**College Of Natural and Computational Science**
**Department Of Mathematics**

# Project On" Euler Method for solving Ordinary Differential Equation Its Application"

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**A Project Submitted to the Department Of Mathematics Wolkite University in Partial Fulfillment of the Requirements of the Bachelor of science Degree in Mathematics**

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Wolkite University
Department of Mathematics

The undersigned here by certify that they have read and recommend to the Department of Mathematics for acceptance of a project entitled Euler's Method For Solving Ordinary Differential Equation Its Application by student Nunu Magersa in partial fulfillment of the requirements for the degree of Bachelor of science,

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Examining committee ______________________

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# Abstract

This project is constructed based on Euler's Method For Solving Ordinary Differential Equation the essence of the Euler's method is to find the approximate values of the solutions.Euler's method gives us a clue to the basic concepts of numerical solution for initial value problem.

# Notation

DE..............................Differential Equation
IVP..........................initial value problem
ODE.......................... ordinary differential equation

# Chapter 1

# Introduction

## 1.1 Background of the study

In this project we present the solution of **initial valu problems(IVP),** using Euler's method. In our real world many problems in different discipline appear in the form of differential equation. not all differential equations can be explicitly solved for y,This can be problematic if we need to know the value of at specific point,This is where method of numerical integration are useful,as they allow us to estimate the value of y based on known initial conditions,one of these methods for numerical integration is Euler's method which will be the subject of the following discussion Although differential equations can be solved analytically using different techniques. Numerical methods applied to solve differential equations when analytical solution is difficult to obtain.In this project we will see some numerical techniques those we apply to solved differential equations by Euler's method for solving initial value problems.**Differential equations(DE),** are used to model problems in science and Engineering that involves the rate of change of one variable with respect to an other most of these problems requires the solution of an initial value problems,that is the solution to differential equation that satisfy the given condition. not the most accurate of method it is one of the simplest,which is useful when beginning to understand these method,essentially the method works by finding the slope at a known,traveling in a small amount h in that direction then calculating the new slope and traveling in the direction of the new slop. In common real-life situation the differential equations that model the problem is too complicate to solve and one can uses numerical methods to the original problems.

## 1.2 Objective of the project

## 1.3 General Objective

The general objective of this project is to solve initial value problem by using Euler's method and intended to achieve the following specific objectives

## 1.4 Specific Objectives

- solve the first order ordinary differential equations, using Euler's method,
- to introduce Euler's method
- to calculate initial value problems of ordinary differential equation by Euler's method
- to solve linear initial value problem using Euler's method
- To discuss Application of Euler's method

# Chapter 2

# preliminaries concept

## 2.1 Ordinary Differential equation

**Definition 2.1.1.** *A differential equation involving one or more derivatives of a dependent variable with respect to only one variable is called an **ordinary differential equation(ODE).***

**Example 2.1.1.** *a,* $\frac{dy}{dx} = x - 5$
*b,*$y'' + xy' + y = 2$

**Definition 2.1.2.** *An equation the derivative of one or more unknown functions or dependent variable with respect to one or more independent variable is said to be **differential equation(DE)***

**Example 2.1.2.** *a.* $\frac{d^2x}{dt^2} + 16x = 0, y(0) = y_0$
*b.*$\frac{dy}{dt} = y - 5, y(0) = y_0$
$\frac{dy}{dt} = 2y - 10, y(0) = y_0$

## 2.2 Initial value problem

*A solutions to differential equations may require satisfied certain defined conditions and such conditions are called initial conditions if they are given at only one point of the independent variable conditions given at only one point of the independent variable are called initial condition.*

*the problem of solving an $n^{th}$ order differential equation together with an initial condition is called an **Initial Value Problem(IVP).***

**Example 2.2.1.** $\frac{dy}{dx} = \frac{4x^2-7x}{3y^2+2} y(1) = 1$
***Solution***this is a separable differential but it is also subject to initial

*condition we start off by getting all the terms on their respective side, and then taking the anti derivative you per anti derivative equation will look like then taking anti derivative you include a c value*
$y^3 + 2y = \frac{4}{3}x^3 - \frac{7}{2}x^2 + C$
*then, using initial condition given we can solve for the value of C* $1^3 + 2 = \frac{4}{3}1^3 - \frac{7}{2}1^2 + C solving for C, we get$
$3 = \frac{-13}{6} + C$
$C = \frac{31}{6}$ *which given us a final answer of*
$y^3 + 2y = \frac{4}{3}x^3 - \frac{7}{4}x^2 + \frac{31}{6}$

## 2.3 Euler's method

**Definition 2.3.1.** *Euler's method is the most elementary approximation techniques for solving in initial value problems . Then it is seldom used in practice the simplicity of its derivation can be used to illustrate the techniques involved in the construction.Euler's method has limited usage because of large error that is accumulated as the process proceeds.The process is very slow and to obtain reasonable accuracy with Euler's method we have to take a smaller value of h.further,the method should not be used for a larges range of x as the values found by this method go on becoming farther and farther away from the true value.*

**Definition 2.3.2.** *Euler's Method is a form of numerical integration — a way to approximate the solution of a first-order differential equation where the initial point on the solution curve is known, but the shape of the curve is unknown.*

*To use Euler's Method, first calculate the slope of the curve at the known starting point* $A_0$ *and use the slope to calculate the tangent line at A0. Take a small step along the tangent line to* $A_1$*, and assume that* $A_1$ *is still on the unknown curve. Follow the same method for several points* $A_2, A_3$*, and so on to create the approximated curve. Euler's Method can be used with relatively little error if the ratio of the interval to the original known value interval* $A_0$ *remains small.*

generally ,we take the following General formula where

## 2.4 Euler's Formula

$h = x_n - x_{n-1}, x_n = x_0 + nh,$
thus,
for $n = 0, y_1 = y_0 + hf(x_0, y_0)$
for $n = 1, y_2 = y_1 + hf(x_1, y_1)$
for $n = 2, y_3 = y_2 + hf(x_2, y_2)$
for $n = 3, y_4 = y_3 + hf(x_3, y_3)$
for $n = 4, y_5 = y_4+hf(x_4, y_4) Generally, wetakethefollowingGeneralformula$
$\vdots$
$y_{n+1} = y_{n-1} + hf(x_{n-1}, y_{n-1})$

## 2.5 The Explicit Euler's method

The ancestor of all the advanced numerical methods in use today was formed by Leon hard Euler in 1768.on writing down the first terms in the Taylor expansion of the solution at $x_0$ and using the prescribed initial value and the differential equation at $x = x_0$ it is noted that $y(x_0 + h) = y(x_0) + hy'(x_0) + \ldots = y_0 + f(x_0, y_0) + \ldots$ choosing a small step size h¿0 and neglecting the higher -order terms represented by the dots, an approximation $y_1 toy(x_1)$ at the later time $x_1 = x_0+h$ is obtained by setting$y_1 = y_0 + hf(x_0, y_0)$. the next idea is to take $y_1$ as the starting value for a further step which then yields an approximation to the solution at $x_2 = x_1 + h$ as $y_2 = y_0 + hf(x_0, y_0)$ continuing in this way,in the (n+1)st step we take $y_n \approx y(x_n)$ as the starting value for computing an approximation at $x_{n+1} = x_n + h$ as
$y_{n+1} = y_n + hf(x_n, y_n)$
and after a sufficient number of steps we reach the final time T.the computational cost f the method lies in the evaluations of the function f.The step size need not be the same in each step and could be replaced by $h_n$ n the formula ,so that $x_{n+1} = x_n + h_n$ It is immediate that the quality of the approximation $y_n$ depends on two aspects the error made by truncating the Taylor expansion and the error introduced by continuing from approximate solution value. these two aspects are captured in the notions of consistency and stability, respectively, and are fundamental to all numerical methods for ordinary differential equations.

## 2.6 Implicit Euler's method

A minor-looking change in the method already considered by Euler in 1768.makes a big difference taking as the argument of f the new value instead of the previous one yields
$Y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$
form which $y_{n+1}$ is now determined implicitly.Differential equations for which the numerical solution using the implicit Euler method is more efficient than that using the explicit Euler method are called stiff differential equations. they include important applications in the description of processes with multiple time scales (Example fast and slow chemical reactions) and in spatial sem idiscretizations of time-dependent partial differential equations for example for the heat equation, stable numerical solutions are obtained with the explicit Euler method only when temporal step sizes are bounded by the square of the the spatial grid size, where as the implicit Euler method is unconditionally stable

# Chapter 3

# Euler's Method For Solving Ordinary differential Equation

Euler's method is a numerical technique to solve ordinary differential equations of the form
$y' = \frac{dy}{dx} = f(x, y)$with $y(x_0) = y_0$
to solve the equation given for the values of y at
$x = xi = x_0 + ih, i = 1, 2, 3....n$
integral equation with one step size $\int_{x_0}^{x_1} dy = \int_{x_0}^{x_1} f(x, y)dx$
take $f(x, y) = f(x_0, y_0)$in the range $x_0 to x_1$
then we get $y_1 = y_0 + (x_1 - x_0)f(x_0, y_0)$
$y_1 = y_0 + hf(x_0, y_0)$ similarly ,take $f(x, y) = f(x_1, y_1)$
in range $x_1 to x_2$,then we get $y_2 = y_1 + (x_2 - x_1)f(x_1, y_1)$
$y_2 = y_1 + hf(x_1, y_1)$

## 3.1 Algorithm Of Euler's Method

The typical steps of Euler's method are given below:

Step 1: define $f(x_n, y_n)$
Step 2: input initial values $x_0$ *and* $y_0$
Step 3: input step sizes h and number of steps n
Step 4: calculate x and y
for
$n = 1 : N - 1$
$x_{n+1} = x_n + h$
$y_{n+1} = y_n + hf(x_n, y_n)$
Step 5: output $x_{n+1}$ *and* $y_{n+1}$

**Example 3.1.1.** *using Euler's method solve for*
$x = 0.01 of \frac{dy}{dx} = \frac{(x-y)}{(x+y)}$ *given that* $y(x_0) = y(0) = y_0$*=1 and h=0.02*

***Solution*** *:-the values of x with the given step size h are*
$x_0 = 0$
$x_1 = 0.02,$
$x_2 = 0.04,$
$x_3 = 0.06,$
$x_4 = 0.08$
$and\ x_5 = 0.1$
$y_1 = y_0 + hf(x_0, y_0)$
$and\ f(x_0, y_0) = f(0, 1)$
$\frac{(0-1)}{(0+1)} = -1$
$= 1 + 0.02(-1)$
$= 0.98$
$y_2 = y_1 + hf(x_1, y_1), but$
$= f(x_1, y_1)$
$= f(0.02, 0.98)$
$\frac{(0.02-0.98)}{(0.02+0.98)} = -0.96$
$= 0.98 + 0.02(-0.96) = 0.9608$
$y_3 = y_2 + hf(x_2, y_2), but$
$f(x_2, y_2)$
$= F(0.04, 0.9608)$
$\frac{(0.04-0.9608)}{(0.04+0.9608)} = -0.92006$
$= 0.9608 + 0.02(-0.92006) = 0.942398$
$Y_4 = Y_3 + hf(x_3, y_3), but$
$f(x_3, y_3)$
$f = (0.06, 0.942398)$

$\frac{(0.06-0.942398)}{(0.06+0.94398)} = -0.880287$

$= 0.942398 + 0.02(-0.880287) = 0.92479$
$y_5 = y_4 + hf(x_4, y_4), but$
$f(x_4, y_4)$
$f = (0.08, 0.92479)$

$\frac{(0.08-0.92479)}{(0.08+0.92479)} = -0.84076$

$= 0.92479 + 0.02(-0.84076) = 0.907975$
$y_6 = y_5 + hf(x_5, y_5), but$
$f(x_5, y_5)$

$f(0.1, 0.907975)$

$\frac{(0.1-0.907975)}{(0.1+0.907975)} = -0.80158$

$= 0.907975 + 0.02(-0.80158) = 0.89194$

**Example 3.1.2.** *compute* $y_3$, $y_6$ *and* $y_8$ *the following using Euler's method*
$\frac{dy}{dx} = 1 + xy^3$
$y(0) = 1$
$h = 0.1$
***solution***:-*the value of with given step h are*
$x_0 = 0$
$x_1 = 0.1$
$x_2 = 0.2$
$x_3 = 0.3$
$x_4 = 0.4$
$x_5 = 0.5$
$x_6 = 0.6$
$x_7 = 0.7$
$x_8 = 0.8$
$y_1 = y_0 + hf(x_0, y_0), butf(x_0, y_0)$
$f(0, 1)\ 1 + 0(1)^3 = 1$
$1 + 0.1(1)$
$= 1.1\ y_2 = y_1 + hf(x_1, y_1), butf(x_1, y_1)$
$= 1 + (0.1)(1.1)^3$
$1.1 + 0.1(1.1331)$
$= 1.21331$
$y_3 = y_2 + hf(x_2, y_2), butf(x_2, y_2)$
$1 + 0.1(1.21331)^3$
$= 1.34903279$
$y_4 = y_3 + hf(x_3, y_3), butf(x_3, y_3)$
$= 1.34903279 + 0.1(1, 706527170)$
$1.419685507$

$y_5 = y_4 + hf(x_4, y_4), but f(x_4, y_4)$
$= f(0.4, 1.419685507)$
$1.419685507 + 0.1(2.1445543961)$
$= 1.6341409466$
$y_6 = y_5 + hf(x_5, y_5), but f(x_5, y_5)$
$= f(0.5, 1.6341409466)$
$1.6341409466 + 0.1(5.36387165)$
$= 2.1705247$
$y_7 = y_6 + hf(x_6, y_6) but f(x_6, y_6)$
$2.1705247 + 0.1(7.13543850)$
$2.884067985$
$y_8 = y_7 + hf(x_7, y_7), but f(x_7, y_7)$
$2.884067985 + 0.1(17.792467672)$
$= 4.6633146$

## 3.2 Advantage and Disadvantage of Euler method

**Advantages of Euler's Method**:
Euler's Method is simple and direct Can be used for linear and nonlinear IVPs
**Disadvantages Of Euler method**:
it is less accurate and numerically unstable. Approximation error is proportional to the step size h. Hence, good approximation is obtained with a very small h.

# Chapter 4

# Application Of Euler's Method

In mathematics and computational science, the Euler method is a first order numerical procedure for solving ordinary differential equation with a given initial value.It is the basic explicit method for numerical integration of ordinary differential equation and is the simplest Runge-kutta method. the Euler method is a first order method .wech means that the local error (error per step)is proprtional to the square of the step size, and the global error (error at a given time )is proportional to the step size . The Euler method often serves as the basis to construct more complex method,

**Example 4.0.1.** *predictor corrector method*

**Example 4.0.2.** *As an application consider the following initial value problem* $\frac{dy}{dx} = \frac{x}{y}$ $y(0) = 1$ *which was chosen because we know the analytical solution and we can use if for check,Its etact or analytical solution is found to be* $y(x) = \sqrt{x^2 + 1}$ *Therefore, we will be able to compare the approximate solution and the exact solution here we wish to approximate* $y(0, 3)$ *using the Euler's methods with step sizes* $h = 0.1$ *and* $h = 0.05$ *we find by hand calculation* $x_0 = 0 x_1 = 0.1 x_2 = 0.2 x_3 = 0.1$ $y_1 = 1$
$y_1 = y_0 + hf(x_0, y_0) = y_0 h x_0, y_0 = 1$
$y_2 = y_1 + hf(x_1 y_1) = 1.01$
$y_3 = y_2 + hf(x_2, y_2) = 1.0298$
$since y(0, 3) = \sqrt{(0.3)^2 + 1} = 1.044030$ *we find that*
*Error* $\frac{(y(0.3) - y_3)}{y(0.3)} .100 = 1.35$*similarly for the step size*$h = 0.05$ *we find that the error is Error*$\frac{(y(0.3) - y_6)}{y(0.3)} .100 = 0.67$

# Conclusion And Summary

under the topic of Euler,s methods the conclusions could be made. Numerical method applied to solve differential equation when analytically solution is difficult

Euler's method is considered to be one of the oldest and simplest methods to find the numerical solution of ordinary differential equation or the initial value problems. Here, a short and simple algorithm and flowchart for Euler's method has been presented, which can be used to write program for the method in any high level programming language. Through Euler's method, you can find a clear expression for y in terms of a finite number of elementary functions represented with x. The initial values of y and x are known, and for these an ordinary differential equation is considered. I think that we have adequately demonstrated the concepts underlying the Euler's Method algorithm. We have seen the derivation of the required formulas from both a graphical and a formulaic point-of-view. So, Euler's method is a nice method for approximating fairly nice solutions that don't change rapidly. However, not all solutions will be this nicely behaved. There are other approximation methods that do a much better job of approximating solutions. Also notice that we don't generally have the actual solution around to check the accuracy of the approximation.

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