3) on the right by  $\hat{m}(t)$  and taking the expectation value of the resulting expression, we get

$$\langle \hat{a}^\dagger(t)\hat{m}(t) \rangle = \frac{2\kappa g}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{m}^\dagger\hat{m} + \frac{4\varepsilon g}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)}\hat{m}^2 + \frac{2\kappa\sqrt{N}}{\kappa^2 - 4\varepsilon^2} \left[ \hat{F}_a^\dagger(t)\hat{m} + \hat{F}_a(t)\hat{m} \right]. \quad (3.4)$$

Now adding Eq. (2.98) and Eq. (2.99), we readily get

$$\frac{d}{dt}\langle \hat{m}(t) \rangle = -\frac{1}{4}\eta_0\langle \hat{m}(t) \rangle - \frac{1}{4}\eta_0\langle \hat{m}_a(t) \rangle. \quad (3.5)$$

A formal solution of Eq. (3.5) can be written as

$$\hat{m}(t) = \hat{m}(0)e^{-\frac{\eta_0}{4}t} + \int_0^t e^{-\frac{\eta_0}{4}(t-t')} \left[ -\frac{\eta_0}{4}\hat{m}_a(t') + \hat{F}_m(t') \right] dt', \quad (3.6)$$

where  $\hat{F}_m(t')$  is atomic noise operator.

Multiplying Eq. (3.6) on the left by  $\hat{F}_a^\dagger(t)$  and taking the expectation value of the resulting expression, we have

$$\begin{aligned} \langle \hat{F}_a^\dagger(t)\hat{m}(t) \rangle &= \langle \hat{F}_a^\dagger(t)\hat{m}(0) \rangle e^{-\frac{\eta_0}{4}t} + \int_0^t e^{-\frac{\eta_0}{4}(t-t')} \left[ -\frac{\eta_0}{4}\langle \hat{F}_a^\dagger(t)\hat{m}_a(t') \rangle \right. \\ &\quad \left. + \langle \hat{F}_a^\dagger(t)\hat{F}_m(t') \rangle \right] dt'. \end{aligned} \quad (3.7)$$

Taking into account the fact that a noise operator  $\hat{F}$  at a certain time should not affect the atomic variable at earlier time and assuming that the cavity mode and atomic mode operators are not correlated, we get

$$\langle \hat{F}_a^\dagger(t)\hat{m}(t) \rangle = 0. \quad (3.8)$$

In account of this result, Eq. (3.4) takes the form

$$\langle \hat{a}^\dagger(t) \hat{m}(t) \rangle = \frac{2\kappa g N}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)} [\langle \hat{N}_a(t) \rangle + \langle \hat{N}_b(t) \rangle] + \frac{4\varepsilon g N}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)} \langle \hat{m}_c(t) \rangle. \quad (3.9)$$

Following a similar procedure, one can establish that

$$\langle \hat{m}^\dagger(t) \hat{a}(t) \rangle = \frac{2\kappa g N}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)} [\langle \hat{N}_a(t) \rangle + \langle \hat{N}_b(t) \rangle] + \frac{4\varepsilon g N}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)} \langle \hat{m}_c^\dagger(t) \rangle, \quad (3.10)$$

$$\langle \hat{m}(t) \hat{a}(t) \rangle = \frac{4\varepsilon g N}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)} [\langle \hat{N}_b(t) \rangle + \langle \hat{N}_c(t) \rangle] + \frac{2\kappa g N}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)} \langle \hat{m}_c(t) \rangle, \quad (3.11)$$

$$\langle \hat{m}^\dagger(t) \hat{a}^\dagger(t) \rangle = \frac{4\varepsilon g N}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)} [\langle \hat{N}_b(t) \rangle + \langle \hat{N}_c(t) \rangle] + \frac{2\kappa g N}{\sqrt{N}(\kappa^2 - 4\varepsilon^2)} \langle \hat{m}_c^\dagger(t) \rangle. \quad (3.12)$$

We next seek to evaluate  $\langle \hat{F}_a^\dagger(t) \hat{a}(t) \rangle$ . To this end, a formal solution of Eq. (2.98) can be written as

$$\hat{a}(t) = \hat{a}(0) e^{-\frac{\eta}{2}t} + \int_0^t e^{-\frac{\eta}{2}(t-t')} \left[ \frac{g}{\sqrt{N}} \hat{m}(t') + \frac{2g\varepsilon}{\kappa\sqrt{N}} \hat{m}^\dagger(t') + \sqrt{N} [\hat{F}_a(t') + \hat{F}_a^\dagger(t')] \right] dt'. \quad (3.13)$$

where  $\eta = \frac{\kappa^2 - 4\varepsilon^2}{\kappa}$ .

Multiplying Eq. (3.13) on the left by  $\hat{F}_a^\dagger(t)$  and taking the expectation value of the resulting expression, we get

$$\begin{aligned} \langle \hat{F}_a^\dagger(t) \hat{a}(t) \rangle &= \langle \hat{F}_a^\dagger(t) \hat{a}(0) \rangle e^{-\frac{\eta}{2}t} + \frac{g}{\sqrt{N}} \int_0^t e^{-\frac{\eta}{2}(t-t')} \langle \hat{F}_a^\dagger(t) \hat{m}(t') \rangle + \frac{2g\varepsilon}{\kappa\sqrt{N}} \int_0^t e^{-\frac{\eta}{2}(t-t')} \langle \hat{F}_a^\dagger(t) \hat{m}^\dagger(t') \rangle \\ &\quad + \sqrt{N} \int_0^t e^{-\frac{\eta}{2}(t-t')} (\langle \hat{F}_a^\dagger(t) \hat{F}_a(t') \rangle + \langle \hat{F}_a^\dagger(t) \hat{F}_a^\dagger(t') \rangle) dt'. \end{aligned} \quad (3.14)$$

In view of Eq. (3.8) and its adjoint along with the fact that a noise operator  $\hat{F}$  at a certain time should not affect the atomic variable at earlier time, Eq. (3.14) becomes

$$\langle \hat{F}_a^\dagger(t) \hat{a}(t) \rangle = \sqrt{N} \int_0^t e^{-\frac{\eta}{2}(t-t')} (\langle \hat{F}_a^\dagger(t) \hat{F}_a(t') \rangle + \langle \hat{F}_a^\dagger(t) \hat{F}_a^\dagger(t') \rangle) dt'. \quad (3.15)$$

Similarly

$$\langle \hat{F}_a(t) \hat{a}(t) \rangle = \sqrt{N} \int_0^t e^{-\frac{\eta}{2}(t-t')} (\langle \hat{F}_a(t) \hat{F}_a(t') \rangle + \langle \hat{F}_a(t) \hat{F}_a^\dagger(t') \rangle) dt', \quad (3.16)$$

$$\langle \hat{a}^\dagger(t) \hat{F}_a(t) \rangle = \sqrt{N} \int_0^t e^{-\frac{\eta}{2}(t-t')} (\langle \hat{F}_a(t) \hat{F}_a(t') \rangle + \langle \hat{F}_a^\dagger(t) \hat{F}_a(t') \rangle) dt', \quad (3.17)$$

$$\langle \hat{a}^\dagger(t) \hat{F}_a^\dagger(t) \rangle = \sqrt{N} \int_0^t e^{-\frac{\eta}{2}(t-t')} (\langle \hat{F}_a^\dagger(t) \hat{F}_a^\dagger(t') \rangle + \langle \hat{F}_a(t) \hat{F}_a^\dagger(t') \rangle) dt', \quad (3.18)$$

$$\langle \hat{a}^\dagger(t) \hat{F}_a(t) \rangle + \langle \hat{F}_a^\dagger(t) \hat{a}(t) \rangle + \langle \hat{F}_a(t) \hat{a}(t) \rangle + \langle \hat{a}^\dagger(t) \hat{F}_a^\dagger(t) \rangle = 2\sqrt{N}(N_0\kappa + \kappa(N_0 + 1) - 2\kappa M_0). \quad (3.19)$$

Now substituting Eqs.(3.9),(3.10),(3.11),(3.12), and(3.19) into Eq. (3.2) we easily arrive at

$$\begin{aligned} \frac{d}{dt} \langle \hat{a}^\dagger \hat{a} \rangle &= -\left(\frac{\kappa^2 - 4\varepsilon^2}{\kappa}\right) \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle + \frac{4\kappa g^2}{(\kappa^2 - 4\varepsilon^2)} [\langle \hat{N}_a(t) \rangle + \langle \hat{N}_b(t) \rangle] \\ &+ \frac{16\varepsilon g^2}{(\kappa^2 - 4\varepsilon^2)} \langle \hat{m}_c(t) \rangle + \frac{16\varepsilon^2 g^2}{\kappa(\kappa^2 - 4\varepsilon^2)} [\langle \hat{N}_b(t) \rangle + \langle \hat{N}_c(t) \rangle] \\ &+ 4N\kappa(N_0 - M_0 + \frac{1}{2}). \end{aligned} \quad (3.20)$$

The steady-state solution of this equation is expressible as

$$\begin{aligned} \langle \hat{a}^\dagger \hat{a} \rangle &= \frac{\gamma_c}{\kappa} \left[ \left(\frac{\nu}{\gamma_c}\right)^2 [\langle \hat{N}_a(t) \rangle + \langle \hat{N}_b(t) \rangle + \frac{4\varepsilon}{\kappa} \langle \hat{m}_c(t) \rangle] + \left(\frac{2\Gamma}{\gamma_c}\right)^2 [\langle \hat{N}_b(t) \rangle + \langle \hat{N}_c(t) \rangle] \right] \\ &+ \frac{4N\kappa^2}{(\kappa^2 - 4\varepsilon^2)} (N_0 - M_0 + \frac{1}{2}). \end{aligned} \quad (3.21)$$

Following a similar procedure, one can establish that

$$\begin{aligned} \langle \hat{a} \hat{a}^\dagger \rangle &= \frac{\gamma_c}{\kappa} \left[ \left(\frac{\nu}{\gamma_c}\right)^2 [\langle \hat{N}_b(t) \rangle + \langle \hat{N}_c(t) \rangle + \frac{4\varepsilon}{\kappa} \langle \hat{m}_c(t) \rangle] + \left(\frac{2\Gamma}{\gamma_c}\right)^2 [\langle \hat{N}_a(t) \rangle + \langle \hat{N}_b(t) \rangle] \right] \\ &+ \frac{4N\kappa^2}{(\kappa^2 - 4\varepsilon^2)} (N_0 - M_0 + \frac{1}{2}), \end{aligned} \quad (3.22)$$

$$\langle \hat{a}^2 \rangle = \frac{\gamma_c}{\kappa} \left[ \left[ \left( \frac{\nu}{\gamma_c} \right)^2 + \left( \frac{2\Gamma}{\gamma_c} \right)^2 \right] \langle \hat{m}_c(t) \rangle + \frac{2\kappa}{\varepsilon} \left( \frac{\Gamma}{\gamma_c} \right)^2 (N + \langle \hat{N}_b \rangle) \right] + \frac{4N\kappa^2}{(\kappa^2 - 4\varepsilon^2)} (N_0 - M_0 + \frac{1}{2}). \quad (3.23)$$

By putting Eqs.(2.82)-(2.85) into Eqs.(3.21)-(3.23) we readily obtain

$$\begin{aligned} \bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle &= \frac{\gamma_c}{\kappa} N \left[ \left( \frac{\nu}{\gamma_c} \right)^2 \left\{ \frac{(\Omega + \Gamma_+)(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right. \right. \\ &\quad \left. \left. + \frac{4\varepsilon}{\kappa} \left[ \frac{\nu(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right\} \right. \\ &\quad \left. + \left( \frac{2\Gamma}{\gamma_c} \right)^2 \left[ 1 - \frac{\Gamma_+(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right] + \frac{4N\kappa^2}{\kappa^2 - 4\varepsilon^2} (N_0 - M_0 + \frac{1}{2}). \end{aligned} \quad (3.24)$$

For the case where the squeezing parameter is not present ( $r = 0$ ) and coupled to vacuum reservoir, Eq.(3.24) can be written us

$$\begin{aligned} \bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle &= \frac{\gamma_c}{\kappa} N \left[ \left( \frac{\nu}{\gamma_c} \right)^2 \left\{ \frac{(\Omega + \Gamma_+)(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right. \right. \\ &\quad \left. \left. + \frac{4\varepsilon}{\kappa} \left[ \frac{\nu(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right\} \right. \\ &\quad \left. + \left( \frac{2\Gamma}{\gamma_c} \right)^2 \left[ 1 - \frac{\Gamma_+(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right] + \frac{2N\kappa^2}{\kappa^2 - 4\varepsilon^2}, \end{aligned} \quad (3.25)$$

Eq.(3.24) can also be written in the case  $\varepsilon = N_0 = M_0 = 0$

$$\langle \hat{a}^\dagger \hat{a} \rangle = \frac{\gamma_c}{\kappa} N \left[ \frac{2\Omega^2}{\gamma_c^2 + 3\Omega^2} \right] + 2N, \quad (3.26)$$

$$\begin{aligned}
\langle \hat{a}\hat{a}^\dagger \rangle &= \frac{\gamma_c}{\kappa} N \left[ \left( \frac{\nu}{\gamma_c} \right)^2 \left[ 1 - \frac{\Gamma_+(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right. \right. \\
&\quad \left. \left. + \frac{4\varepsilon}{\kappa} \left[ \frac{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right] \right. \\
&\quad \left. + \left( \frac{2\Gamma}{\gamma_c} \right)^2 \frac{(\Omega + \Gamma_+)(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] + \frac{4N\kappa^2}{\kappa^2 - 4\varepsilon^2} (N_0 - M_0 + \frac{1}{2}),
\end{aligned} \tag{3.27}$$

$$\begin{aligned}
\langle \hat{a}^2 \rangle &= \frac{\gamma_c}{\kappa} N \left[ \left[ \left( \frac{\nu}{\gamma_c} \right)^2 + \left( \frac{2\Gamma}{\gamma_c} \right)^2 \right] \frac{\nu(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right. \\
&\quad \left. + \frac{2\kappa}{\varepsilon} \left( \frac{\Gamma}{\gamma_c} \right)^2 \left\{ 1 + \frac{\Omega(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right\} \right] + \frac{4N\kappa^2}{\kappa^2 - 4\varepsilon^2} (N_0 - M_0 + \frac{1}{2}).
\end{aligned} \tag{3.28}$$

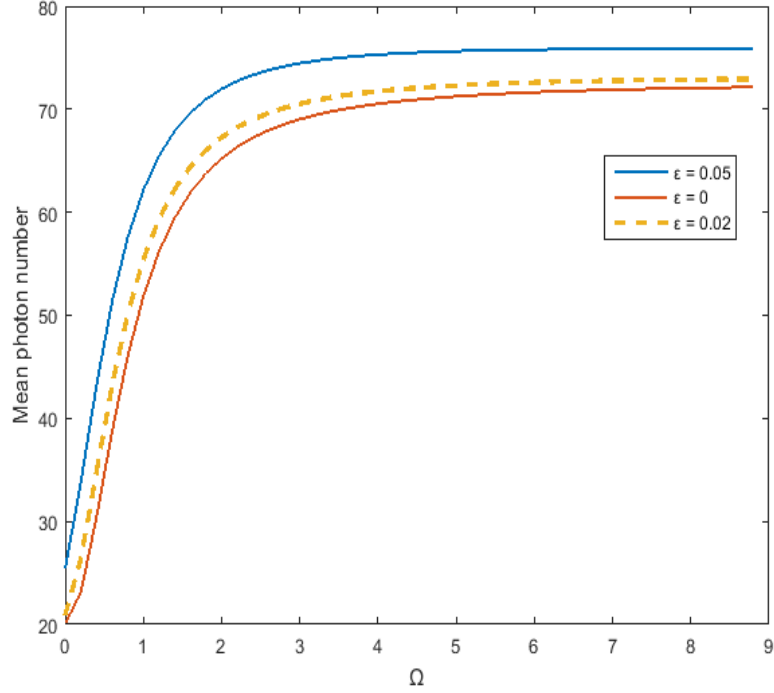


Figure 3.1: Plots of the steady-state mean photon number [Eq.(3.24)] versus  $\Omega$ , for  $\kappa=0.8$ ,  $\gamma_c=0.4$ ,  $N=50$ ,  $r=0.5$ , and for different values of  $\varepsilon$ .

We can see that the intensity of cavity light increases as the parametric amplifier increases Fig.(3.1). Hence, the presence of parametric amplifier enhances the brightness of the cavity light and leads to a significant increase in the mean photon number. In

addition, the maximum mean photon number  $\bar{n} = 20.32$  when  $\varepsilon=0$ ,  $\bar{n} = 20.92$  when  $\varepsilon = 0.02$ , and  $\bar{n} = 25.4$  when  $\varepsilon = 0.05$ . This all results occur when the three-level laser is operating at  $\Omega = 0$ . From the plots in Fig.(3.1), we observe that the mean photon number of the cavity light increases due to the coupling of the top and bottom levels of the three-level atom by the coherent light.

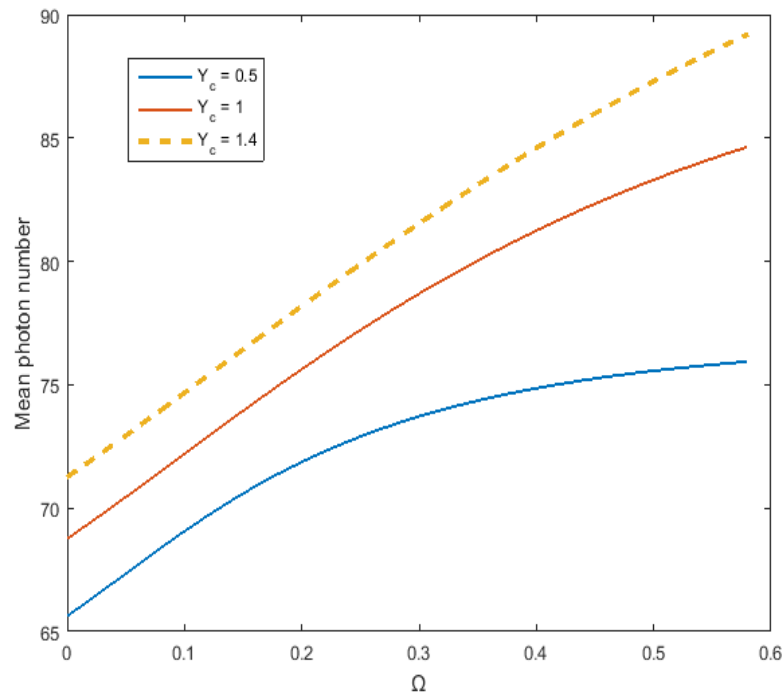


Figure 3.2: Plots of the steady-state mean photon number [Eq.(3.24)] versus  $\Omega$ , for  $\kappa=0.8$ ,  $\varepsilon=0.02$ ,  $N=50$ ,  $r=0.5$ , and for different values of  $\gamma_c$ .

From the plots in Fig.(3.2) one easily see that the mean photon number increases as the rate of photon emission ( $\gamma_c$ ) and the amplitude ( $\Omega$ ) of coherent light increases and vice versa.

From the plots in Fig.(3.3) one easily see that as the squeezing parameter, and the amplitude ( $\Omega$ ) of coherent light increases the mean photon number increases, and vice versa. The mean photon number obtained when the squeeze parameter ( $r = 0$ ) Eq.(3.25) is due to the atom interaction with cavity mode, and parametric interaction with cavity mode.

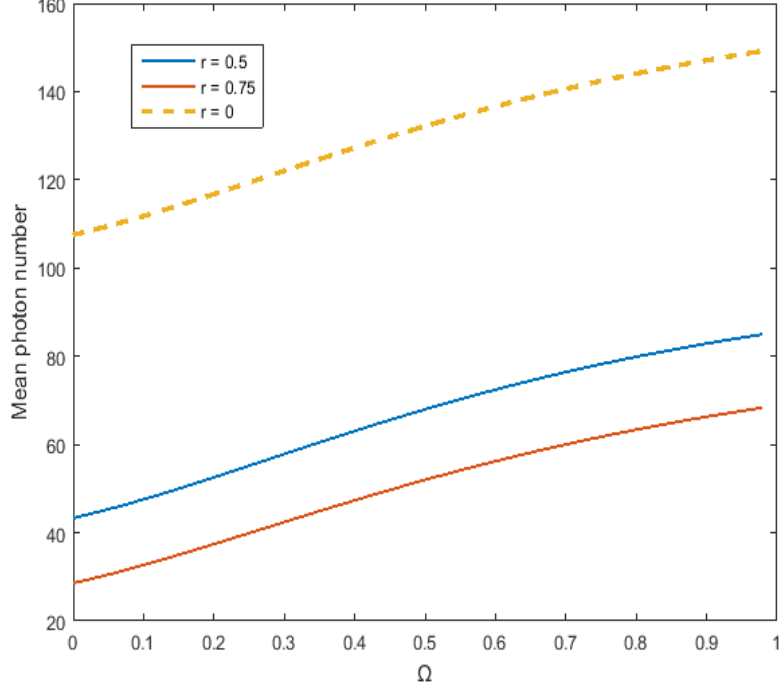


Figure 3.3: Plots of the steady-state mean photon number [Eq.(3.24)] versus  $\Omega$ , for  $\kappa=0.8$ ,  $\varepsilon=0.02$ ,  $N=50$ ,  $\gamma_c=0.4$ , and for different values of  $r$ .

## 3.2 Photon Number Variance

The variance of the photon number for the cavity light is expressible as

$$(\Delta n)^2 = \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2. \quad (3.29)$$

Using the fact that  $\hat{a}$  is a Gaussian variable with zero mean and decoupling the first term, the photon-number variance of the cavity mode can be put in the form [28]

$$(\Delta n)^2 = \langle \hat{a}^\dagger \hat{a} \rangle \langle \hat{a} \hat{a}^\dagger \rangle + \langle \hat{a}^{\dagger 2} \rangle \langle \hat{a}^2 \rangle. \quad (3.30)$$

By putting Eqs.(3.21)-(3.23) into Eq.(3.30)we obtain

$$\begin{aligned}
(\Delta n)^2 = & \left[ \frac{\gamma_c}{\kappa} \left[ \left( \frac{\nu}{\gamma_c} \right)^2 [\langle \hat{N}_a(t) \rangle + \langle \hat{N}_b(t) \rangle + \frac{4\varepsilon}{\kappa} \langle \hat{m}_c(t) \rangle] + \left( \frac{2\Gamma}{\gamma_c} \right)^2 [\langle \hat{N}_b(t) \rangle + \langle \hat{N}_c(t) \rangle] \right] \right. \\
& + \frac{4N\kappa^2}{\kappa^2 - 4\varepsilon^2} (N_0 - M_0 + \frac{1}{2}) \left[ \frac{\gamma_c}{\kappa} \left[ \left( \frac{\nu}{\gamma_c} \right)^2 [\langle \hat{N}_b(t) \rangle + \langle \hat{N}_c(t) \rangle + \frac{4\varepsilon}{\kappa} \langle \hat{m}_c(t) \rangle] \right. \right. \\
& + \left. \left. \left( \frac{2\Gamma}{\gamma_c} \right)^2 [\langle \hat{N}_a(t) \rangle + \langle \hat{N}_b(t) \rangle] \right] + \frac{4N\kappa^2}{\kappa^2 - 4\varepsilon^2} (N_0 - M_0 + \frac{1}{2}) \right] \\
& + \left[ \frac{\gamma_c}{\kappa} \left[ \left[ \left( \frac{\nu}{\gamma_c} \right)^2 + \left( \frac{2\Gamma}{\gamma_c} \right)^2 \right] \langle \hat{m}_c(t) \rangle + \frac{2\gamma_c}{\varepsilon} \left( \frac{\Gamma}{\gamma_c} \right)^2 (N + \langle \hat{N}_b \rangle) \right] \right. \\
& \left. + \frac{4N\kappa^2}{\kappa^2 - 4\varepsilon^2} (N_0 - M_0 + \frac{1}{2}) \right]^2. \tag{3.31}
\end{aligned}$$

Substituting Eqs.(2.82)-(2.85) into Eq.(3.31) we easily obtain

$$\begin{aligned}
(\Delta n)^2 = & \bar{n} \left[ \frac{\gamma_c}{\kappa} N \left[ \left( \frac{\nu}{\gamma_c} \right)^2 \left[ 1 - \frac{\Gamma_+(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right. \right. \right. \\
& + \left. \left. \frac{4\varepsilon}{\kappa} \left[ \frac{\nu(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right] \right. \\
& + \left. \left( \frac{2\Gamma}{\gamma_c} \right)^2 \frac{(\Omega + \Gamma_+)(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] + \frac{4N\kappa^2}{\kappa^2 - 4\varepsilon^2} (N_0 - M_0 + \frac{1}{2}) \left. \right] \\
& + \left[ \frac{\gamma_c}{\kappa} N \left[ \left[ \left( \frac{\nu}{\gamma_c} \right)^2 + \left( \frac{2\Gamma}{\gamma_c} \right)^2 \right] \frac{\nu(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right. \right. \\
& + \left. \left. \frac{2\kappa}{\varepsilon} \left( \frac{\Gamma}{\gamma_c} \right)^2 \left\{ 1 + \frac{\Omega(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right\} \right] \right. \\
& \left. + \frac{4N\kappa^2}{\kappa^2 - 4\varepsilon^2} (N_0 - M_0 + \frac{1}{2}) \right]^2. \tag{3.32}
\end{aligned}$$

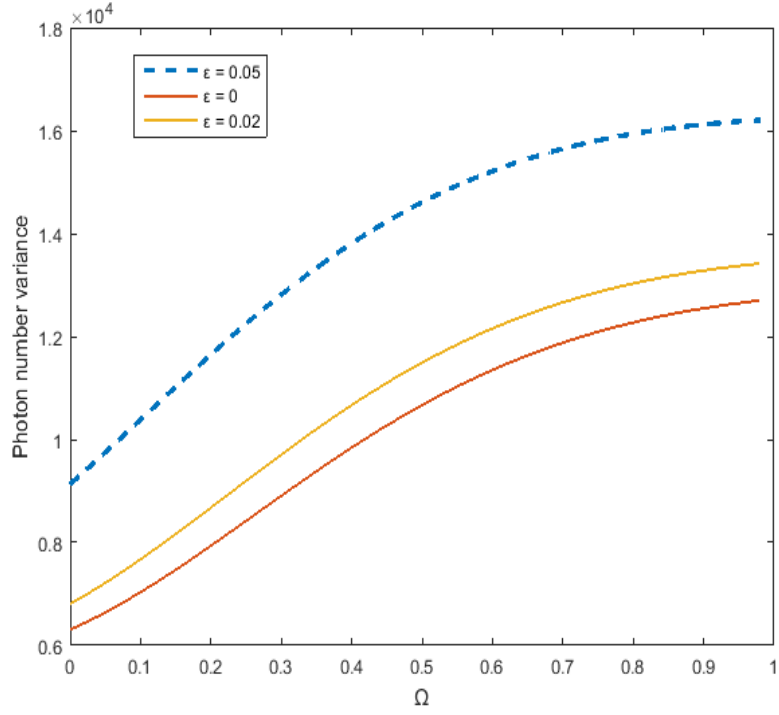


Figure 3.4: Plots of the photon number variance [Eq.(3.32)] versus  $\Omega$ , for  $\kappa=0.8$ ,  $r=0.5$ ,  $N=50$ ,  $\gamma_c=0.4$  and for different value of  $\varepsilon$ .

The photon number variance of cavity light increases as the parametric amplifier increases as shown in Fig.(3.4). In addition, the maximum photon number variance  $(\Delta n)^2 = 6.306 \times 10^4$  when  $\varepsilon=0.01$ ,  $(\Delta n)^2 = 9.135 \times 10^3$  when  $\varepsilon = 0.05$ , and  $(\Delta n)^2 = 6.807 \times 10^3$  when  $\varepsilon = 0.02$ . These all results occur when the three-level laser is operating at  $\Omega = 0$ .

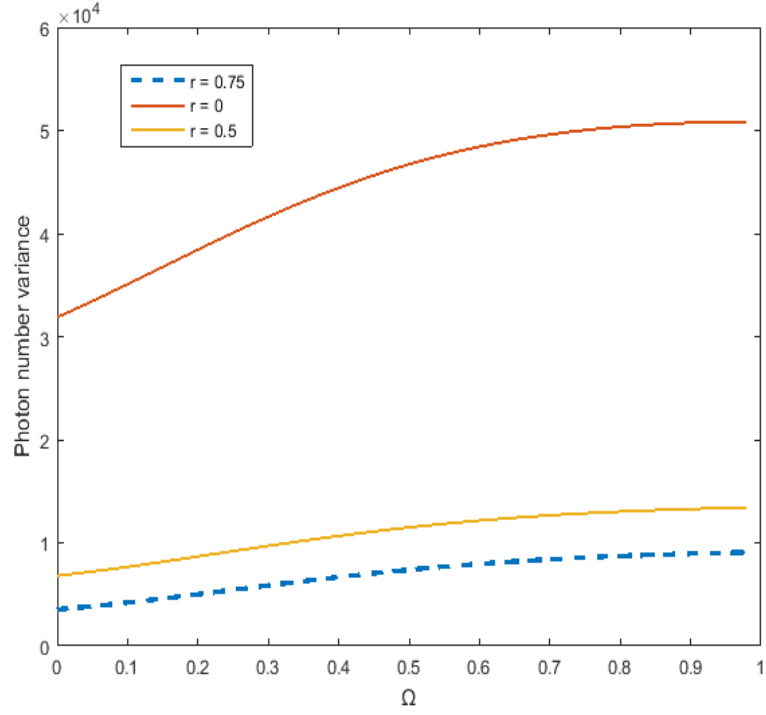


Figure 3.5: Plots of the steady-state photon number variance [Eq.(3.32)] versus  $\Omega$ , for  $\kappa=0.8$ ,  $\varepsilon=0.02$ ,  $N=50$ ,  $\gamma_c=0.4$  and for different value of  $r$ .

The photon number variance decreases as the squeezing parameter increases, as shown in Fig.(3.5). The maximum photon number variance  $(\Delta n)^2 = 3.185 \times 10^4$  when  $r=0$ ,  $(\Delta n)^2 = 6.269 \times 10^3$  when  $r=0.5$ , and  $(\Delta n)^2 = 3.54 \times 10^3$  when  $r=0.75$ , and all results increase for large value of driving coherent light.

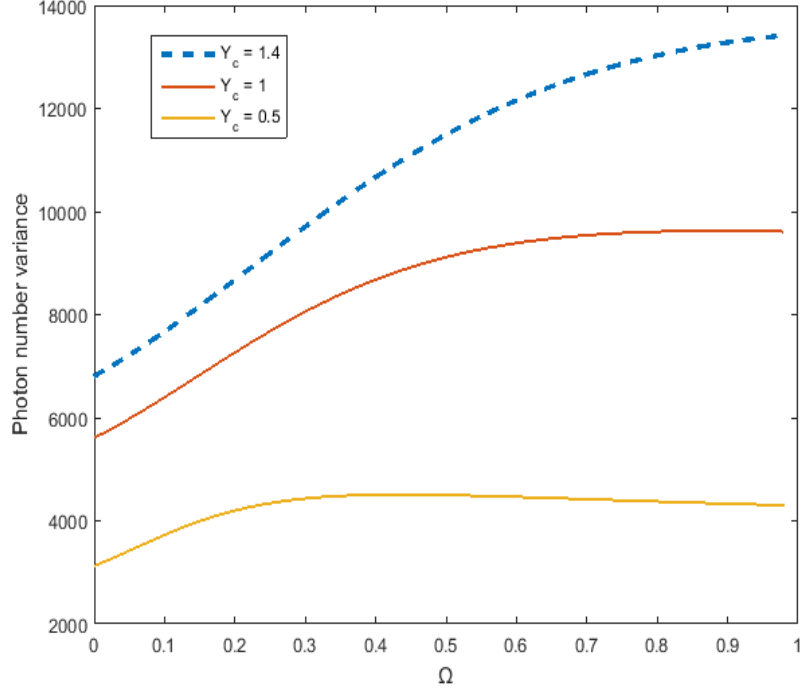


Figure 3.6: Plots of the photon number variance [Eq.(3.32)] versus  $\Omega$ , for  $\kappa=0.8$ ,  $\varepsilon=0.02$ ,  $N=50$ ,  $r=0.5$  and for different value of  $\gamma_c$ .

For some values of amplitude of driving coherent light( $\Omega$ ), as the rate of photon emission  $\gamma_c$  increases the photon number variance also increases, as seen in Fig.(3.6). The photon number variance  $(\Delta n)^2 = 6.807 \times 10^3$  when  $\gamma_c=1.4$ ,  $(\Delta n)^2 = 3.123 \times 10^3$  when  $\gamma_c=0.5$ , and  $(\Delta n)^2 = 5.608 \times 10^3$  when  $\gamma_c=1$ , all results occur  $\Omega=0$ .

# Chapter 4

## Quadrature Fluctuations

The quadrature variance and quadrature squeezing of cavity light produced by a coherently driven degenerate three-level laser with closed cavity coupled to a squeezed vacuum reservoir via a single-port mirror are investigated in this chapter.

### 4.1 Quadrature Variance

The quadrature variance of the cavity light is now calculated over the whole frequency range. We obtain the global quadrature variances for light modes  $\hat{a}$  by using the steady-state solutions of the equations of evolution of the expectation values of the atomic operators and the quantum Langevin equations for the cavity mode operators. The cavity light squeezing properties are characterized by plus and minus quadrature operators defined by

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a}, \quad (4.1)$$

and

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \quad (4.2)$$

It can be readily established that

$$[\hat{a}_-, \hat{a}_+] = 2i \frac{\gamma_c}{\kappa} \left[ \frac{\kappa^2}{\kappa^2 - 4\varepsilon^2} \right] (\langle \hat{N}_a \rangle - \langle \hat{N}_c \rangle). \quad (4.3)$$

An interesting application of the commutator algebra is to derive a relation giving the uncertainties product of two operators  $\hat{A}$  and  $\hat{B}$ , i.e,  $\Delta A \Delta B \geq \frac{1}{2} | \langle [\hat{A}, \hat{B}] \rangle |$ . Hence, in view of this, the uncertainty relation of the two quadrature operators can be written as

$$\Delta a_+ \Delta a_- \geq \frac{\gamma_c}{\kappa} \left[ \frac{\kappa^2}{\kappa^2 - 4\varepsilon^2} \right] (\langle \hat{N}_a \rangle - \langle \hat{N}_c \rangle), \quad (4.4)$$

where  $\Delta a_+$  and  $\Delta a_-$  are the uncertainties in the plus and minus quadratures. On account of Eqs. (2.83) and Eq. (2.85), Eq. (4.4) takes the form

$$\Delta a_+ \Delta a_- \geq \frac{\gamma_c}{\kappa} \left[ \frac{\kappa^2}{\kappa^2 - 4\varepsilon^2} \right] N \times \left| \frac{(\Omega + 2\Gamma)\Omega}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} - 1 \right|. \quad (4.5)$$

It is interesting to consider some special cases. We first consider the case in which the nonlinear crystal is removed from the cavity and the cavity mode is coupled to vacuum reservoir. Thus, upon setting  $\varepsilon = 0$ , Eq.(4.5) becomes

$$\Delta a_+ \Delta a_- \geq \frac{\gamma_c}{\kappa} N \times \left| \frac{\gamma_c^2 + 2\Omega^2}{\gamma_c^2 + 3\Omega^2} \right|. \quad (4.6)$$

In addition, we consider the case in which the top and bottom levels of the atoms do not coupled by the pump mode. Hence, upon setting  $\Omega=0$  in Eq. (4.6), we see that

$$\Delta a_+ \Delta a_- \geq \frac{\gamma_c}{\kappa} N, \quad (4.7)$$

which is the minimum uncertainty relation for vacuum state. Next, we proceed to calculate the quadrature variance of the cavity light. The variance of the plus and minus quadrature operators of the cavity light are defined by

$$(\Delta a_{\pm})^2 = \pm \langle (\hat{a}^\dagger \pm \hat{a})^2 \rangle \mp [\langle \hat{a}^\dagger + \hat{a} \rangle]^2, \quad (4.8)$$

so that on account of Eq. (4.8), we have

$$(\Delta a_{\pm})^2 = \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle \pm \langle \hat{a}^{\dagger 2} \rangle \pm \langle \hat{a}^2 \rangle. \quad (4.9)$$

With the help of Eqs. (3.21), (3.22), and (3.23), one can readily establish that

$$\begin{aligned} (\Delta a_{\pm})^2 &= \frac{\gamma_c}{\kappa} \left[ \left( \frac{\nu}{\gamma_c} \right)^2 [N + \langle \hat{N}_b(t) \rangle] + \frac{8\varepsilon}{\kappa} \langle \hat{m}_c(t) \rangle \pm 2 \langle \hat{m}_c(t) \rangle \right] + \left( \frac{2\Gamma}{\gamma_c} \right)^2 [N + \langle \hat{N}_b(t) \rangle \pm 2 \langle \hat{m}_c(t) \rangle] \\ &\pm \frac{4\kappa}{\varepsilon} \left( \frac{\Gamma}{\gamma_c} \right)^2 [N + \langle \hat{N}_b(t) \rangle] \Bigg] + \frac{16N\kappa^2}{(\kappa^2 - 4\varepsilon^2)} \left( N_0 - M_0 + \frac{1}{2} \right). \end{aligned} \quad (4.10)$$

Substituting Eq.(2.82) and (2.84) into Eq.(4.10) we obtain

$$\begin{aligned} (\Delta a_{\pm})^2 &= \frac{\gamma_c}{\kappa} N \left[ \left( \frac{\nu}{\gamma_c} \right)^2 \frac{\kappa\nu^2 + \kappa 2\Gamma_+(\Omega + \Gamma) + \kappa(\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma) + [8\varepsilon\nu + \kappa\Omega \pm 2\kappa\nu](\Omega + 2\Gamma)}{\kappa\nu^2 + 2\kappa\Gamma_+(\Omega + \Gamma) + \kappa(\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right. \\ &+ \left. \left( \frac{2\Gamma}{\gamma_c} \right)^2 \left[ \frac{\nu^2 + \Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma) + [\Omega \pm 2\nu](\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right. \\ &\pm \left. \frac{4\kappa}{\varepsilon} \left( \frac{\Gamma}{\gamma_c} \right)^2 \left[ \frac{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma) + \Omega(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right] \\ &+ \frac{16N\kappa^2}{(\kappa^2 - 4\varepsilon^2)} \left( N_0 - M_0 + \frac{1}{2} \right). \end{aligned} \quad (4.11)$$

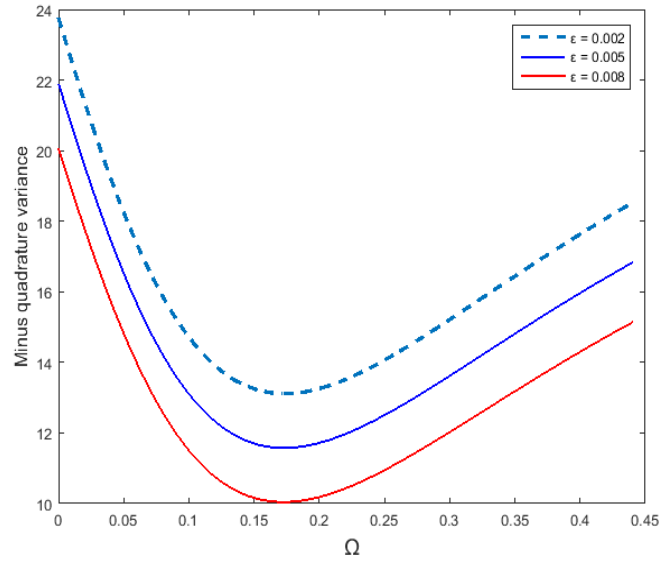


Figure 4.1: Plots of  $(\Delta a_-)^2$  [Eq.(4.11)] versus  $\Omega$ , for  $\kappa=0.8$ ,  $\gamma_c=0.4$ , and  $N=50$ , and for different values of the amplitude of the pump mode that drives the NLC,  $\varepsilon$ .

From the plot in fig.(4.1), we notice that squeezing exhibits in a different manner from a single cavity radiation, for values of  $\Omega$  between 0 and 0.45, with differing degree of squeezing. Moreover, it is possible to realize that the degree of squeezing increases with the parametric amplifier.

## 4.2 The quadrature Squeezing

We seek to calculate the quadrature squeezing of the cavity light relative to the quadrature variance of the cavity vacuum state. We then define the quadrature squeezing of the cavity light by

$$S = \frac{(\Delta a_{\pm})_v^2 - (\Delta a_{-})^2}{(\Delta a_{\pm})_v^2}. \quad (4.12)$$

Substituting Eq.(4.7),(4.11) into (4.12) we obtain

$$S = 1 - \left[ \left( \frac{\nu}{\gamma_c} \right)^2 \frac{\kappa \nu^2 + \kappa 2\Gamma_+(\Omega + \Gamma) + \kappa(\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma) + [8\varepsilon\nu + \kappa\Omega \pm 2\kappa\nu](\Omega + 2\Gamma)}{\kappa \nu^2 + 2\kappa\Gamma_+(\Omega + \Gamma) + \kappa(\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right. \\ \left. + \left( \frac{2\Gamma}{\gamma_c} \right)^2 \left[ \frac{\nu^2 + \Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma) + [\Omega \pm 2\nu](\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right. \\ \left. \pm \frac{4\kappa}{\varepsilon} \left( \frac{\Gamma}{\gamma_c} \right)^2 \left[ \frac{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma) + \Omega(\Omega + 2\Gamma)}{\nu^2 + 2\Gamma_+(\Omega + \Gamma) + (\Gamma_+ - 2\Gamma)(\Omega + 4\Gamma)} \right] \right]. \quad (4.13)$$

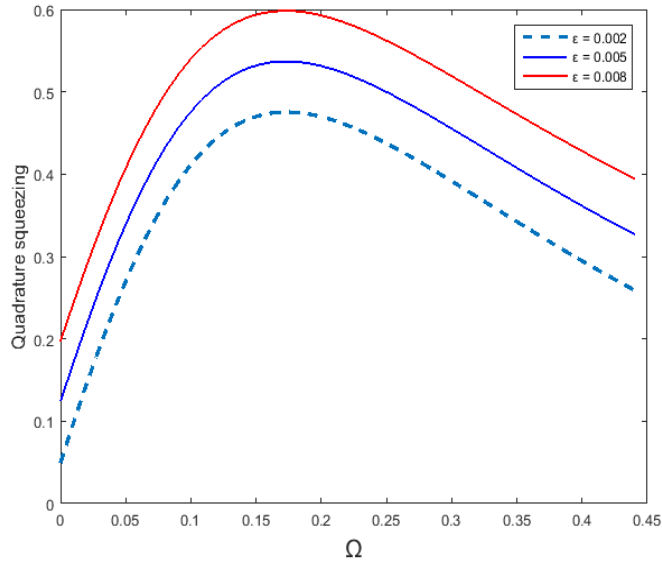


Figure 4.2: Plots of quadrature squeezing vs  $\Omega$  [Eq.(4.13)], for  $\kappa=0.8$ ,  $\gamma_c=0.4$ , and for different value of the amplitude of pump mode that drives the NLC,  $\varepsilon$ .

We note that, unlike the mean photon number and photon number variance, the

quadrature squeezing does not depend on the number of atoms. This means that the cavity light quadrature squeezing is independent of the number of photons, but it is dependent on the amplitude of the driving coherent light and the parametric amplifier. Moreover, we note that the interaction of cavity mode with parametric amplifier increases the quadrature squeezing. The plots in Fig.(4.2) shows that the maximum quadrature squeezing is 47.5%, 54%, and 60% for  $\varepsilon= 0.002$ ,  $\varepsilon= 0.005$ , and  $\varepsilon= 0.008$  respectively. The squeezing of the cavity mode increases for  $(\Omega < 0.2)$  but decreases for  $(\Omega > 0.2)$ . In addition, we have observed that the quadrature squeezing increases with the amplitude of the driving coherent light until it reaches a maximum value of 60% below the vacuum-state level.

# Chapter 5

## Conclusion

In this thesis, we investigated the quantum and statistical properties of cavity radiation produced by a coherently pumped degenerate three-level atom whose cavity contain a degenerate parametric amplifier with closed cavity coupled to a squeezed vacuum reservoir. With the use of the Hamiltonian that describes a wide range of interactions, we determined the master equation for a three-level atom in a squeezed vacuum reservoir. Again using the interaction Hamiltonian and the Heisenberg equation of motion we obtained the quantum Langevin equations for cavity mode operators, as well as the equations of evolution of the expectation values of atomic operators, using a large-time approximation technique. The approximated solutions of the quantum Langevin equations for cavity mode operators, as well as the correlation features of cavity mode operators, would used to find the solutions of the equation of evolution of the expectation value of atomic operator. Using the steady state solution, the photon statistics and quadrature squeezing of cavity light were calculated.

We have found that the light generated by the three-level laser is in a squeezed state, and the squeezing occurs in the minus quadrature. The maximum quadrature squeezing of the light generated by the three level laser is found to be 60% below the vacuum-state level for certain conditions. The brightness of the cavity light enhanced in the presence of both the stimulated emission rate and the parametric amplifier. We have seen that one effect of the coupling of the top and bottom levels is to increase the mean and photon

number variance. We found that, quadrature squeezing is dependent on the amplitude of the driving coherent light and the parametric amplifier, not on the number of atoms. We have shown that the effect of the squeezed parameter is to decrease the mean and variance of the photon number, but the presence of parametric amplifier  $\varepsilon$  increases them. We have also seen that for a three-level atoms, the presence of the parametric amplifier enhances the squeezing of the cavity light generated by the systems under consideration.

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