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TIME SERIES ANALYSIS OF INFLATION IN ETHIOPIA: UNIVARIATE CASE

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This is to certify that the paper prepared by Atalelech Ku'u and Misganu Mare entitled: the modeling and forecasting the volatility of the inflation in Ethiopia and submitted in partial fulfillment of the requirements for the Degree of bachelor of Science complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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ABSTRACT

Inflation refers to a situation in which the economy's overall price level is rising where as Inflation rate is the percentage change in the price level from the previous period. The aim of this research paper focused to fit a Univariate time series model which can be used to forecast the overall inflation behavior in Ethiopia. The secondary data based on the Consumer Price Index (CPI) was used on monthly observations from January 2001 to December 2018 reported from Central Statistical Agency(CSA).

The Univariate time series model was employed for modeling inflation. We apply ADF test to detect Stationarity of the series. Model selection is one of the fundamental tasks of scientific inquiry and the partial autocorrelation, autocorrelation functions, AIC and BIC were used to identify the appropriate model. Thus, the estimated conditional mean and variance for in-sample series have been achieved by combination of AR (1) with GARCH (1, 1) model with the minimum AIC and BIC rank sum value for the log return series. In all the cases the coefficients of the ARCH terms are significant at 5% level of significance implying that there is clustering of volatility of overall inflation. That is, large changes in log returns of prices of goods and services are likely to be followed by further large changes. Similarly, the significance of the coefficients of the GARCH terms in each case at the 5% level of significance indicates that the present conditional variance is dependent on its past variances.

The forecasting accuracy of the model was checked using like MZ-R^2*, RMSE, MAE and MAPE. The general level of price of goods and services is rising over time at the country level. It creates uncertainty when the average price level of goods and services changes significantly and becomes unstable. Therefore, focus should be given on policies that will achieve price stability in the country level. STATA12 software has been used to analyze the data.*

Keywords: *Inflation, ARIMA Model, GARCH model, Model Identification, forecasting.*

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ACRONYMS AND ABBREVIATION

ACF	Autocorrelation Function
ADF	Augmented Dickey Fuller
AIC	Akaike Information Criterion
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroskedastic
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criterion
CPI	Consumer Price Index
CSA	Central Statistical Agency
GED	Generalized Error Distribution
GARCH	Generalized Autoregressive Conditional Heteroskedastic
GDP	Gross domestic product
IMF	International Monetary Fund
MA	Moving Average
NBE	National Bank of Ethiopia
PACF	Partial Autocorrelation Function
PP	Phillips Perron
PPI	Producer Price Index
SAR	Seasonal Autoregressive
SARIMA	Seasonal Autoregressive Integrated Moving Average
SMA	Seasonal Moving Average
TGARCH	Threshold Generalized Autoregressive Conditional Heteroskedastic

USD	United States Dollar
LM	Lagragian Multiplier

CHAPTER ONE

1 . INTRODUCTION

1.1 Background of the Study

Inflation refers to a continuous rise in price levels of goods and services leading to a fall in currency's purchasing power. A country is said to be under pressure of inflation when the prices of most goods and services continue to scale upward for a long duration of time. On the other hand, deflation is defined as a sustained decline in an aggregate measure of prices such as the consumer price index or the GDP deflator (Romer, 2012).

There are three key terms in the definition of inflation. First, inflation refers to the movement in the general level of price not the changes in one price relative to other prices. Second, the prices are those of goods and services not asset. The third and most important thing is that the rise in price has to be continuous over a long period of time (Labonte, 2011).

Globally, inflation co-moves across countries. This means that price movements are transmitted to local markets and then global inflation can help to forecast country inflation. Understanding the world's need for price stability, the possible growth halting effects that emanate from the rising levels of inflation in most African countries is becoming an issue of increasing concern. The issue seems to be more important in poorer economies since inflation mainly manifests itself in rising food prices (IMF, 2013).

On the year 2011, the inflation level in East Africa has reached as high as 20 percent on average. Such a rise in inflation is becoming a great challenge for the region's growth and development process. In Ethiopia, inflation rate reached 34 percent and Uganda recorded the second highest level of inflation in the region hitting 30.5 percent in October 2011. This trend of increase in inflation in the region is suspected to be harmful for the developing economies, especially for their poor residents (Alain, 2012).

The history and trend of inflation in Ethiopia prior to 2002 has remained more or less stable. However, in the post 2002/03 the situations have been dramatically changed. Inflation has increased. During the same period, the economy has recorded fast growth rate on average 10.5% GDP growth and continue growing consecutively for the last eight years (WB, NBE, 2010).

In Ethiopia raw inflation figures are reported monthly using the CPI by the Central Statistical Agency. The CPI is an estimation of the price changes for a typical basket of goods. In other words, the prices of everyday goods such as housing, food, education, clothing, etc., are compared from one month to the next and the difference represents the CPI. The CPI published by CSA composed of the weighted average of two sub indexes that reflect the development of prices of goods production in certain sectors of economy, namely food and non food prices.

1.2 Overview of Inflation Rate in Ethiopia

Ethiopia is one of Africa's largest countries with an estimated population of 108,494,634 people in 2018. One of the world's oldest civilizations, Ethiopia is also one of the world's poorest countries. At $390, Ethiopia's per capita income was much lower than the Sub-Saharan African average of $1,176 in 2010 fiscal year.

In Ethiopia, the trend of inflation showed more or less stable prior to 2002 with annual average inflation rate of 5.2 percent during 1980-2003. However, in the post 2002/03 the situations have been dramatically changed and it has increased with the significant increase in output prices. The 12 months moving average inflation rate shows the longer term inflationary situation every year (Biresaw et al, 2013).

The growing domestic supply-demand gap, in the context of the surge in growth, contributed to a rise of inflation. The country level overall inflation rate rose by 32.0 percent in July 2012 as compared to the one observed in a similar period a year ago. The country level food inflation increased by 39.2 percent as compared to the one observed a year ago. The country level non-food inflation rate increased by 21.5 percent in July 2012 as compared to the one observed in July 2011 (CSA, 2012).

In the same manner, Inflation rate based on last year's similar month comparison in July 2018 general year on year inflation has increased by 14.0 percent as compared to the one observed in July 2017. The rise in general inflation rate is due to the fact that the general consumer price index (CPI) of 193.0 percent observed in July 2018 was higher than the corresponding 169.2 percent general consumer price index (CPI) observed in July 2017.

1.3 Statements of the Problem

The problem of inflation have local, national and an international reasons today. When we see current world economy, every country even the developed countries are affected by these problems. The price of general commodity has been increasing in Ethiopia as well as in the world. It is obvious that increasing the average price affects the entire commodity of all people of Ethiopia.

Inflation is viewed as being undesirable because of some serious economic and social effects. Inflation impacts on income distribution making a random redistribution of real income. Those receiving fixed money income are usually disadvantaged because often their income are not adjust upward fast enough to compensate for the effect of continually rising price. Their real income was fall. Individuals whose income rises more rapidly than the inflation rate was experience increasing real incomes.

If the rate of inflation is high, individuals with money tend to buy real assets such as property, gold and antique, which often increase in value faster than the rate of inflation. Inflation tends to increase spending and encourage borrowing at the expense of savings. If price are rising quicker than income, individual was tend to buy at current prices before goods and services become more expensive and less affordable.

Therefore, it is better to study independently to get a more detailed picture on the trend of inflation and forecasts should be updated whenever new information becomes available. Lack of enough production of all types of commodities, increasing the values of dollar and lack of export material are major motivating reasons to investigate inflation in Ethiopia.

1.4 Objective of the Study

1.4.1 Main Objective

The main objective of this study is to fit a Univariate time series model for overall inflation from January 2000 to December 2018 in Ethiopia.

1.4.2 Specific Objectives

- To see the trend of overall inflation in Ethiopia over the study period.
- To fit an appropriate ARIMA or/and a (G) ARCH model for the overall inflation.
- To identify the short run and long run behavior of inflation at the country level.
- To predict in- sample forecasts for overall inflation in the country.

1.5 Significance of the Study

This study was purposeful to know how much inflation was serious at the country level. In addition to this, anyone can prepare him/herself to know what are the causes and consequence of high inflation rate. Given the country experienced the highest inflation levels in East Africa it is clear that there is a need for further study in the area. Therefore, this study may be beneficial since it performs an empirical investigation which was giving an insight on the inflation.

In particular, this study is also more significant for searching away how the inflation was totally alleviated by giving recommendation to the concerned bodies how to solve the problem. This can be an alternative method to forecast overall inflation by fitting an appropriate model and it helps policy makers to take action in order to control the overall inflation. Furthermore, the result could serve as reference material for other researchers in related topics or problems.

1.6. Scope of this study

This study is focus on the investigate inflation rate in Ethiopia by explicitly modeling, as well as the general consumer price index and also this study is indicated time series trend and appropriate model for mean AR(1) and for variance GARCH(1,1) this model is used to forecast the future condition of inflation.

CHAPTER TWO

2 . LITERATURE REVIEW

2.1 Definition and some characteristics of inflation

Inflation originally referred to the debasement of the currency. In economics Inflation is a rise in general level of prices of goods and services in the economy in a period of a time. However, economic debates about the relationship between money supply and price levels have led to its' primary use today.

In describing inflation, it can also be described as a decline in the value of money, loss of purchasing power in the medium of exchange which is also the monetary unit of account. When the general price of level raises each unit of currency buys fewer goods and Services. Many researchers have undertaken a variety of researches regarding the determinants of inflation. Nevertheless, they did not agree as to the specific variables that causes inflation in the country. This implies that the issue of inflation requires an intensive study with sound methodology so that it may be easy to control and predict it.

According to Jonathan Kearns (2016), forecasts of global inflation have predictive power for global inflation at a medium horizon (12 months) but not at a longer horizon. Global inflation forecasts, and forecast errors, are correlated with survey forecasts and errors of oil and food prices, and global GDP growth, but not financial variables. For some countries, forecasts of global inflation improve the accuracy of forecasting regressions that include survey forecasts of country inflation. However, for most countries, lagged or forecast of global inflation does not improve the accuracy of survey forecasts of country inflation. Whatever information global inflation may include about country inflation, for most countries it seems that survey forecasts of country inflation have historically already incorporated that information.

2.2 Empirical Literatures

There were numerous attempts to model inflation in developing countries. Among the studies on modeling inflation, the study by Loungani and Swagel (2001) is the one that could serve as a starting point for understanding inflation in developing countries. The author's present stylized facts about inflation behavior in developing countries, focusing primarily on the relationship between the exchange rate regime and the sources of inflation.

Meyler, et al. (1998) outlined autoregressive integrated moving average (ARIMA) time series models for forecasting Irish inflation. They considered two alternative approaches to the issue of identifying ARIMA models - the Box Jenkins approach and the objective penalty function methods. The emphasis is on forecast performance, which suggests that ARIMA forecast has outperformed.

Hamad (2007) used ARIMA models in studying and analyzing monthly time series for the inflation rate in Saudi Arabia for the period February 1980 to September 2004, in order to identify the pattern of change in the rate of inflation, then build a model from ARIMA family that helps to predict the values of the rate of inflation in the short term. The study used statistical time series stationarity tests such as Dickey - Fuller (ADF) and autocorrelation function (ACF). The model of ARIMA showed some seasonal effects on the inflation rate during the study period.

Richard and Stephen (2012) discussed the time series analysis of Canada's inflation rate from 1995 to the end of 2011. Data was obtained shown significant lags in the autoregressive process at time lag 5,7,12 and 24 and the moving average process at time lag 1,5,12 and 24. Following the Box-Jenkins approach it reached an ARMA model of monthly inflation. The Appropriate model was an SARMA (0, 0, 0) (2, 0, 2)12.

Bokhari and Feridum (2006) empirical study is another investigation which aims at modeling and forecasting inflation in Pakistan. For this purpose a number of econometric approaches are implemented and their results are compared. In ARIMA models, adding additional lags for p and or q necessarily reduced the sum of squares of the estimated residuals. Results further indicate that the VAR models do not perform better than the ARIMA (2, 1, 2) models and, the two factor model with ARIMA (2, 1, 2) slightly performs better than the VAR model. Related investigation also done by Muhammad et al.(2006) and the main focus of the study was to forecast the monthly inflation on short-term basis.

Faisal (2012) examined the volatility of inflation rate in Bangladesh using time series GARCH model. He used monthly inflation rates spanning the period 1990-2011. According to them, the main objective of an inflation rate policy is to determine an appropriate inflation rate and ensure its stability and over the years, efforts put by the Government to achieve this have not yielded positive results. He thus sought to build a forecasting model that would adequately capture the volatility of inflation rate return series using GARCH model and the outcome of his research was to assist the government to manage the exposure of the inflation rate volatility in the short run, inform investors on future behavior of inflation rates thus helping them in decision making and help end users of volatility models such as importers, exporters, etc.

Uwilingiyimana et al (2015) conducted a study on forecasting inflation in Kenya using two models, the ARIMA (1,1, 12) and GARCH (1, 2) and a combination of the two model ARIMA (1, 1, 12)-GARCH (1, 2). The study revealed the combination between ARIMA (1, 1, 12)-GARCH (1, 2) model provided the best and improved results for estimating and forecasting accuracy compared to the other forecasting models.

Otu et al (2014) discussed the application of SARIMA Models in Modeling and Forecasting Nigeria's Inflation Rates. They employed Box and Jenkins to build the Autoregressive Integrated Moving Average (ARIMA) monthly inflation rates for the period November 2003 to October 2013 with a total of 120 data points. They found that the Seasonal ARIMA (1, 1, 1) * (0, 0, 1)12 was the best model to forecast Nigeria's inflation rate.

Awogbemi and Oluwaseyi (2011) results showed that ARCH and GARCH models are better models because they give lower values of AIC and BIC as compared to the conventional Box and Jenkins ARMA models for inflation in Nigeria. The researchers also observed that since volatility seems to persist in all the commodity items, people who expect a rise in the rate of inflation (the 'bullish crowd') will be highly favored in the market of the said commodity items.

Durevall et al (2010), using monthly data from 2000-2009, model inflation in Ethiopia by including error correction mechanisms for food and non-food prices. In contrast to other studies on inflation, they specify separate long-run relationships for the monetary, domestic food, and external food and non-food sectors, though they ignore long-run effects of energy prices. Their findings indicate that the external sector largely determines inflation in the long run. Specifically, domestic food prices adjust to changes in world food prices, measured in local currency (EBT), and non-food prices adjust to changes in world producer prices. Domestic food supply shocks also have a strong effect on inflation but it is a short-run effect. The evolution of money supply does not affect food prices directly, though money supply growth significantly affects non-food price inflation in the short run.

Ahmed (2007) examined the determinants of inflation in Ethiopia and concludes "structural changes" such as increasing bargaining power of farmers and monetary expansion are the main reasons of inflation in Ethiopia. He argues that monetary expansion is largely dictated by credit expansion in both the public and private sector. Credit expansion is explained on the public side, by decline in foreign finance flow, including a reduction foreign aid.

Tadelle (2008) investigated the nature of inflation in Ethiopia and constructed a model that can be used to forecast future values. The exponential smoothing model was employed and the forecasting performance of winter (additive) models was found to be better. Two alternative approaches for model identification were considered, namely, the Box-Jenkins methodology and Penalty function criteria. For Ethiopian monthly inflation data covering the period 1997 to 2006 ARMA model was fitted. He suggests that SARIMA (1, 0, 10) * (12, 0, 12)12 model using CPI was found better for forecasting inflation in Ethiopia.

Yohannes et al (2009) used monthly data to estimate error correction models to identify the relative importance of several factors contributing to overall inflation, and its three major components, cereal prices, food prices and non-food prices. The main finding of the study indicates: in the long run, domestic food and non-food prices are determined by the exchange rate and international food and goods prices.

Eden (2012) modeled inflation volatility and analyzed its effect on economic growth in Ethiopia. Co-integrated VAR model and granger causality test were used to see the relationship between inflation, inflation uncertainty and growth. From the co-integrated VAR model, she concluded that the growth rate of GDP affects inflation positively in the long run and negatively in the short run. The granger causality result also indicates that inflation granger causes inflation uncertainty positively and inflation uncertainty granger causes output growth negatively.

CHAPTER THREE

3 . DATA AND METHODOLOGY

3.1 Study Area and Sources of Data

Ethiopia is one of the developing countries located in Eastern Africa. It borders Sudan and South Sudan on the West, Eritrea on the North, Djibouti and Somalia on the East, and Kenya on the South. Its geographical location is 3^0 and 18^0 N latitude, and 38^0 and 48^0 longitude, just north of the equator with a total land-area of about 122.2 million hectares. The country is divided into nine regions and two administrative cities; Addis Ababa and Dire-dawa (yemataw, 2013).

Generally, there are two sources of data, this are primary and secondary source of data. The primary source of data was obtained directly from the respondents through questionnaire while the secondary data were obtained from documents, books, internets and the governmental sector in the study area. We use secondary Sources of data which was reported from January 2001 to December 2018 by the Central Statistical Agency on monthly bases at the country level. The data analysis was done by using the STATA 12 statistical software.

3.2 Component of Time series

When the data are arranged on the basis of their time of occurrence, we would find that the variable under investigation is fluctuating from time to time. The fluctuations are caused by composite force that is constantly at work. This force has four components: Trend, cyclical variation, seasonal component and irregular or random movement.

3.2.1 Seasonal component

Seasonal component is also known as periodicity. This type of component is generally annual in period and arises for many series, whether weekly, monthly or quarterly measured, when similar patterns of behavior are observed at particular times of the year. It describes any regular fluctuations with a period of less than one year.

3.2.2 Cyclical Variation

It refers to long-term oscillation about trend over a long period of time. In a large number of time series of economic data, it has been observed that there is somewhat periodic up and down movement.

3.2.3 Trend component

A trend is evolutionary movement, either upward or downward, in the value of the variable. This type of component is present when a series exhibits steady upward growth or a downward decline, at least over several successive time periods, when allowance has been made for the other components. This may be loosely defined as 'long-term change in the mean level'. A difficulty with this definition is deciding what is meant by "long term‟.

3.2.4 Irregular component

The phrase irregular fluctuations are often used to describe any variation that is left over when other components of the series (trend, seasonal and cyclical) have been accounted for. As such, they may or may not be random.

3.3 Models of Time Series

There are two mathematical models, which is commonly used for the decomposition of a time series into component parts. These are additive model and multiplicative model.

i. Additive Model: $yt = Mt + St + Ct + E_t$ (3.1)

Where, yt = Observations at time t

Ct = Cyclical component at time t

Mt = Trend component at time t

Et = Random component at time t

St = Seasonal component at time t

It is an appropriate model if we assume that all components are independent of one another and the magnitude of the seasonal fluctuations does not vary with the level of the series.

ii. Multiplicative Model: $yt = Mt * St * Ct * Et$ (3.2)

Where, yt = Observations at time t

Ct = Cyclical component at time

Mt = Trend component at time t

Et = Random component at time t

St = Seasonal component at time t

3.4 Stationary time series

Time series Stationarity is an important point to be described in time series analysis. A series is said to be stationary if the mean and autocovariances of the series do not depend on time. If both are constant over time, then the series is said to be a stationary process i.e. is not a random walk (has no unit root), otherwise, the series is described as being a non-stationary process i.e. a random walk (has unit root). Sometimes, seasonality can also account for non-stationary of a series. A stochastic process $\{Yt\}$ can be broadly classified as weak (covariance) or strong (strictly) stationary process.

{Yt} is said to be weakly (covariance) stationary if the first and second moments are time invariant, i.e., stochastic process $\{Yt\}$ has a mean that is independent of time, $\mathrm{E(Yt)} = \mathrm{E(Ys)} = \mu \;\; \forall t \neq s$ and $Cov(Yt, Ys) = cov(Yt + j, Ys + j) = \tau_{t,s} \quad \forall_{t,s,\ j\geq 1}$ which are independent of time. That is, the covariance does not depend on the magnitude of time t and time s; instead depend on the distance between t and s.

Formally, {Yt} is strictly stationary if the probabilistic behavior of every collection of values $\{\mathrm{Yt1}, \mathrm{Yt2}, \dots, \mathrm{Ytj}\}$ is identical to that of the time shifted set $\{\mathrm{Yt1} + \mathrm{k}, \mathrm{Yt2} + \mathrm{k}, \dots, \mathrm{Yt\,j} + \mathrm{k}\}$. Meaning, $P[Yt1 \leq C1, Yt2 \leq C2, \dots .. Yti \leq Ci] = P[Yt1 + j \leq C1, Yt2 + j \leq C2, \dots .. Yti + j \leq Ci]$ for $\mathrm{j} \in [0, \pm 1, \pm 2, \dots \dots ..]$.

3.4.1 Testing Stationarity

Stationarity of a time series is an important phenomenon because it can influence the behavior of the series unless it is detected with standard tests (unit root tests) and properly handled. In non stationary series, the effect of a shock never dies away and it leads to spurious regressions (i.e., one can regress completely unrelated series then find inflated t-ratio which suggests whether a coefficient of one variable is significant or not to explain the other and high R^2 which indicates how good one term is at predicting another) and forged results of standard tests.

3.4.2 Visual Inspection

The opening stride in the analysis of time series is usually to plot the data and obtain simple descriptive measures of the main property of the series via a visual inspection of the time series plot. This may reveal one or more of the following characteristics: seasonality, trends either in the mean level or the variance of the series, long- term cycles, and so on. If any such patterns are present, then these are signs of non-Stationarity.

3.4.3 Unit-Root Test

The development of unit root theory, initially proposed by Dickey and Fuller (1979, 1981) has spawned a generation of unit root research. Unit root theory is the cornerstone to the methodology used for testing the Stationarity or non-Stationarity of a time series. This study will apply the most commonly used and unarguably powerful unit root tests: Augmented Dickey- Fuller (ADF) test due to Dickey and Fuller (1979, 1981), and the Phillip-Perron (PP) test due to Phillips (1986) and Phillips and Perron (1988). These test procedures are developed for models with and without intercept terms as well as trend terms. The following discussion outlines the basic features of unit root tests (Hamilton, 1994).

Consider an AR (1) process: $Y_t = \rho(Y_{t-1}) + \acute{X}_t\,\delta + \varepsilon_t$ Where X_t are optional exogenous regressors which may consist of constant or a constant and trend, ρ and ε are parameters to be estimated and ε_t is assumed to be white noise.

If $|\,\rho\,| \geq 1$, Y is a non-stationary series and the variance of Y increases with time and approaches infinity. On the other hand, if, $|\,\rho| < 1$, Y is a stationary series. Thus, the hypothesis of (trend) Stationarity can be evaluated by testing whether the absolute value of ρ is strictly less than one. The hypotheses are: H0: The series are not stationary ($|\rho\,| = 1$) versus HA: The series are stationary ($|\rho| < 1$).

3.4.3.1 Augmented Dickey-Fuller (ADF) Unit-Root Test

The standard Dickey-Fuller test is conducted after subtracting Y_{t-1} from both side of the equation as follows.

$$Y_t - Y_{t-1} = \rho(Y_{t-1}) - Y_{t-1} + \acute{X}_t\,\delta + \varepsilon_t \qquad (3.3)$$

$\Delta Y_t = (\rho - 1)(Y_{t-1}) + \acute{X}_t\,\delta + \varepsilon_t$, which in turn can be expressed as $\Delta Y_t = \alpha(Y_{t-1}) + \acute{X}_t\,\delta + \varepsilon_t$ Where $\alpha = \rho - 1$.

The null and alternative hypothesis can then be stated as H0: α = 0 against HA: α < 0. The test statistic is the conventional t-ratio for α: $t_{\alpha} = \frac{\tilde{\alpha}}{(SE(\tilde{\alpha}))}$ Where $\tilde{\alpha}$ is the estimator of α and SE($\tilde{\alpha}$) is the standard error of $\tilde{\alpha}$.

The simple Dickey-Fuller unit root test described above is valid only if the series is an AR (1) process. If the series is correlated at higher order lags, the assumption of white noise disturbances $\boldsymbol{\varepsilon}$t is violated. The Augmented Dickey-Fuller (ADF) test constructs a parametric correction for higher-order correlation by assuming that the series follows an AR(P) process and adding lagged difference terms of the dependent variable Y to the left-hand side of the test regression: $\Delta Y_t = \alpha(Y_{t-1}) + \acute{X}_t\,\delta + \beta_1 \Delta Y_{t-1} + \beta_2 \Delta Y_{t-2} + \ldots + \beta_p \Delta Y_{t-p} + \cup_t$.

This augmented specification is then used to test for unit root using the t-ratio. An important result obtained by Fuller (1979) is that the asymptotic distribution of the t -ratio for β is independent of the number of lagged first differences included in the ADF regression. Moreover, while the assumption that Y follows an AR process may seem restrictive.

3.4.3.2 Phillips-Perron (PP) Unit-Root Test

Phillips and Perron (1988) propose an alternative (nonparametric) method of controlling serial correlation when testing for a unit root. The PP method estimates the non-augmented DF test and modifies the t-ratio of the α coefficient. So that serial correlation does not affect the asymptotic distribution of the test statistic. The PP test is based on the statistic

$$\hat{t}_{\alpha} = t_{\alpha(\frac{\gamma o}{f o})^{1/2}} \frac{T(fo-\gamma o)(SE(\hat{\alpha}))}{2 f_o^{1/2} S} \tag{3.4}$$

Where $\hat{\alpha}$ is the OLS estimate of α, t_{α} is the t- ratio of $\alpha, SE(\hat{\alpha})$ is the coefficient standard error and S is the standard error of the test regression. In addition, γ_o is consistent estimate of the error variance (calculated as $(T-K)\,{S^2}/{T}$, whre K is the number of regressors). The remaining term, $f_{o,}$ is an estimator of the residual spectrum at frequency zero.

3.5 Univariate Time Series Models

Time series is broadly defined as any series of measurements taken at different times. Although the ordering is usually through time, particularly in terms of some equally spaced time intervals. There are various objectives for studying time series. These include the understanding and description of the generating mechanism, the forecasting of future values, and optimal control of a system. The intrinsic nature of a time series is that its observations are dependent or correlated with the order of observation. Therefore, the order of observation matters when analyzing time series data. The development of inflation rate as a science has given rise to growing statistical applications on inflation information. For instance, time series analysis is used in order to evaluate the temporal behavior of inflation rate.

Linear time series analysis provides a natural framework to study the dynamic structure of a time series$\{\boldsymbol{y_t}\}$. A time series model was typically describe the path of a variable $\boldsymbol{y_t}$ in terms of contemporaneous (and perhaps lagged) factors,$\boldsymbol{x_t}$, disturbances (innovations), $\boldsymbol{\varepsilon_t}$, and its own past lags,$\boldsymbol{y_{t-1}}, \boldsymbol{y_{t-2}}, \cdots$.

A Univariate time-series model describes the behavior of a variable in terms of its own past values and disturbance term. Thus, the general expression for the Univariate time series model is $\boldsymbol{y_t = f(y_{t-1}, y_{t-2}, \dots, \varepsilon_t)}$. To make the above equation operational, three things must be specified: the functional form, the number of lags, and a structure for the disturbance term.

3.5.1 ARIMA Models

Autoregressive (AR) models: Autoregressive models are based on the idea that the current value of the series, Y_t, can be explained as a function of p past values, y_{t-1}, y_{t-2}. . . y_{t-p}, where p determines the number of steps into the past needed to forecast the current value.

An autoregressive model of order p, abbreviated AR (p), can be written as: highest order p is referred to as the order of the model.

$$y_t = \boldsymbol{\mu} + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p y_{t-p} + \varepsilon_t \tag{3.5}$$

Where $\{\varepsilon_t\}$ is white noise, i.e., $\{\varepsilon_t\} \sim$ WN (0, $\boldsymbol{\sigma_e^2}$), and ε_t is uncorrelated with Ys for each s < t.

Since AR is autoregressive, writing equation above in terms of the lag operator L, the above equation given as shown below $y_t = (\varphi_1 L + \varphi_2 L^2 + \cdots + \varphi_p L^P)\, y_t + \varepsilon_t$.

Moving average (MA) Models: As an alternative to the autoregressive representation in which the Y_t on the left-hand side of the equation are assumed to be combined linearly, the moving average model of order q, abbreviated as MA (q), assumes the white noise (w_t) *on* the right-hand side of the defining equation are combined linearly to form the observed data. A series is said to follow a moving average process of order q, or simply MA (q) process if

$$yt = \mu + \varepsilon_t - \theta_1 \varepsilon t - 1 - \theta_2 \varepsilon t - 2 - \cdots - \theta q \varepsilon t - q \tag{3.6}$$

Where θ_1, θ_2, …, θ_q are the MA parameters. MA (q) models immediately define stationary; every MA process of finite order is stationary. In order to preserve a unique representation, usually the requirement is imposed that all roots of θ (B) = 1- θ_1 B - θ_2 B^2 - ···- θ_q B^q = 0 are greater than one in absolute value. If all roots of θ (B) =0 lie outside the unit circle, the MA process has an autoregressive representation of generally infinite order $\theta_j y_{t-j} = w_t$ with $|\theta_j| < \infty$.

MA process as with an infinite order autoregressive representation are said to be invertible. A characteristics feature of MA (q) is that their ACF, $\boldsymbol{\theta}$ becomes statistically insignificant after j=q. The property of the ACF should be reflected in the correlogram, which should 'cut off' after lag q. The PACF converges to zero geometrically.

Autoregressive –Moving average (ARMA): We now proceed with the general development of autoregressive, moving average, and mixed autoregressive moving average (ARMA), models for stationary time series. In most case, it is best to develop a mixed autoregressive moving average model when building a stochastic model to represent a stationary time series. The order of an ARMA model is expressed in terms of both *p* and *q*. The model parameters relate to what happens in period *t* to both the past values and the random errors that occurred in past time periods. A general $\boldsymbol{ARMA}\ (\boldsymbol{p}, \boldsymbol{q})$ model can be written as follow:

$$\boldsymbol{yt = \mu + \varphi 1 yt - 1 + \varphi 2 yt - 2 + \cdots, + \varphi p yt - p + \varepsilon t - \theta 1 \varepsilon t - 1 - \theta 2 \varepsilon t - 2 - \cdots - \theta q \varepsilon t - q} \qquad (3.7)$$

By using backward shift operator B to obtain $\boldsymbol{\varphi}(\mathrm{B})\boldsymbol{y_t} = \boldsymbol{\theta}(\mathrm{B})\boldsymbol{\varepsilon_t}$

The ARMA model is stable i.e., it has a stationary 'solution' if all roots of $\boldsymbol{\varphi}$(B =0 are less than one in absolute value. The representation is unique if all roots of $\boldsymbol{\theta}$(B) =0 lie outside the unit circle and (*B*) not have common roots.

Stable ARMA models always have an infinite order MA representation. If all roots of (*B*) are larger than one in absolute value, it has an infinite order AR representation. The process is invertible only when the roots of $\boldsymbol{\theta}(\boldsymbol{B})$ lie outside the unit circle .Furthermore, a process is said to be causal when the roots of $\boldsymbol{\varphi}(\boldsymbol{B})$ lie outside the unit circle.

To have ARMA (*p, q)* model, both ACF and PACF should show a pattern of decaying to zero. The autocorrelation of an ARMA (*p, q*) process is determined at greater lags by the AR (*p)* part of the process as the effect of the MA part dies out. Thus, eventually the ACF consists of mixed damped exponentials and sine terms. Similarly, the partial autocorrelation of an ARMA *(p, q*) process is determined at greater lags by the MA (*q*) part of the process. Thus, eventually the partial autocorrelation function will also consist of a mixture of damped exponentials and sine waves.

Autoregressive Integrated Moving Averages (ARIMA) Models: Autoregressive integrated moving average (ARIMA) models are specific subset of Univariate modeling, in which a time series is expressed in terms of past values of itself (the autoregressive component) plus current and lagged values of a 'white noise' error term (the moving average component). ARIMA models are Univariate models that consist of an autoregressive polynomial, degree of differencing (d), and a moving average polynomial.

A process (Y_t) is said to be an autoregressive integrated moving average process, denoted by ARIMA (p, d, q) if it can be written as: $(B)\ ^{d}y_t= (B)\ w_t$ Where, $B^d= (1-B)^d$ with $B^d y_t$ and d^{th} consecutive differencing.

If E $(\nabla^d y_t) = 0$, we write the model as $(B)^d y_t= \alpha + (B)\ w_t$ where α is a parameter related to the mean of the process $\{Y_t\}$, by $\alpha = \mu\ (1-_{1}-\ldots-_{p})$ and this process is called a white noise process, that is, a sequence of uncorrelated random variables from a fixed distribution (often Gaussian) with constant mean $E(Y_t)= \mu$ usually assumed to be "zero" and constant variance. If d=0, it is called ARMA (p, q) model while when d=0 and q=0, it is referred to as autoregressive of order *p* model and denoted by AR *(p)*. When p=0 and d=0, it is called Moving Average of order *q* model, and is denoted by MA (q).

Building ARIMA Models: To identify an ARIMA model for a particular time series data, Box and Jenkins (1976) proposed a methodology that consists of four phases: (i) Model identification (ii) Diagnostic checking for the identified model (iii) Estimation of model parameters and (iv) Application of the model (i.e. forecasting).

i) Model Identification: The purpose of the identification stage is to determine the differencing required achieving Stationarity and also the order of both the seasonal and the non- seasonal AR and MA operators for the residual series. There are a number of identification methods proposed in the literature. The autocorrelations function (ACF) and the partial autocorrelation functions (PACF) are the two most useful tools in any attempt at time series model identification.

Autocorrelation Function (ACF*)*: The sample ACF (r_k) measures the amount of linear dependence between observations in a time series that are separated by a lag k. To use the ACF in model identification, estimate r_k and then plot r_k series against lag k up to a maximum lag of about five times the seasonality interval and this should be less than to one fourth of the series under study.

Table 3.1: Behavior of ACF and PACF for ARMA models

	AR(P)	MA(q)	ARMA(P, q)
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

Table 3.2: Behaviors of ACF and PACF for pure SARMA model

	$AR(P)_S$	$MA(Q)_S$	$ARMA(P, Q)_S$
ACF	Tails off after lags k_s	Cuts off after lag Q_S	Tails off after lag k_S
PACF	Cuts off after lags P_S	Tails after lags k_S	Tails off at lags P_S

Partial Autocorrelation Function (PACF)*:* Partial autocorrelation function can also be used for determining the possible order of seasonal autoregressive, non-seasonal autoregressive, moving average and seasonal moving average that should be incorporated in the model by the help of Table-3.1 and 3.2 above. When the process is a pure SARIMA (*p, d, 0) x (P, D, 0*) model, cuts off and is not significantly different from zero after lag p + SP. If damps out at lags that are multiples of s, this suggests that the incorporation of a seasonal moving average component in to the model. The failure of the partial autocorrelation function to truncate at other lags may imply that a non seasonal MA term is required. To obtain an estimate for partial autocorrelations (PACF) at lag *k*, we can employ successive autoregressive estimation procedure. The first step is to model the Y_t series by finite autoregressive models of order K given by $Y_t = \boldsymbol{\mu} + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \cdots, + \varphi_p y_{t-p}$ Where φ_k is the k^{th} autoregressive coefficient and $k = 1, 2, ..., K$. Estimate of these coefficients by ordinary least squares or maximum likelihood estimation method gives the k^{th}- sample partial autocorrelation.

ii) Diagnostic Checking: After fitting a provisional time series model, we can assess its adequacy in various ways. The usual approach is to extract from the data, a sequence of residuals to correspond to the underlying, last unobservable, white noise sequence, and to check that the statistical properties of these residuals are indeed consistent with white noise. Most diagnostic tests deal with the residual assumptions in order to determine whether the residuals from fitted model are independent, have a constant variance, and are normally distributed. Several diagnostic statistics and plots of the residuals can be used to examine the goodness of fit of the tentative model to the historical data.

The first approaches that can be used to evaluate the adequacy of a model are the plot of the errors over time. If visual inspections of the errors reveal that they are randomly distributed over time, then we have a good model. The autocorrelations function (ACF) of the series can also be used to examine whether the residual of the fitted model is white noise or not.

iii) Parameter Estimation: After choosing the most appropriate model, the model parameters are estimated by using several estimation procedures. The estimation-stage results were is used to check: (i) parameter estimates (ii) the appropriateness of coefficient estimates which includes the statistical significance of estimated coefficient and standard error and correlation matrix.

In maximum likelihood methods, the likelihood function is maximized in order to obtain the parameter estimates. The likelihood of a set of data is the probability of obtaining that particular set of data, given its distribution. Akaike (1978) introduced a criterion called Akaike Information Criterion (AIC) in the literature. The AIC is a mathematical selection criterion of model building. When there are several competing models to choose from, select the model that gives the minimum of the AIC defined as AIC = –2ln (L) + 2k Where ln (L) denotes the maximum log likelihood estimator for the error variance and k is the number of seasonal and non-seasonal autoregressive and moving average parameters to be estimated in the model, k = p + q + P+ Q + 1 and n is the number of observations. The optimal order of the model is chosen by the value of k, which is a function of p and q, P and Q so that the value of k yielding the minimum AIC specifies the best model. We need to select the model that has fulfilled all the diagnostic checks and has as few parameters as possible in terms of parsimony.

3.5.2 GARCH Models

One of the most important issues before applying the GARCH models is to first examine the residuals of the series of a given data for evidence of heteroscedasticity. The LM test for the squared residuals of the fitted model proposed by Engle (1982) was conducted for testing heteroscedasticity (ARCH effect).

The autoregressive AR (p), moving average MA (q) and autoregressive moving average ARMA(p, q) models are applicable when the innovation term $\boldsymbol{\varepsilon_t}$ maintains a constant variance (homoscedasticity). If the error term $\boldsymbol{\varepsilon_t}$ is conditionally heteroskedastic, then ARCH/GARCH models are applicable to model the conditional variance of the innovation term. If the model involves just one lag of $\boldsymbol{\varepsilon^2}_{\boldsymbol{t-1}}$, it is known as ARCH(1) model. The generalized ARCH (GARCH) model is a generalization of the ARCH model in that it includes lagged variances in the conditional variance equation. GARCH models have the advantage of capturing long lags in the shocks by using fewer parameters than ARCH models.

The GARCH (1, 1) model is defined as:

$$\boldsymbol{\sigma^2}_{\boldsymbol{t}} = \boldsymbol{\alpha_o} + \boldsymbol{\alpha_1}\boldsymbol{\varepsilon^2}_{\boldsymbol{t-1}} + \boldsymbol{\beta_1}\boldsymbol{\sigma^2}_{\boldsymbol{t-1}} \qquad (\mathbf{3.8})$$

The GARCH model with q number of lagged variance term and p number of lagged squared error terms denoted as GARCH (p, q),and can be written as:

$$\boldsymbol{\sigma^2}_t = \boldsymbol{\alpha_o} + \sum_{i=1}^{p} \boldsymbol{\alpha_i}\, \boldsymbol{\varepsilon^2}_{t-i} + \sum_{j=1}^{q} \boldsymbol{\beta_j}\, \boldsymbol{\sigma^2}_{t-j} \ , \boldsymbol{\alpha_o} > \mathbf{0}, \boldsymbol{\alpha_i} \geq \mathbf{0}, \boldsymbol{\beta_j} \geq \mathbf{0} \qquad (\mathbf{3.9})$$

Where the volatility term $\boldsymbol{\sigma^2}_{t-j}$ denote the variance and $\boldsymbol{j}$ represents the number of lags, and theterm $\boldsymbol{\varepsilon^2}_{t-i}$is the squared error for the period $\boldsymbol{t-i}$.

We can write the above equation more compactly:$\boldsymbol{\sigma^2}_t = \boldsymbol{\omega} + \boldsymbol{\alpha}(\boldsymbol{L})\boldsymbol{\varepsilon^2}_t + \boldsymbol{\beta}(\boldsymbol{L})\boldsymbol{\sigma^2}_t$ Where $\boldsymbol{\alpha}(\boldsymbol{L}) = \boldsymbol{\alpha_1 L} + \cdots + \boldsymbol{\alpha_p L^P}\, \boldsymbol{and}\ \boldsymbol{\beta}(\boldsymbol{L}) = \boldsymbol{\beta_1 L} + \cdots + \boldsymbol{\beta_q L^q}$.

To ensure that the conditional variance is well defined in a GARCH (p, q) model all the coefficients in the corresponding linear ARCH (∞) should be positive rewriting the GARCH (p, q) modeled as an ARCH (∞):

$$\boldsymbol{\sigma^2}_t = \left(\mathbf{1} - \sum_{j=1}^{q} \boldsymbol{\beta_j L_j}\right)^{-\mathbf{1}} + \boldsymbol{\omega} + \sum_{i=1}^{p} \boldsymbol{\alpha_i}\, \boldsymbol{L_i} = \boldsymbol{\omega}^* + \sum_{k=1}^{\infty} \boldsymbol{\phi_k}\, \boldsymbol{\varepsilon^2}_{t-k-1} \qquad (3.10)$$

$\boldsymbol{\sigma^2}_t \geq \mathbf{0}$, if $\boldsymbol{\omega}^* \geq \mathbf{0}$and all $\boldsymbol{\phi_k} \geq \mathbf{0}$.The non-negativity of $\boldsymbol{\omega}^*$and $\boldsymbol{\phi_k}$is also a necessary condition for the non-negativity of $\boldsymbol{\sigma^2}_t$(Rossi, 2004).

The coefficients $\boldsymbol{\alpha}$ of lagged squared returns are interpreted as how fast the model react to, for example, market events, the $\boldsymbol{\beta}$ coefficients of lagged conditional variance determines the degree of persistence in the volatility. A large value of $\boldsymbol{\alpha}$ indicates that the conditional variance decays slowly, and that the volatility is persistent. On the other hand, if the $\boldsymbol{\alpha}$ value is relatively higher than the $\boldsymbol{\beta}$ value, then the volatility is more extreme. The sum,$\sum \boldsymbol{\alpha_i} + \sum \boldsymbol{\beta_i}$, measures the persistence of volatility. Any shock to volatility is permanent if, $\sum \boldsymbol{\alpha_i} + \sum \boldsymbol{\beta_i} = \mathbf{1}$ that is, past volatility is significant in predicting future volatility. Volatility is explosive if$\sum \boldsymbol{\alpha_i} + \sum \boldsymbol{\beta_i} > \mathbf{1}$, meaning, a shock to volatility in one period was lead to even greater volatility in the next period. If$\sum \boldsymbol{\alpha_i} + \sum \boldsymbol{\beta_i} < \mathbf{1}$, volatility is neither permanent nor explosive and past volatility prediction is not as such important(Sinha, 2007). The sum $\sum \boldsymbol{\alpha_i} + \sum \boldsymbol{\beta_i}$ should be less than one for the above GARCH (p, *q*) to be stationary. Making estimations that include this contagion effect requires an extended model.

Normality Test: One of the most commonly applied tests for normality is the Jarque-Bera test. The null hypothesis for Jarque-Bera test is the observations are normal and reject the null hypothesis when the Jarque-Bera test is greater than a chi-square distribution with two degree of freedom.

The standardized third and fourth moments of a distribution are known as its skewness and kurtosis. Skewness measures the extent to which a distribution is not symmetric about its mean value whereas kurtosis measures the peakedness or flatness of the distribution of the series.

H_o: the distribution is normal vs H_1: not nomally distributed

$$JB = n\left[\frac{skewness^2}{6} + \frac{(kurtosis-3)^2}{24}\right] \tag{3.11}$$

Where $skewness = \frac{\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^2\right)^{\frac{3}{2}}}$ and $kurtosis = \frac{\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^2\right)^{2}}$

If $JB > x^2{}_{(\alpha,2)}$ then the decision rejects the null hypothesis which means that the data do not follow the normal distribution.

Basic Properties of GARCH Model

i. Uniqueness and Stationarity: According to(Bougerol, 1992), a necessary and sufficient condition for the GARCH equation to have a unique and stationary solution is

$$\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j < 1$$

ii. Mean Zero: In any model in which σ_t is measurable with respect to ψ_{t-1} (which is the case in GARCH model specified prior), the mean of ε_t is zero. That is,$E[\varepsilon_t] = 0$

iii. Lack of Serial correlation: In GARCH even though it is conditional, heteroscedasticity is inevitable. Hence, the auto covariance of any pair of elements from the series is expected to be zero resulting in lack of serial correlation. For $k > 0, \varepsilon_t$, is not correlated with ε_{t+k} : $E[\varepsilon_t, \varepsilon_{t+k}] = 0$

iv. Unconditional variance: In order to compute $E[\varepsilon_t{}^2]$, it is useful to consider an alternative representation of $\varepsilon_t{}^2$. First define the sequence, $\sigma_t{}^2 = \varepsilon_t{}^2 - v_t$.

Model Selection of ARCH/GARCH Models: When there are multiple adequate models, the selection criterion is normally based on the likelihood function and the number of free parameters of the fitted model or on forecast errors calculated from out of-sample forecast.

In this study, we will apply the Akaike information criterion (AIC) and Bayesian information criteria (BIC) for the model selection purpose. By definition *AIC* = –2ln (*L*) + 2*k* and *BIC* = –2*ln* (*L*) + *k ln* (*T*), where *L* is the maximized value of the likelihood function and k is the number of (free) parameters in the model (i.e., *k* = *p* + *q*+1). Given a set of candidate models, the model with the minimum AIC and BIC value is taken as the best-fit model (Brockwell & Davis, 2009; Burnham & Anderson, 2002).

Parameter Estimation: In order to estimate the unknown parameters of the GARCH family models, the maximum likelihood (ML) method is employed with various distributional assumptions for the error terms. In this study, three distributions of the error terms are considered, namely, the normal, student-t and GED.

Model Checking: For a properly specified GARCH model, the standardized residuals should be identically and independently distributed as standard normal, even if the student-t and generalized error distribution (GED) are assumed (Tsay, 2010). We apply the Breusch–Godfrey test (Godfrey, 1996) or Ljung_ Box goodness of fit test for serial correlation and Jarque-Bera test (Jarque and Bera, 1987) or q-q plot to test for normality.

3.6 Forecasting

Forecasting is an important application of time series analysis which defined as Process of predicting a future event. The series of conditional correlation forecasts for the period of January 2000 to December 2018 is compared to the realized values for the same period. Starting with an initial estimation from period T a forecast is made for the following months (T + 1). The estimation window is moved one month and another forecast is made for (T+ 2).

More generally expressed, the estimation period reaches between the first observation at time $\boldsymbol{t}$ and the last observation of the first period at time $\boldsymbol{T}$. This means that the length of the period $\boldsymbol{t}$ is $\Delta \boldsymbol{t} = \boldsymbol{T} - \boldsymbol{t}$. The last observation of the evaluation period is denoted T *. The forecast loop continues until the starting point $\boldsymbol{t}$ and the ending point $\boldsymbol{T}$ of the first period both have moved a number of steps equal to $\boldsymbol{T} * -\boldsymbol{T} - \mathbf{1}$, which results in a series of $\boldsymbol{T} * -(\boldsymbol{T} + \mathbf{1})$ forecasts.

Forecasts with AR (1) Process: For this process, it holds that $Y_t = \mu + \phi_1 Y_{t-1} + \varepsilon_t$, with $|\phi_1| < 1$. The optimal k-step-forecast is the conditional mean of Y_{T+k}, i.e.

$E[Y_{T+k}] = E[\hat{\mu} + \hat{\phi}_1 Y_{T+k-1} + \varepsilon_{t+k}] = \hat{\mu} + \hat{\phi}_1 E[Y_{T+k-1}] + 0$. We get the following first order difference equation for the prediction function which can be solved recursively:

$E[Y_{T+1}] = \hat{Y}_{T+1} = E[\hat{\mu} + \hat{\phi}_1 Y_T + \varepsilon_{T+1}] = \hat{\mu} + \hat{\phi}_1 E[Y_T] = \hat{\mu} + \hat{\phi}_1 Y_T$.

$E[Y_{T+2}] = \hat{Y}_{T+2} = E[\hat{\mu} + \hat{\phi}_1 Y_{T+1} + \varepsilon_{T+2}] = \hat{\mu} + \hat{\phi}_1 E[Y_{T+1}] = \hat{\mu} + \hat{\phi}_1 \hat{Y}_{T+1}$.

In general, $E[Y_{T+k}] = \hat{Y}_{T+k} = \hat{\mu} + \hat{\phi}_1 \hat{Y}_{T+k-1}$, for $k \geq 1$.

Forecasts with GARCH (1, 1) Process: To forecast the conditional variances of a GARCH (1,1) process, we get the optimal forecasts for the period t +k with k > 0 as $\sigma^2_{t+k|t} = E[\varepsilon^2_{t+k|I_t}]$ results $\varepsilon^2_{t+k} = \alpha_o + (\alpha + \beta)\varepsilon^2_{t+k-1} + v_{t+k} - \beta v_{t+k-1}$. Thus, for the one step ahead forecast we get

$$\sigma^2_{t+1|t} = E[\varepsilon^2_{t+k|I_t}] = \alpha_o + (\alpha + \beta)\varepsilon^2_t - \beta v_t = \alpha_o + (\alpha)\varepsilon^2_t + \beta\sigma^2_t$$

$$\sigma_{t+k|t}^2 = \alpha_o \frac{1-(\alpha+\beta)^{k-1}}{1-\alpha-\beta} + (\alpha+\beta)^{k-1}\beta\sigma_{t+1|t}^2$$

If the forecast horizon grows above all limits, if $\alpha + \beta < 1$, we have $\lim_{t\to\infty} \sigma_{t+k|t}^2 = \frac{\alpha_o}{1-\alpha-\beta} = V(\varepsilon_t)$ thus, the conditional variance of converges towards its unconditional variance. This is no longer true for an IGARCH process. In this case we have $\alpha + \beta = 1$, implying that the conditional variance grows linearly with the forecast horizon. The conditional variance for period t, which defines the information set for the forecasts, has a permanent influence.

CHAPTER FOUR

4 . RESULTS AND DISCUSSION

For this study, the time series data for monthly overall Inflation based on consumer price Index (CPI) were used at the country level. The major source of the dataset was Central Statistical Agency (CSA) which is reported in monthly bases. The data spans the time period observed from January 2001 to December 2018 with a total of 216 observations.

The analysis of data series has further been organized into Preliminary analysis, Model fitting, Model diagnostic, predicting in-sample forecast and evaluating the accuracy of the forecast for the period of January 2001 to December 2018. The statistical software package used for most of the analysis in the study was STATA 12.

4.1 Descriptive Statistics

The opening stride in the analysis of time series is usually to obtain sample descriptive measures of the main property of the series via a visual inspection. Regardless of which technique is used, the first step in any time series analysis is to plot the observed values against time. This may reveal one or more of the following characteristics: seasonality, trends either in the mean level or the variance of the series, long- term cycles, and so on. If any such patterns are present, then these are signs of non Stationarity.

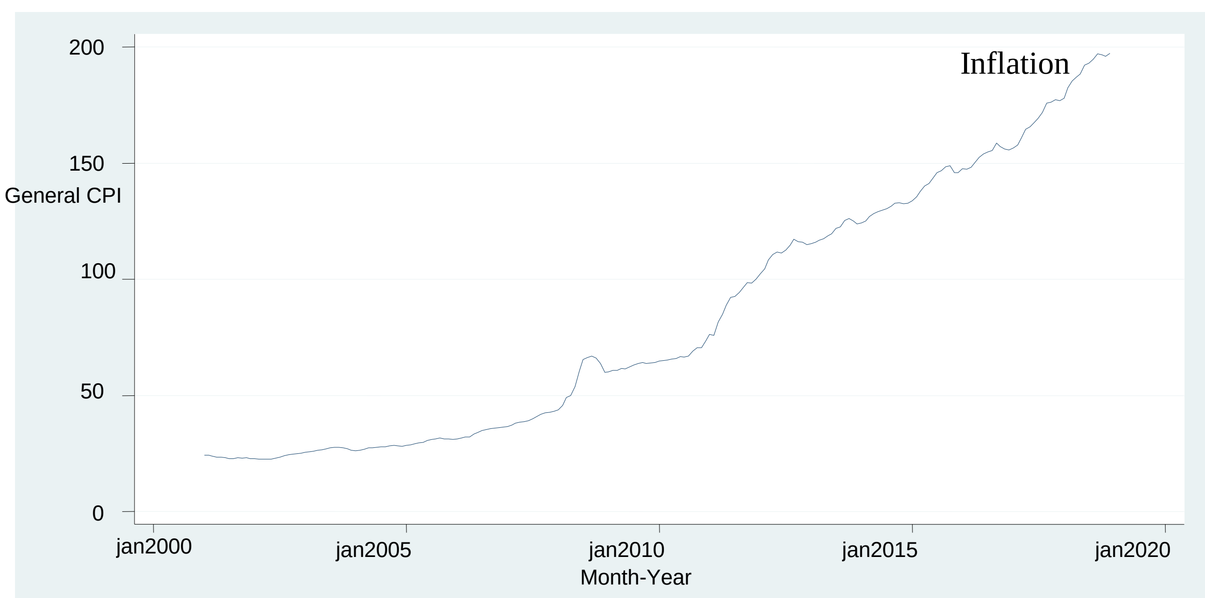

Figure 4.1: General trend of Ethiopia's overall inflation from January 2001- December 2018

The dominance of the upward trend of overall inflation plot in figure 4.1 indicates that the dramatic increase in price of goods and services continues rising to the end of this study period.

This implies that the mean may vary over time and the variance may also unstable for overall inflation series based on general CPI. Now, it is important to investigate the possible transformation of the series to be stationary as far as possible.

In practice, econometricians usually transform financial prices into log return forms. This is because often log return series are found to be stationary such that analysis is possible (Brooks, 2014). Thus, the natural logarithmic return series $\mathbf{r_t = log(\frac{y_t}{y_{t-1}})}$ were computed for monthly consumer price index of the series $\mathbf{y_t}$.

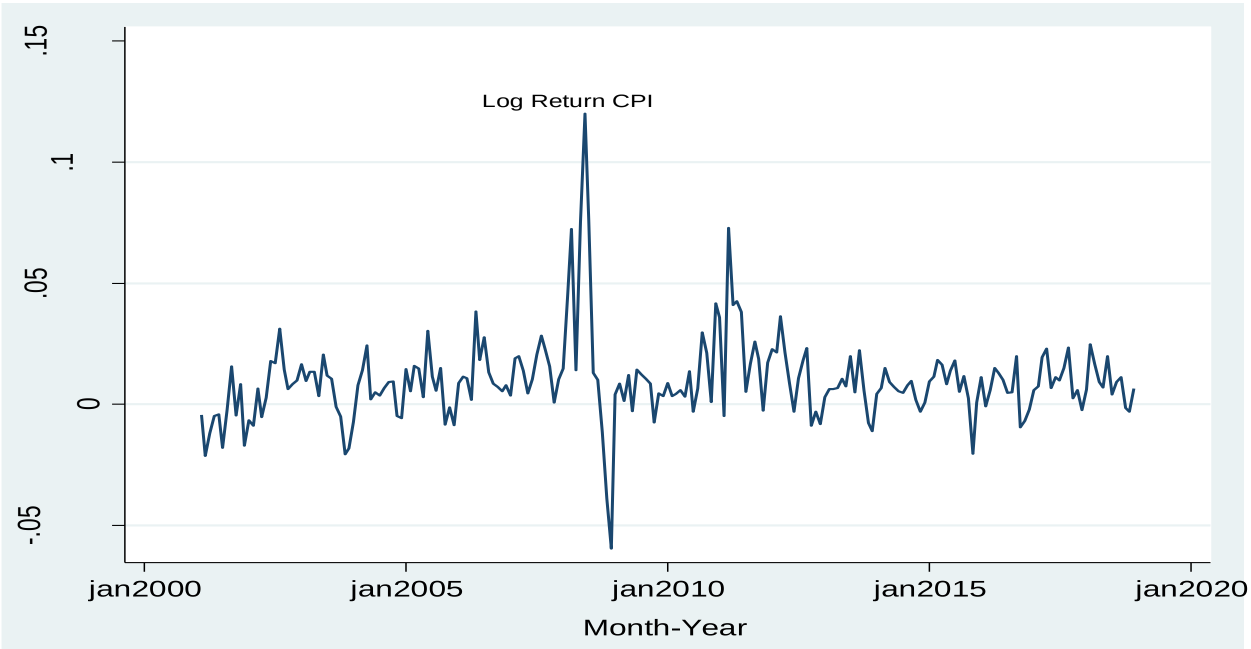

Figure 4.2: Time series plot for log returns of monthly Inflation from January 2001- December 2018

The time series plot in Figure 4.2 seems to be stable after the log returns of the Inflation series. From the log returns series, volatility clustering can be clearly observed i.e. where large changes are followed by other large changes of either sign and small changes are followed by small changes. This may be an indication of the residuals or innovation terms have been conditionally heteroskedastic and can be represented by GARCH family models. Furthermore, the mean reverting (the series tend to remain around a certain value) property can also be seen clearly where the log returns revolve around a constant.

4.2 Test of Stationarity and Features of Log Return Series

Many of the various methods in time series analysis assume that the data with respect to the mean, variance and autocorrelation structure do not change overtime. Hence, the first logical step to make inference is checking the Stationarity of the time series data.

The stationary of the series was tested by using an Augmented Dickey Fuller (ADF) test. The hypothesis to be tested is Ho: the series is non-stationary against H1: the series is stationary. The null hypothesis of the test of non-stationary (unit root) is rejected if the absolute value of the t-statistic is greater than the critical value or p-value is less than a given level of significance.

Table 4.1: ADF Unit Root Test for General CPI and Log returns CPI Result

Variables	Test statistics	Critical values			P-value
		1%	5%	10%	
General CPI	-1.836	-4.002	-3.435	-3.135	0.6870
Log Returns CPI	-7.925	-2.344	-1.652	-1.286	0.00

Since the p-value is greater than the level of significance, the null hypothesis of the unit root test is accepted at 1%, 5% and 10% level of significance. Hence, we do not reject the null hypothesis and indicate that, unit root exist and as a result the general CPI data for inflation is nonstationary.

The log returns from table 4.1 is indicated that the p- value of the test statistic is less than 0.01. Hence, we accept the alternative hypothesis and conclude that the log returns series is stationary.

Table 4.2 shows the summary statistics for General CPI and log returns CPI. It can be seen that the average monthly inflation in Ethiopia based on general CPI is 84.33 with the minimum and maximum values of 22.41 and 197.4 respectively. Table 4.2 also revealed that the average monthly log-returns CPI was 0.0097 with the minimum and maximum values of -0.060 and 0.120 respectively. The estimated unconditional standard deviation of the log returns CPI is also 0.0174.

Table 4.2. General Summary Statistics Results

Variable	Obs.	Mean	Minimum	Maximum	Std. Dev.	Skewness	Kurtosis
General CPI	216	83.33	22.41	197.40	54.85	0.49	1.85
Log Returns	215	0.0097439	-0.060	0.120	0.0174	1.0881	7.7051

Furthermore, literatures on financial time series indicate that the distribution of returns exhibits skewness, leptokurtosis and volatility (Cornew et al., 1984). Hence, Table 4.2 shows that monthly log returns CPI has positively skewed distribution and a single peak at the mean (leptokurtic). This may indicate that the normality assumption could be violated for the log returns CPI series.

4.3 Model Identification

This focuses on two classes of models: ARMA and GARCH. This part was providing a short explanation of what these models and their respective properties are. For these definitions of general models and later in chapter three, when explaining the general functions of the applied tests and measures the $\{\boldsymbol{yt}, \boldsymbol{yt-1}, \boldsymbol{yt-2}, \ \dots, \boldsymbol{yT}\}$ realization of the generic time series $\boldsymbol{yt}$ is used. It should not be confused with $\boldsymbol{rt}$, which is defined as the logarithmic price return CPI.

4.3.1 Specification of Mean Equation

To specify a mean equation of the series, it is better to compare some$\boldsymbol{AR}(\boldsymbol{p})$, $\boldsymbol{MA}(\boldsymbol{q})$ and $\boldsymbol{ARMA}(\boldsymbol{p}, \boldsymbol{q})$ models and takes the best one.

The autoregressive and moving-average orders are selected by examining the sample autocorrelation and partial autocorrelation functions. To use the sample autocorrelation and partial autocorrelation functions for tentative model parameters identifications, for ACF and PACF the values of p and q. If a time series is characterized by seasonal fluctuations, then the correlogram would also exhibit oscillations at the same frequency, means if $\boldsymbol{rt}$ follows sinusoidal patterns, then does so the autocorrelation function ($\boldsymbol{r_k}$).

For an AR series, the sample partial autocorrelation function (PACF) cuts off at lag $\boldsymbol{p}$.The sample PACFs of the log return series has single significant positive peak at lag 1. This suggests that $\boldsymbol{AR}(\boldsymbol{1})$ model could be identified. Similarly for a $\boldsymbol{MA}(\boldsymbol{q})$ model, it is seen that the log return series has significant ACFs at lags 1, 2 and 3 with exponentially decreasing pattern. Graphs of sample ACFs and PACFs are also shown figure 2 and figure 3 from APPENDIX B.

Computing different model selection statistics in order to suggest the best of all the ARMA alternatives at the in-sample stage for log return series is useful. Hence, the two statistics, Akaike Information Criterion (AIC) and Bayesian Information or Schwarz Criterion (BIC) are used. So that different combinations of the $\boldsymbol{AR}(\boldsymbol{P})$ and $\boldsymbol{MA}(\boldsymbol{q})$ models were done for the log return series and then a model with minimizing the AIC and BIC values was selected as shown in Table 1 from Appendix A. The AIC and BIC are ranked relative to the different ARMA models. The two ranks (rank of AIC and rank of BIC) for each model are added to give a single sum. Finally, the lower the rank sum is the better the model (see Table 4.3).

Hence, the estimated mean model for in-sample results have been achieved by ARMA (1, 0) or AR (1) model with the minimum AIC and BIC rank sum value for the log return series. Therefore, the conditional mean equation for the series is given as follows:

$$\boldsymbol{r_t = \mu + \varphi_1 r_{t-1} + \varepsilon_t} \tag{4.1}$$

Where $\boldsymbol{\mu}$ is the constant and $\boldsymbol{\varphi_1}$ is the coefficient polynomial of the autoregressive order one process; $\mathbf{r_t}$ is the log return series at time $\mathbf{t}$, and $\boldsymbol{\varepsilon_t}$ is the random disturbance term.

Table 4.3: Model selection criteria for ARMA (p, q)

Model	AIC		BIC		Rank
	Values	Rank	Values	Rank	sum
ARMA(1,0)	-1201.5	4	-1191.388	1	5
ARMA(2,0)	-1199.728	6	-1186.245	3	9
ARMA(1,1)	-1199.827	5	-1186.344	2	7
ARMA(1,2)	-1198.487	7	-1181.634	4	11
ARMA(1,3)	-1201.72	3	-1181.496	5	8
ARMA(2,3)	-1203.563	1	-1179.968	6	7
ARMA(3,3)	-1202.334	2	-1175.396	7	9

4.3.2 Test for ARCH Effects

The presence of ARCH effect was tested using the squared standardized residuals from the selected $\boldsymbol{AR(1)}$ model. Considering the chosen mean model, the squared Standardized residual plot can be an initial insight to judge the heteroskedastic characteristics of the disturbance term.

Figure 4 from APPENDIX B provides that the squared standardized residual plot of the series generated from the $\boldsymbol{AR(1)}$ model. From the figure, there exist a prolonged period of low volatility and prolonged period of high volatility in the series. This suggests that the squared residuals are conditionally heteroskedastic and can be represented by GARCH family models.

From the Ljung box test, the resulting p-values from the Ljung box test for squared log returns at different lags suggest that the ARCH effects are significant. Thus, the ARCH LM (Ljung- box test) test revealed that there is a significant ARCH effect in the residuals and GARCH family models are appropriate to capture the volatility of the series.

Table 4.4: Summary Statistics for LM test of Log Returns at different lags

lags(p)	$\chi^2(\mathbf{p})$	Df	P-value
1	49.397	1	0.0000
2	52.271	2	0.0000
3	62.358	3	0.0000
4	62.935	4	0.0000
5	69.987	5	0.0000
6	70.048	6	0.0000
7	72.644	7	0.0000
8	72.451	8	0.0000
9	72.293	9	0.0000
10	72.042	10	0.0000
11	72.320	11	0.0000
12	72.350	12	0.0000

H0: no ARCH effects vs. H1: ARCH (p) disturbance

4.3.3 GARCH Model Specification

If the presence of ARCH effect is found to be significant, then the PACFs are helpful to determine the order of an ARCH model as used in the AR model. But, to reduce the computational burden, a GARCH model with low lags can be helpful and the GARCH (1, 1) model is the most convenient specification (Bollerslev and Taylor, 1992). Thus, the GARCH (1, 1) model was compared to higher-order symmetric GARCH models based on AIC and BIC values and find that it is a more parsimonious model representation of the conditional variance process.

Different p and q values are compared with GARCH (1, 1) model using AIC and BIC to estimate and evaluate the forecasts of the competing GARCH models. Finally, a GARCH model for different distributions like normal distribution, student's t-distribution and generalized error distribution are evaluated and the parameters are estimated for the chosen GARCH model are presented (Geda et al.).

Note that the AIC and BIC of the GARCH models are obtained by estimating the mean and variance equations simultaneously. The row ranks method can be used in the ranking of the various GARCH models based on the two statistics, AIC and BIC. As a result, the appropriate model will be a model which contains the lower the rank sum. In Table 2 from APPENDIX A, GARCH (1, 1) model under Student (t=8) and generalized error distributions were chosen for the conditional variance process and can be expressed as:

$$\boldsymbol{\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2} \tag{4.2}$$

Where $\boldsymbol{\sigma_t^2}$ is the conditional variance of the residuals, $\boldsymbol{\varepsilon_t = \sqrt{\sigma_t^2}\, z_t}$ and $\boldsymbol{z_t \sim \mathrm{iid}\ N(0, 1)}$

4.3.4 Estimation of the model parameters

When in modeling variance, the estimated variance must be non-negative; however there is no easy rule that ensures non-negativity for the general GARCH (p, q) process. But (Nelson and Cao, 1992) has given easy checkable condition for GARCH (1, 1) model. The non-negativity condition for GARCH (1, 1) model: $\boldsymbol{\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2}$.

The estimated parameters of the model to be non-negative, the following condition should be satisfied such that $\boldsymbol{\alpha_0 > 0, \alpha_1 \geq 0 \text{ and } \beta_1 \geq 0}$. So that, looking the parameter estimates of ARCH and GARCH terms from Table 4.5 satisfy the non-negativity rule as described above.

The estimated parameter coefficients of the log return series as shown in Table 4.5 were done assuming that the error terms follow student's t-distribution ($\boldsymbol{df = 8}$) and generalized error distribution (GED). The sufficient condition for the variance to be positive definite are the usual GARCH (1, 1) model conditions are satisfied (see Table 4.5).

The sum of the estimated coefficients, $\hat{\alpha}_1 + \hat{\beta}_1$ measures the persistence of volatility in a GARCH model. Persistence determines how rapidly the variance forecast converges to the unconditional variance. Concerning on the log return series for Inflation $(\hat{\alpha}_1 + \hat{\beta}_1) < 1$ depicted that volatility is neither permanent nor explosive, meaning previous month volatility is not significant in predicting future month volatility of inflation. In other words, the forecasted conditional variance reverts to the unconditional variance as the forecast horizon increases.

Table 4.5: parameter estimates from AR (1) - GARCH (1, 1) model

Series	Student-t	GED
Mean equation		
$\hat{\boldsymbol{\mu}}$	0.0084314 (0.000)	0.0088657 (0.000)
$\hat{\varphi}_1$	0.4350605 (0.000)	0.4261121 (0.000)
Variance equation		
$\hat{\alpha}_o$	0.0000363 (0.026)	0.0000386 (0.067)
$\hat{\alpha}_1$	0.3895285 (0.005)	0.4305174 (0.021)
$\hat{\beta}_1$	0.4094829 (0.019)	0.3943343 (0.072)
$\hat{\alpha}_1 + \hat{\beta}_1$	0.7990114	0.8248517

Values in brackets are p-values at 5% level of significance.

4.3.5 Checking the adequacy of the model

Checking the adequacy of the model under the proposed distributions is important before using the model for prediction. To check the adequacy of the AR (1) model, standardized residuals are used as the classical test based on the null hypothesis that standardized residuals are uncorrelated. Similar procedures are performed on the squared residuals to test the adequacy of the GARCH (1, 1) model. A sample ACF has the approximate upper and lower confidence bounds. Most of the sample ACFs is within or near their two standard-error limits (the two horizontal lines) indicating that autocorrelations that fall inside the limits are not significantly different from zero.

Since none of the sample ACFs shows any significant serial correlations, i.e., both in the standardized residual and its square are white noise series, then the fitted models appear to be adequate in describing the linear dependence in the conditional mean and volatility equations.

Thus, figure 5 from APPENDIX B depicted that the AR (1) model is adequate as the standardized residuals are white noise. Similarly, Figure 6 from APPENDIX B revealed that the squared residuals are not serially correlated which is an indication of GARCH (1, 1) model is adequate to fit the volatility of the log return series. Hence, GARCH (1, 1) model under the two distributions are adequate and the Q-statistic (Ljung_ Box goodness of fit test) in table 4.6 confirms this fact.

Table 4.6: Ljung_ Box test for white noise of residuals in AR (1) - GARCH (1, 1) model

Residuals	Student _t distribution	GED distribution
Standardized Residuals	19.8333 (0.0703)	30.2071 (0.8694)
Squared stand. Residuals	8.0733 (0.7794)	31.0454 (0.8441)

Values in brackets are p-values at 5% level of significance.

4.3.5.1 Checking for normality of standardized errors

Once the selected model is adequate in modeling the volatility of a financial time series variable, it is necessary to check the normality of the residuals. A plausible method to test for normality of the standardized residual would be to construct the statistic $\hat{\varepsilon}_t / \sqrt{\hat{h}_t}$ which is the model disturbance at each point in time t divided by the conditional standard deviation at that point in time. The q-q plot of the standardized residuals from each AR (1) - GARCH (1, 1) models are given from APPENDIX B. Therefore, the output in all of the q-q plots have approximately normally distributed standardized errors, except for deviations at the end tails.

4.3.5.2 Determining the distribution using forecast error measures

By presenting different forecast accuracy measures, we can compare the forecast error measures by taking the monthly log return for the period of January 2019 to April 2019 out of sample data under the two distributions.

Table 4.7: Forecast error measures under student's t and generalized error distributions

Statistics	T-GARCH(1, 1)	GED-GARCH(1, 1)
$\mathrm{MZ.R}^2$	0.1870568	0.1753581
MAE	0.0004454	0.0004434
MSE	1.19e-06	1.18e-06
RMSE	0.0007072	0.0007087

The R^2 values from the Mincer-Zarnowitz regression presented in Table 4.7 imply that the realized volatility measure for the series is a better measure of unobserved volatility when the distribution in the model contains large R^2 value. This is also confirmed by the other forecast error measures like values of the MAE, MSE and RMSE should get lower if the computing model is best performing with a given distribution.

It is difficult to compare when MAE, MSE and RMSE have very small value which is close to zero. So when this situation is happened, we can use the value of $\mathrm{MZ.R}^2$. Therefore, the distribution of the error term from Table 4.9 indicates that GARCH (1, 1) is relatively performing better under student's t- distribution based on $\mathrm{MZ.R}^2$ for log return Inflation.

Finally, AR (1) - GARCH (1, 1) under student's t ($\boldsymbol{df = 8}$) distribution was fitted to forecast the log return series and can be expressed as:

Mean Equation: $\hat{\mathrm{r}}_{\mathrm{t}} = 0.0084 + 0.4351\,\hat{\mathrm{r}}_{\mathrm{t-1}}$

Variance Equation: $\hat{\sigma}_{\mathrm{t}}^2 = 0.0000363 + 0.4305\,\hat{\varepsilon}_{t-1}^2 + 0.3943\,\hat{\sigma}_{t-1}^2$

(4.3)

Where $\hat{\sigma}_{\mathrm{t}}^2$ is the estimated conditional variance of the residuals, $\hat{\varepsilon}_{\mathrm{t}} = \sqrt{\hat{\sigma}_{\mathrm{t}}^2}\,\mathrm{z}_{\mathrm{t}}$ and $\mathrm{z}_{\mathrm{t}} \sim \mathrm{iid}\ \mathrm{N}(0, 1)$.

4.3.6 Predicted Forecast for Conditional mean and Variance process

Primarily, one of the fundamental applications of time series is forecasting. In this study, the main goal of forecasting is prediction of future variability of Inflation based on a recent or previous estimated series of data.

Therefore, the predicted forecasts were done using student's t (**df = 8**) distribution. The in-sample forecasts are plotted to the actual general CPI line as green color and the predicted forecasts appear to the pink color line in Figure 4.3.

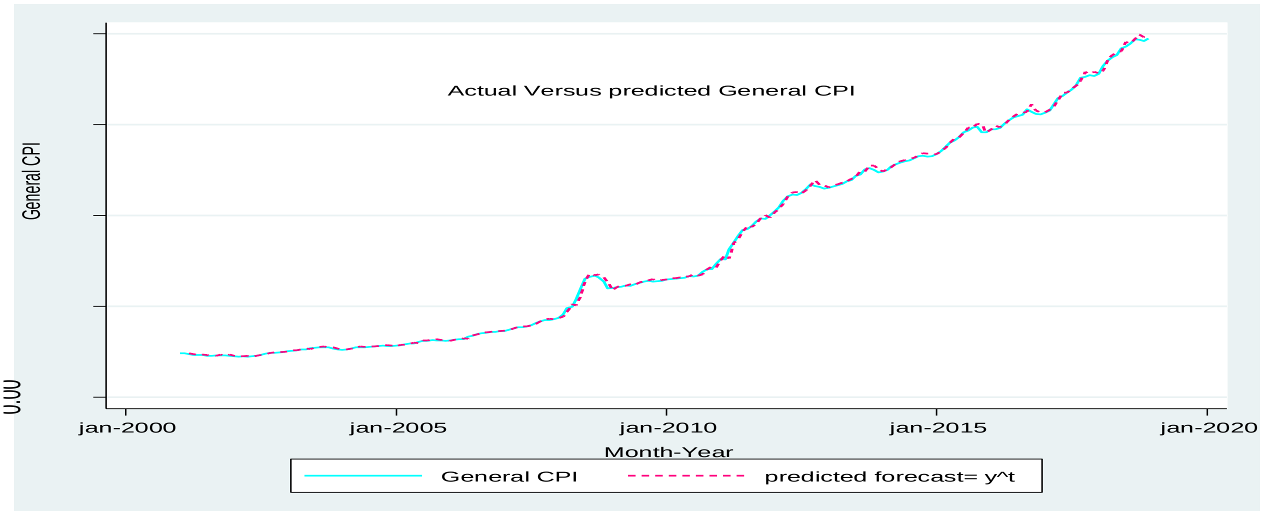

Figure 4.3: Plots of the actual vs. predicted General CPI from AR (1) model

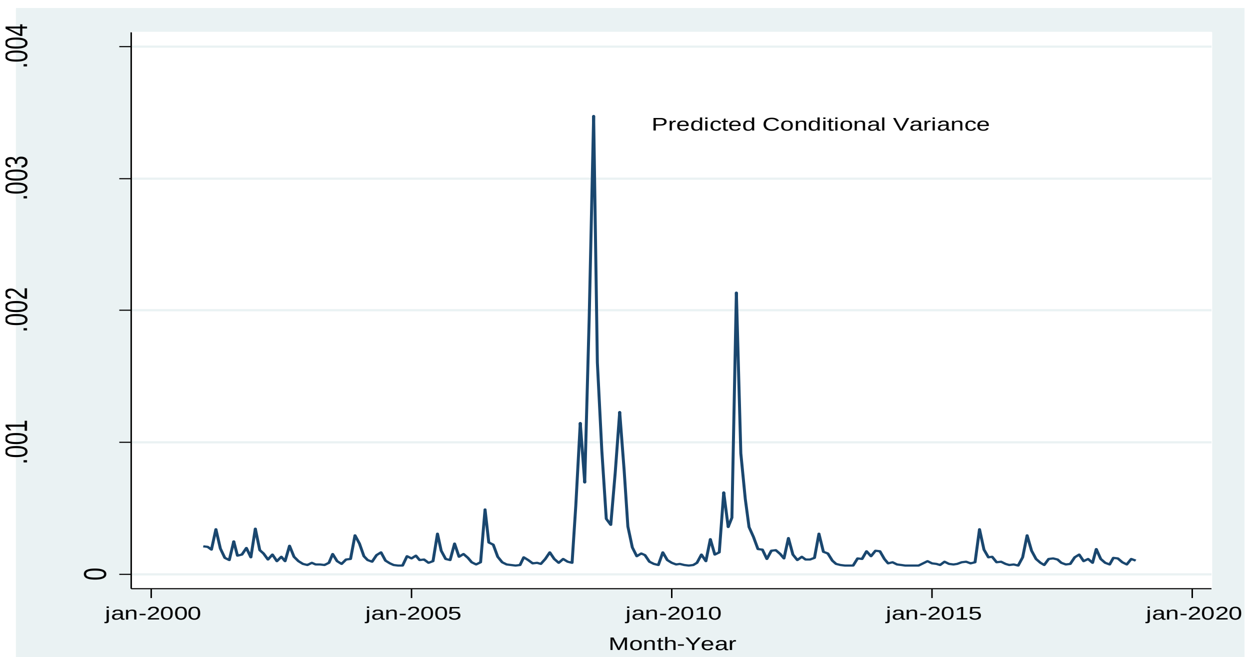

Figure 4.4: Conditional Variance prediction of residuals from GARCH (1, 1) model

4.4 Discussions

This study was undertaken to fit a Univariate time series model which can be used to forecast the overall inflation behavior in Ethiopia. The main objective of discussion is compare the variable explained in literature review and research finding. Among the various alternatives model, the conditional mean and variance using the AIC and BIC model selection procedures, AR (1) and GARCH (1, 1) was selected as respectively for the log return series.

According to this finding, in all the cases the coefficients of the ARCH terms are significant at 5% level of significance implying that there is clustering of volatility of overall inflation. That is, large changes in log returns of prices of goods and services are likely to be followed by further large changes. Similarly, the significance of the coefficients of the GARCH terms in each case at the 5% level of significance indicates that the present conditional variance is dependent on its past variances.

Then this finding is consistence or similar with finding identified by Awogbemi and Oluwaseyi (2011). And also the results showed that ARCH and GARCH models are better models because they give lower values of AIC and BIC as compared to the conventional Box and Jenkins ARMA models for inflation in Ethiopia. The sum of the coefficients of the ARCH and GARCH terms is less than one. These results indicate that the volatility shocks are permanent or persistence for the log return series. Thus, past volatility is important to predict future volatility and the forecasts of the conditional variances converge to the steady state quite slowly.

This finding observed that since volatility seems to persist in all the commodity items, people who expect a rise in the rate of inflation (the ‘bullish crowd’) will be highly favored in the market of the said commodity items.

It can be seen that GARCH model forecasts capture the pattern of volatility. In the long run forecasts of the conditional variances seems to converge to the unconditional variance.

CHAPTER FIVE

5 .CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The main objective of this study is focused on fitting an appropriate time series model for overall Inflation from January 2001 to December 2018 at the country level. Macroeconomic performance is measured on how some indicators like inflation behaves through time. In this regard, the finding of this study showed that the trend of overall inflation has significant increase over the study period. This indicates that the general level of price of goods and services continuous rising over time at the country level.

The overall inflation had persistence volatility behavior for log return series. In the short run, the last month conditional mean and conditional variance or lagged shocks had statistically significant effects on the current month for the log return series. But in the long run, the forecasts of the conditional mean and variances seem to converge to the unconditional mean and unconditional variance of 0.0084 and 0.000207 respectively.

Finally, the study revealed that the combination of AR (1) - GARCH (1, 1) model with Student's t-distribution for error terms was found the best for estimating and forecasting accuracy compared to the other forecasting models in the context of setting this study.

5.2 Recommendations

Firstly, inflation has a negative effect on national output and employment. It creates uncertainty when the average price level of goods and services changes significantly over time and economic decisions become increasing difficult. So that it is recommended for policy makers to take action in order to control the overall inflation and to be stable.

Secondly, the research work has revealed that Volatile inflation was associating with unpredictable movements in the relative prices in the economy. Since, the health of Ethiopia's economy is highly dependent on Birr against the USD and the country is import dependent, as such there must be a national agenda to increase foreign inflows and introduce a policy aimed at exchange rate targeting.

Lastly, it is also advised for researchers to conduct further research to determine main factors contributing to such rising overall inflation in the country.

Limitation of this study

The major problem that was challenged the research like:

- The shortage of time.
- Lack of finance to necessary material for the research.
- There is no internet accessibility.

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APPENDICES

Appendix A: Tables

Table 1: Model selection criteria for ARMA (p, q)

Model	AIC	BIC
ARMA(0,0)	-1128.011	-1121.27
ARMA(1,0)	-1201.5	-1191.388
ARMA(2,0)	-1199.728	-1186.245
ARMA(3,0)	-1199.697	-1182.844
ARMA(0,1)	-1187.603	-1177.491
ARMA(1,1)	-1199.827	-1186.344
ARMA(2,1)	-1197.941	-1181.088
ARMA(3,1)	-1199.257	-1179.034
ARMA(0,2)	-1191.133	-1177.65
ARMA(1,2)	-1198.487	-1181.634
ARMA(2,2)	-1198.434	-1178.21
ARMA(3,2)	-1199.08	-1175.486
ARMA(0,3)	-1199.793	-1182.939
ARMA(1,3)	-1201.72	-1181.496
ARMA(2,3)	-1203.563	-1179.968
ARMA(3,3)	-1202.334	-1175.396

Table 2: AR (1) + GARCH (p, q) model selection on AIC and BIC

Statistics	Residual Distribution	GARCH (1,1)	GARCH (1, 2)	GARCH (2,1)	GARCH (2,2)	Distribution Ranks
AIC	Normal	-1284.994 (1)	-1283.115 (2)	-1283.315 (3)	-1281.416 (4)	3
	Student-t	-1293.742 (1)	-1291.913 (3)	-1292.051 (2)	-1290.19 (4)	2
	Generalized Error	-1296.567	-1294.652	-1294.354	-1869.677	1

		(1)				
BIC	Normal	-1268.141 (1)	-1262.891 (3)	-1263.092 (2)	-1257.822 (4)	3
	Student- t	-1276.889 (1)	-1271.689 (3)	-1271.828 (2)	-1266.595 (4)	1
	Generalized Error	-1276.343 (2)	-1271.058 (3)	-1270.759 (4)	-1854.591 (1)	2
Rank sum	Model Ranks	7	14	13	17	

Table 3: Parameter Estimates in under student's t- distribution

ARCH family regression -- AR disturbances

Sample: 2 - 216		Number of obs =	215
Distribution: t(8)		Wald chi2(1) =	27.91
Log likelihood =	651.8709	Prob > chi2 =	0.0000

LogReturnCPI	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
LogReturnCPI						
_cons	.0084314	.0012053	7.00	0.000	.006069	.0107938
ARMA						
ar L1.	.4350605	.082358	5.28	0.000	.2736418	.5964791
ARCH						
arch L1.	.3895285	.1402396	2.78	0.005	.1146638	.6643931
garch L1.	.4094829	.174258	2.35	0.019	.0679434	.7510223
_cons	.0000363	.0000163	2.23	0.026	4.40e-06	.0000683

Table 4: Parameter Estimates in under generalized error distribution

ARCH family regression -- AR disturbances

Sample: 2 - 216 Number of obs = 215
Distribution: GED Wald chi2(1) = 38.04
Log likelihood = 654.2834 Prob > chi2 = 0.0000

LogReturnCPI	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
LogReturnCPI						
_cons	.0088657	.001062	8.35	0.000	.0067842	.0109472
ARMA						
ar						
L1.	.4261121	.069089	6.17	0.000	.2907002	.561524
ARCH						
arch						
L1.	.4305174	.1871489	2.30	0.021	.0637123	.7973224
garch						
L1.	.3943343	.2194384	1.80	0.072	-.0357571	.8244257
_cons	.0000386	.0000211	1.83	0.067	-2.70e-06	.0000799
/lnshape	.1616677	.1478067	1.09	0.274	-.1280281	.4513634
shape	1.17547	.1737422			.8798286	1.570452

APPENDIX B: Figures

Figure 1: Histogram of the log Returns

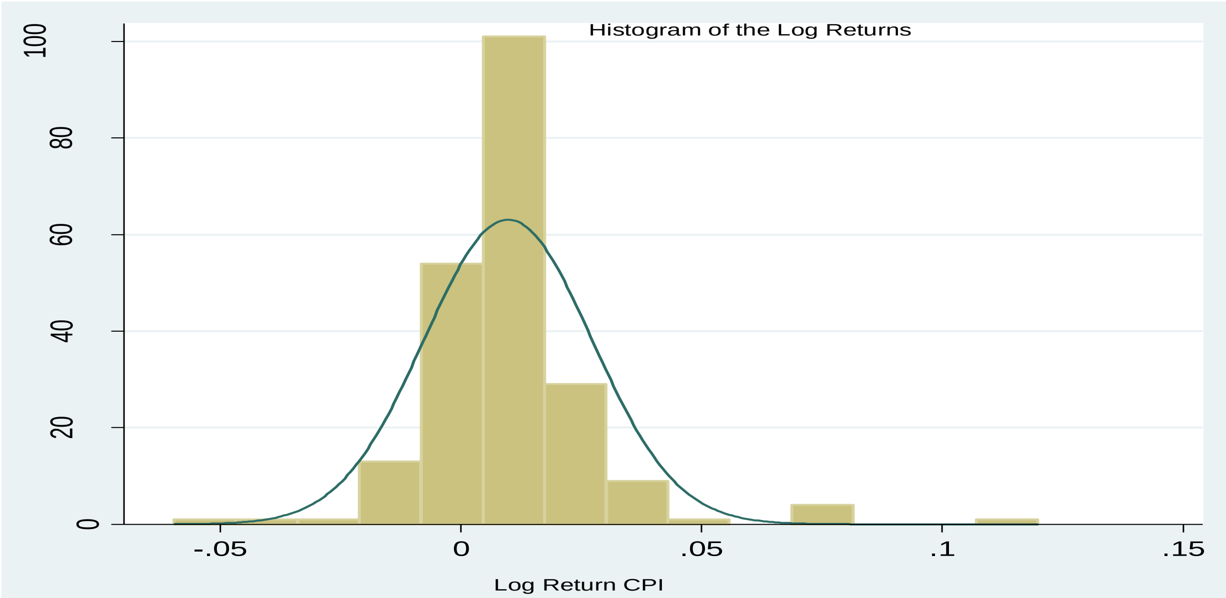


Figure 2: Auto correlation function for inflation based on CPI: period 2001-2018

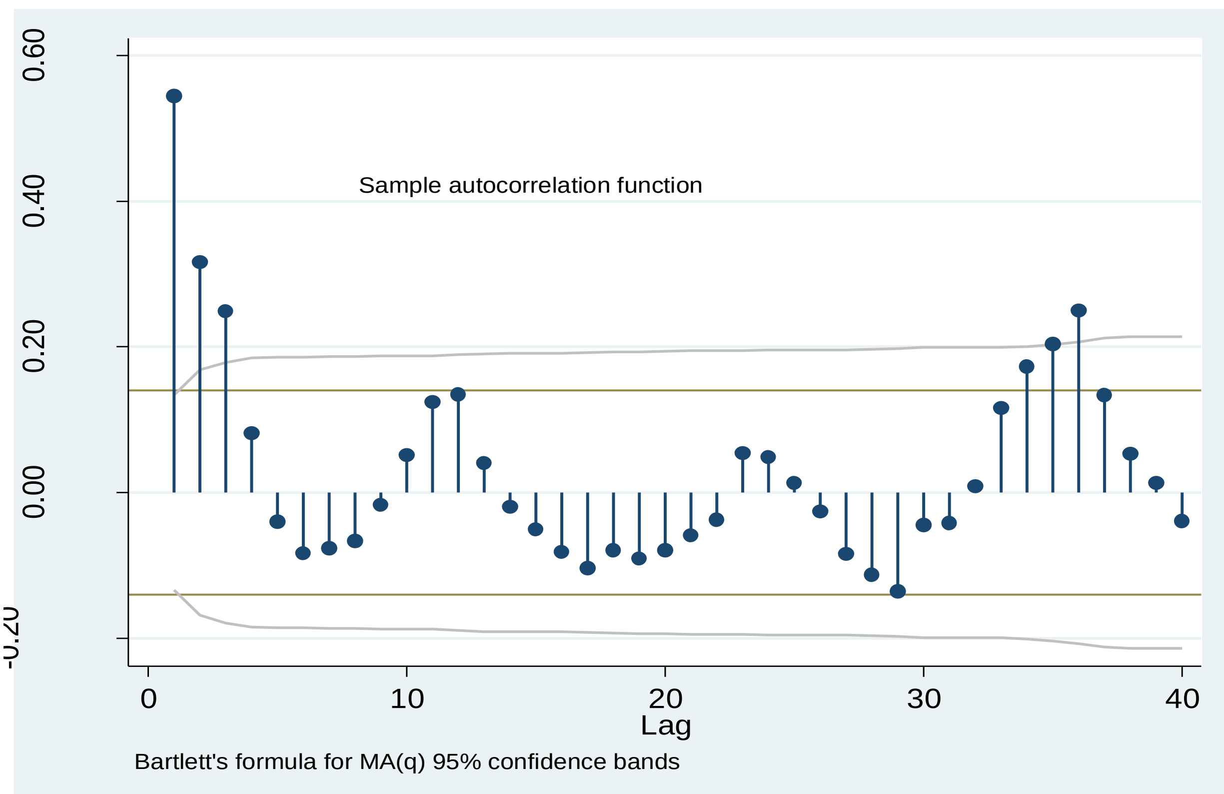


Figure 3: Partial Auto Correlation Function for inflation based on CPI: Period: 2001-2018

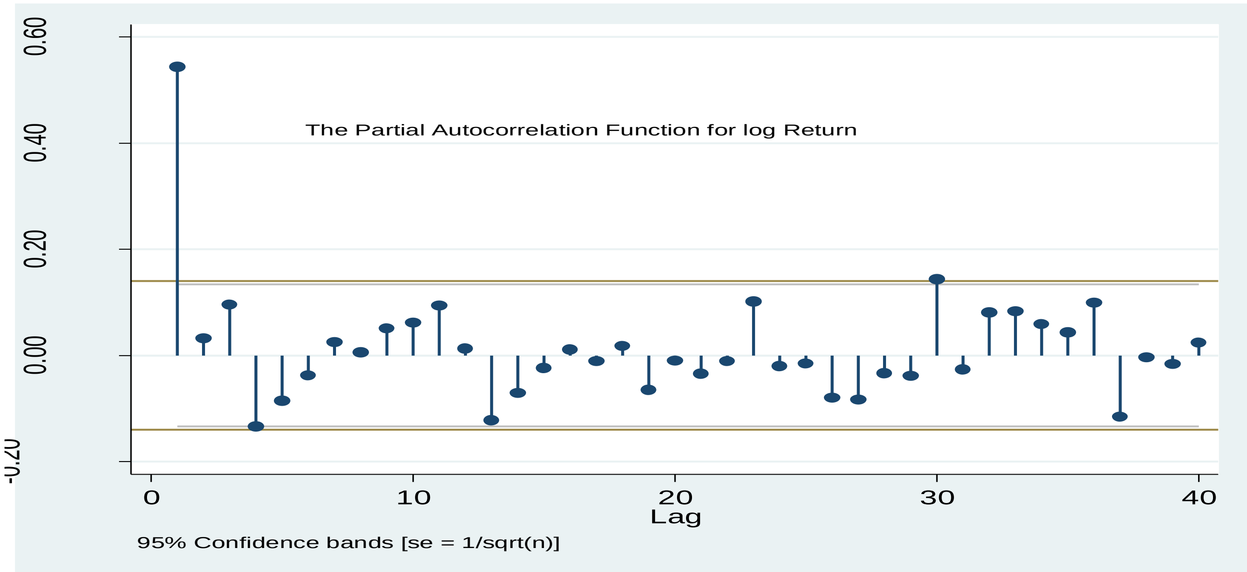


Figure 4: Plot of the squared residuals

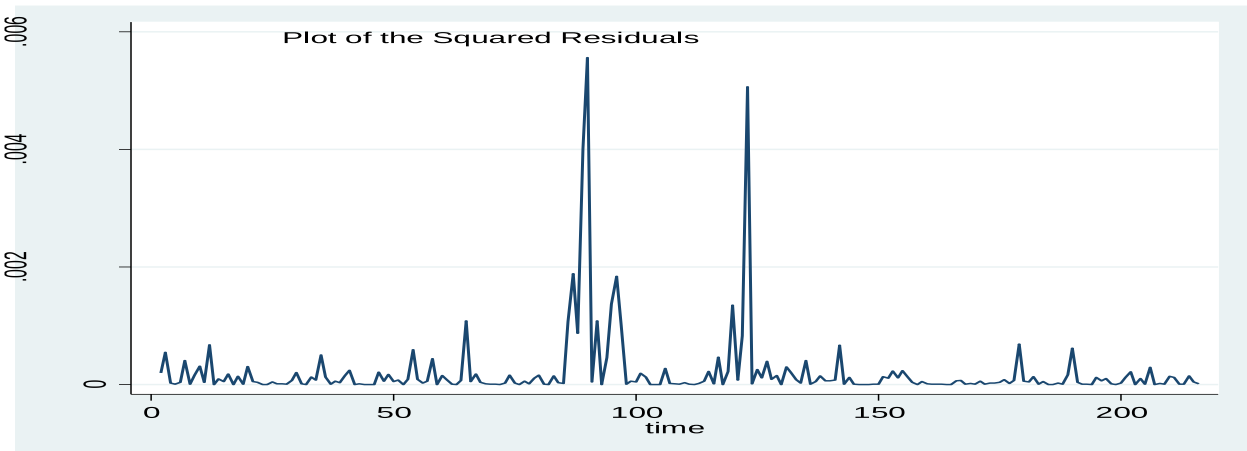


Figure 5: Correlogram of standardized residuals

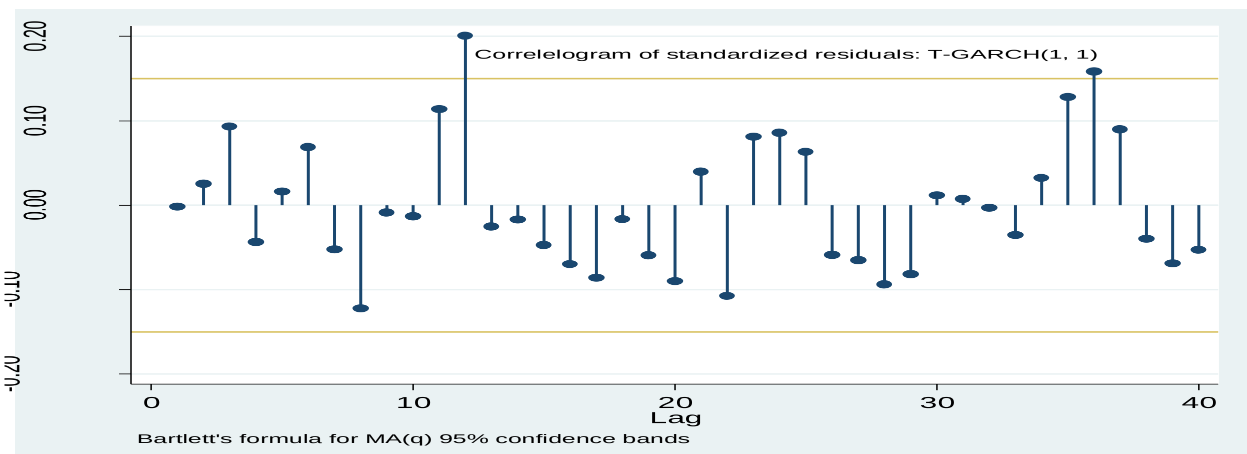


Figure 6: Correlogram of Squared standardized residuals

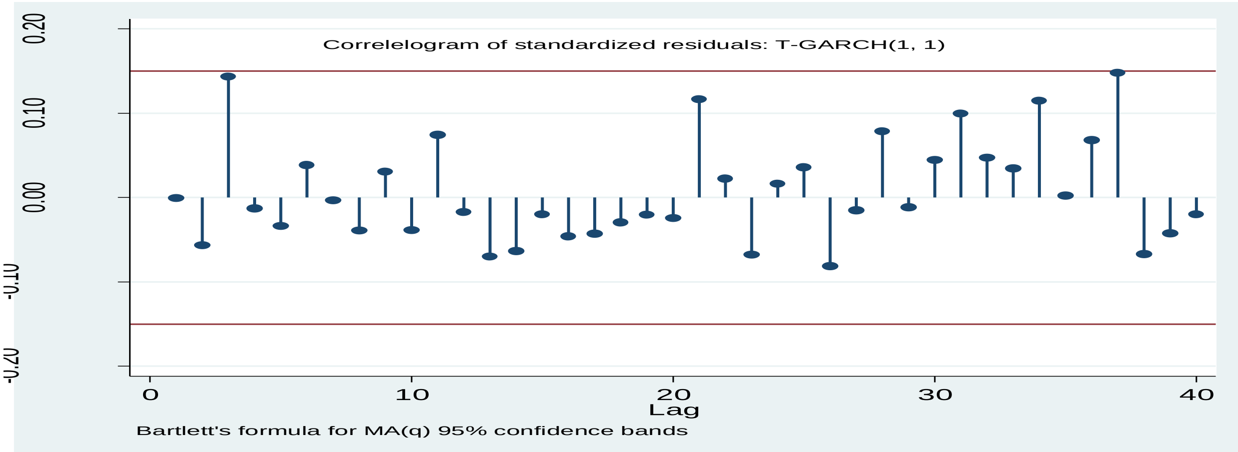


Figure 7: Correlogram of Standardized residuals under GED

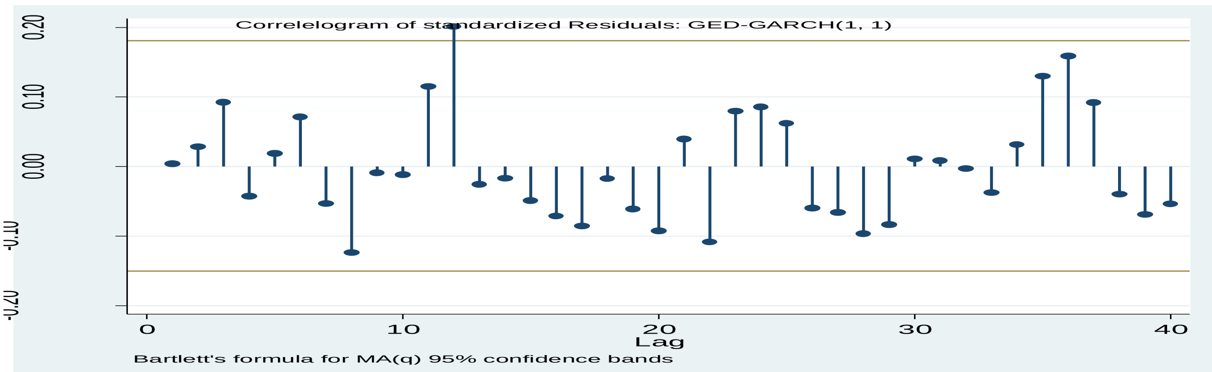


Figure 8: Correlogram of Squared standardized residuals under GED

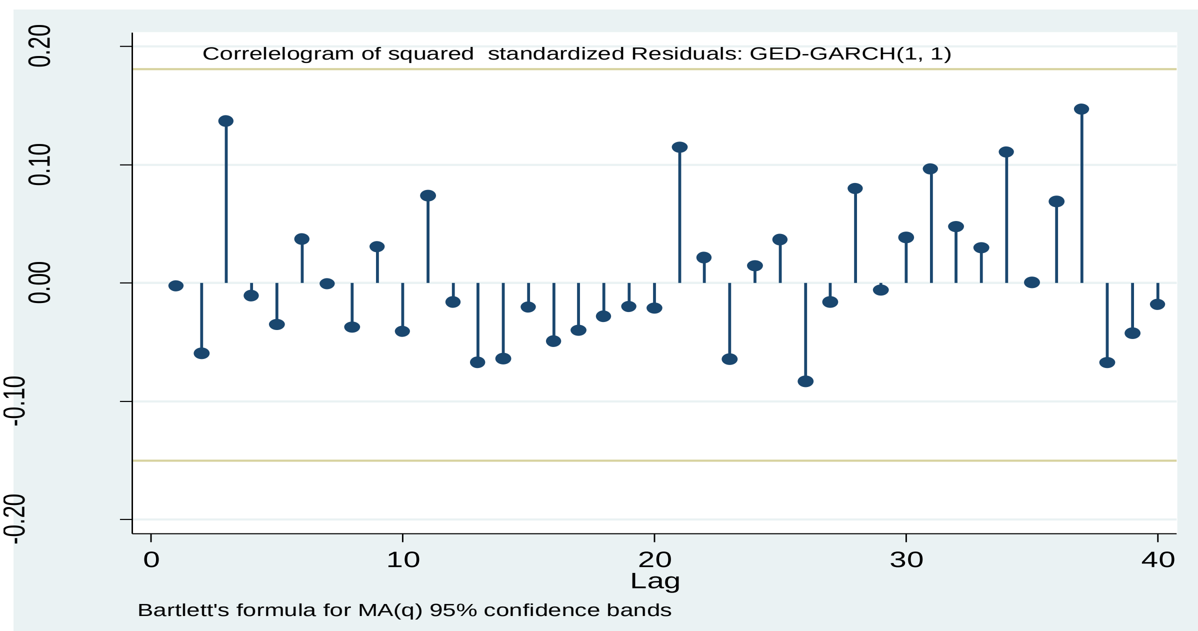
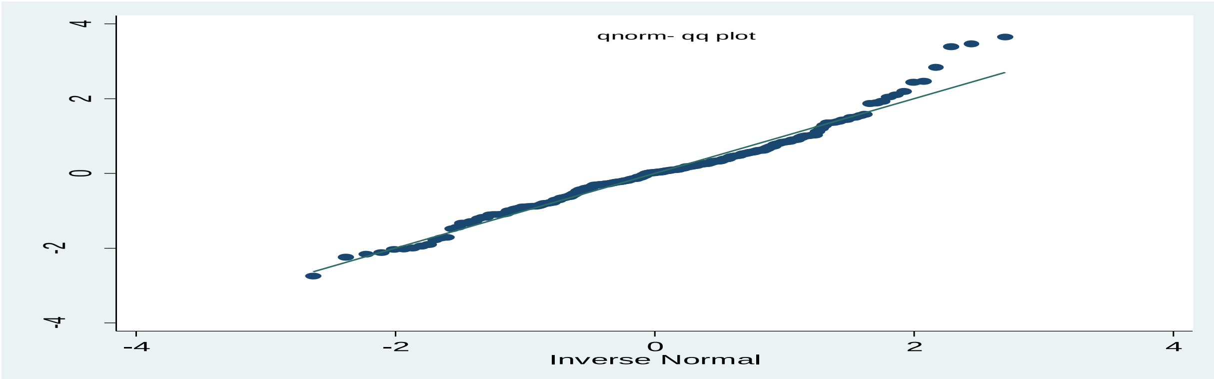


Figure 9: q-q plot of standardized errors under Student's t- distribution



. Figure 10: q-q plot of standardized errors under GED

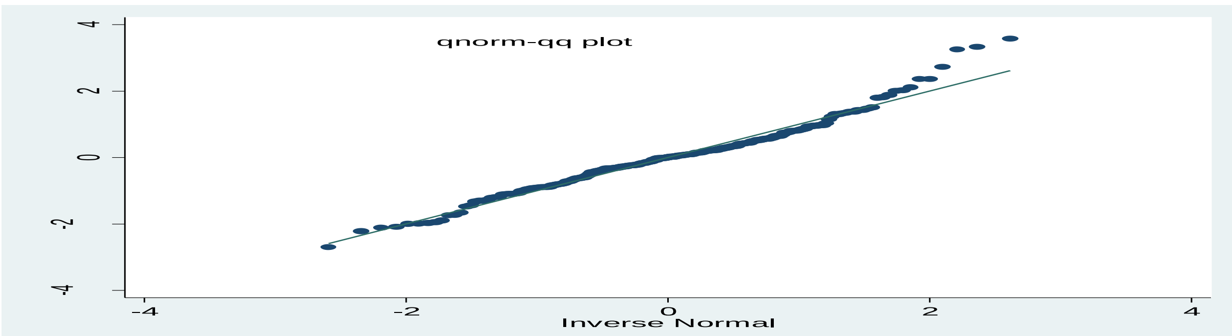


Figure10: Leverage effect of the normalized squared residuals

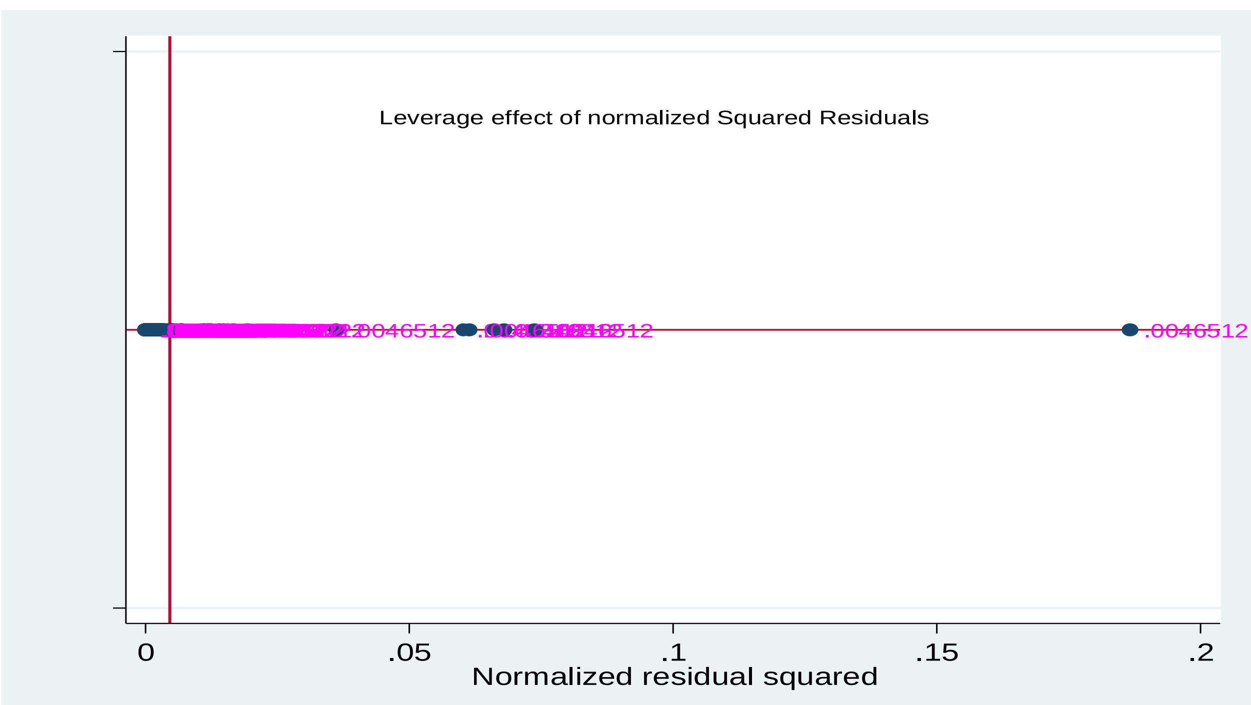
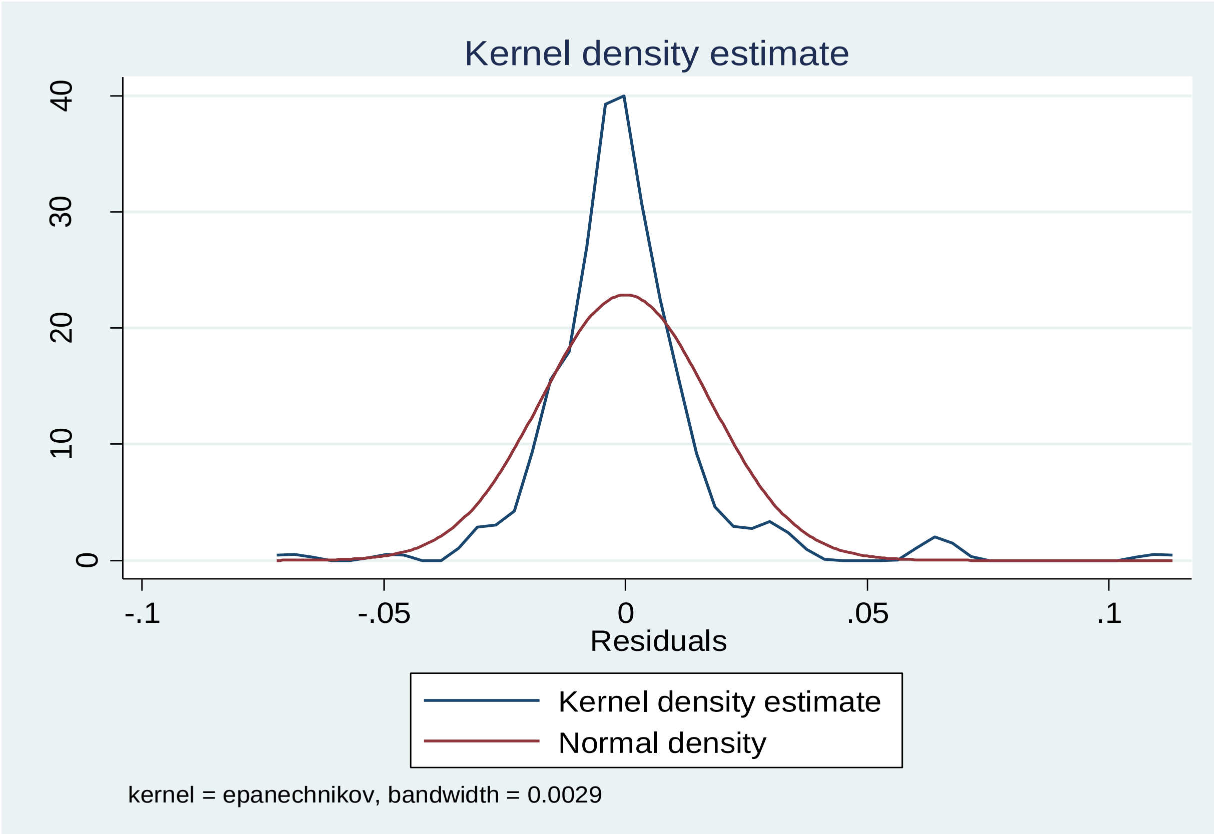


Figure 12: Kernel density plot of residuals



DECLARATION

we, the undersigned, declare that this paper is our original work and has not been presented for a degree in any other university, and that all sources of materials used for the paper have been duly acknowledged.

Declared by;

Name: ________________________________

Signature: _____________________________

Date: _________________________________

Name: ________________________________

Signature: _____________________________

Date: _________________________________

Confirmed by Advisor:

Name: ________________________________

Signature: _____________________________

Date: ____________________________