College of Natural and Computational Science
Department of Mathematics

Project On Numerical Differentiation And Its Application

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Wolkite University
Department of Mathematics

The undersigned hereby certify that they have read and recommend to the Department of Mathematics for acceptance of a project entitled **Numerical Differentiation and Its Application** by wili jobira in partial fulfillment of the requirements for the degree of Bachelor of Science.

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Abstract

This project deals about Numerical Differentiation and Its Application .it contains four chapters.the first chapter is deals about introduction. the second chapter is deals about basic definition.the third chapter is deals about methodology.the fourth chapter is deals about numerical differentiation and its application it contains many subsection under it.

Notations

Δ	Newton forward interpolation formula
∇	Newton back ward interpolation formula

Chapter 1

Introduction

Engineers and scientists are frequently faced with the problem of differentiation or integration of some functions.if the functions have closed from representation and are amenable for standard calculus methods,then differentiation and integration can be carried out. however,in many situations,we many not know the exact functions.We will be knowing,only,the values of the functions at a discrete set of points.in some instances,the functions are known but they are so complicated that analytic differentiation,integration is difficult.in both these situations,we seek the help of obtaining the derivative of a function using a numerical technique is known as numerical differentiation.

The method of finding the value of an integral of the form using numerical techniques is called numerical integration In this section we discuss various numerical differentiation.

Numerical differentiation deals with the following problem:given the function y=f(x)find one of its derivatives at the point$x = x_k$ .Here, the term given implies that we either have an algorithm for computing the function,or possesses a set of discrete data points (x_i, y_i) ,i=1,2,...,n.In other words,we have a finite number of(x,y)data points or pairs from which we can compute the derivative.Numerical differentiation is a method to compute the derivative of a function at some values of independent vari-

able x, when the function f(x)is explicitly unknown, however it is known only for a set of arguments. Like the numerical interpolation discussed a number of for differentiation are derived in this chapter They are:
(a)Derivative based on Newton's forward interpolation formula.
This formula is used to find the derivative for some given x lying near the beginning of the data table.
(b)Derivative based on Newton's backward interpolation formula.
This formula is suitable to find the derivative for a point near the end of the data table.
(c) Derivatives based on Stirling's interpolation formula.
This formula is used to find the derivative for some point lying near the middle of the tabulated value. A method to find the maxima and minima of a given function is also discussed in this chapter.4.4.1

1.1 Background of the study

Numerical differentiation is the process of calculating the value of the derivative of a function at some assigned value of x from the given set of data points $(x_i, y_i) = f(x_i)), i = 0, 1, 2, ...n$ which correspond to the values of an unknown function y=f(x).To find $\frac{dx}{dy}$ we first replace the exact relation y=by the best interpolating polynomial y=f(x) as we know earlier and then differentiate the latter as many times as we desire.The choice of the interpolation formula to be used, will depend on the assigned value of x at which $\frac{dx}{dy}$ is desired.If the points are equally spaced and $\frac{dx}{dy}$ is required near the beginning of the table,we use Newton-Gregory's Forward Interpolation Formula.If we require the table,we employ Newton-Gregory's Back ward Interpolation Formula.If the Of the value of the derivative is required near the middle of x are not equispaced,we use Newton,s Divided difference Interpolation $\frac{dx}{dy}$ Formula or to get the derivative values

Chapter 2

Preliminaries

2.1 Basic Definition

In this chapter we recall some definitions and known result on numerical differentiation and its application.this chapter serves as base and background for the study of subsequent chapter we shall keep on referring back to it as and when required.

Definition: Numerical differentiation:

Numerical differentiation is the process by which we can find the derivative or derivatives of a function at some values of the independent variable when we are given a set of values of that function.

Uses of Numerical differentiation: the numerical differentiation techniques can be used in the following situations:

- Where the function values corresponding to distinct values of the argument are known but the function is unknown . For example, we may knowing the values of f(x)at various values of x, say xi, i = 1,2,3,..., n in a tabulated(arranged) form.
- Where the function to be differentiated is complicated, and so, it is difficult to differentiate by usual procedures. numerical differentiation is the process of calculating the value of the derivative of a function at some assigned value of x from the given set of f(x)data points (xi, yi = f(xi)), i = 0,1,2,..., n in which correspond to the values of an unknown function y = f(x). To find $\frac{dx}{dy}$ we first replace the exact relation y = f(x) by the best interpolating polynomial y = f(x) as we know earlier and then differentiate the latter as many times as we desire. the choice of the interpolation formula to be used, will depend on the assigned value of x at which $\frac{dx}{dy}$ is desired. If the points are equally spaced and $\frac{dx}{dy}$ is required

near the beginning of the table, we use Newton-Gregory's forward interpolation formula.If we require the derivative at the end of the table, we employ Newton-Gregory's backward interpolation formula.If the value of the derivative is required near the middle of the table, we use one of the central difference interpolation formula.If the values of x are not equi-spaced, we use Newton's Divided difference Interpolation $\frac{dx}{dy}$Formula or to get the derivative value Formula for derivative:consider the function y=f(x)which is tabulated for the values(=ih),i=0,1,2,...n

2.1.1 Derivative using newtons forward difference formula

suppose that we are given a set of values i=0,1,2,...n.we want to find the derivative of y=f(x)passing through the(n+1)points,at a point nearer to the starting value at x=Newton,s forward difference interpolation formula is y=y_0+p$\Delta y_0+p\frac{(p-1)}{2!}\Delta^2$ y_0+p$\frac{(p-1)(p-2)}{3!}\Delta^3 y_0$+................(1)

Where p=$\frac{x-x_0}{h}$...........................(2)

On differentiation (1) w,r,t,p and on differentiation (2) w,r,t,x we have $\frac{dp}{dx}\sim\frac{1}{h}$ $\frac{dy}{dx}=\frac{dy}{dp}x\frac{dp}{dx}$ $=\Delta y_0+\frac{2p-1}{2}\Delta^2 y_0+\frac{3p^2-6p+2}{6}\Delta^3 y_0$+4p$^3-\frac{18p^2+22p-6}{24}$ $\Delta^4 y_0$+.................(3)

Equation 3 gives the values of $\frac{dy}{dx}$at any point x which may be any where in the

Ax=x_0 and p=0.hence putting p=0 equation(3)gives $\frac{dy}{dx}$x$\sim x_i=\frac{dy}{dp}$p$\sim$1=$\frac{1}{h}\Delta y_0+\frac{1}{2}\Delta^2 y_0$

Again on differentiation(3) we get

$\frac{d^2y}{dx^2}=\frac{d\frac{dy}{dx}}{dx}d=\frac{d}{dp}(\frac{dy}{dx}).\frac{dp}{dx}=\frac{1}{h^2}\Delta^2 y_0$+(p-1)$\Delta^3 y_0+\frac{6p^2-18p+11}{24}\Delta^4 y_0$+.........(4)

From which we obtain($\frac{d^2y}{dx^2}$) x$\sim x_0=\frac{1}{h^2}$ $\Delta^2 y_0$-$\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0$+.........at x=$x_0$ and p=0.........(5)

Similarly($\frac{d^3y}{dx^3}$)x=$x_0=\frac{1}{h^3}\Delta^3 y_0$-$\frac{3}{2}\Delta^4 y_0$+...........(6)

2.1.2 Derivative Using Newton,s Backward Difference Formula

Newtons backward difference interpolation formula is y(x)=yn+p$\nabla y_n\frac{(p+1)}{2!}\nabla^2 y_n$ $\frac{(p+1)(p+2)}{3!}\nabla^3 y_n$+................(7)

where p=$\frac{x-xn}{h}$................(8)

On differentiation (7) w,r,t,p,we $\frac{dy}{dp}$=∇y_n+$\frac{2p+1}{2}\nabla^2 y_n$+$\frac{6p^2+6p+2}{6}\nabla^3 y_n$+ $\frac{4p^3+18p^2+22p+6}{6}\nabla^4 y_n$+.............

On differentiation (8) w,t,we $\frac{dp}{dx}$~$\frac{1}{h}$ $\frac{dy}{dx}$ =$\frac{dy}{dp}$=$\frac{1}{h}$(∇y_n+$\frac{2p+1}{2}\nabla^2 y_n$+$\frac{3p^2+6p+2}{6}\nabla^3 y_n$ +$\frac{4p^2+18p^2+22p+6}{24}\nabla^4 y_n$+)..........(9)

Equation(9)gives the value of at$\frac{dy}{dx}$ any point x which may be any where in the interval

At x=x_n and p=0,hence putting p=equation(9)gives

($\frac{dy}{dy}$)x~x_n=($\frac{dy}{dx}$)x_n=$\frac{1}{h}\nabla y_n$+$\frac{1}{2}\nabla^2 y_n$+$\frac{1}{2}\nabla^2 y_n$+$\frac{1}{3}\nabla^3 y_n$+$\frac{1}{4}\nabla^4 y_n$+............(10)

Again on differentiation(9) we obtain($\frac{d^2y}{dx^2}$)=$\frac{d\frac{dy}{dx}}{dx}$=$\frac{d}{dp}$($\frac{dy}{dx}$).$\frac{dp}{dx}$=$\frac{1}{h^2}$ ∇y_n(p+1)∇y_n +$\frac{6p^2+18p+11}{12}\nabla^4 y_n$+...................(11)

From which we obtain ($\frac{d^2y}{dx^2}$)x~x_n=$\frac{1}{h^2}$ $\nabla^2 y_n$-$\nabla^3 y_n$+$\frac{11}{12}\nabla^4 y_n$+$\frac{5}{6}\nabla^5 y_n$+.........at x=$x_n$ and p=0

similarly ($\frac{d^3y}{dx^3}$)x~x_n =$\frac{1}{h^2}\nabla^3 y_n$-$\frac{3}{2}\nabla^4 y_n$+...........(12)

2.1.3 Derivatives using Central Difference Formula

The Stirling's Formula is given by y=y_0+$\frac{p}{2}$(Δy_0+Δy_-1)+$\frac{p^2}{2}\Delta^2 y_-$1+$\frac{p^3-p}{12}$($\Delta^3 y_-$1 +$\Delta^3 y_-$2)+$\frac{p^4-p^2}{24}\Delta^4 y_-$2+.....................(13)

Where p=$\frac{x-x_0}{h}$ on differentiation (13) w.r.t.x we have $\frac{dy}{dx}$=$\frac{dy}{dy}$.$\frac{dp}{dx}$=$\frac{1}{h}$.$\frac{dp}{dx}$=[$\frac{1}{2}$(Δy_0 +Δy_-1)+p$\Delta^2 y_-$1+$\frac{3p^2-1}{12}$($\Delta^3 y_-$1

+$\Delta^3 y_-$2)+$\frac{2p^3-p}{12}$$\Delta^4 y_-$2+$\frac{5p^4-15p^2-4}{240}$($\Delta^5 y_-$2 +$\Delta^5 y_-$3)+.........]

At x=x_0 and p=0 hence putting p=0 we get ($\frac{dy}{dx}$)x$\sim x_0$=$\frac{1}{h^2}$[$\Delta^2 y_-$1-$\frac{1}{12}$$\Delta^4 y_-$2+$\frac{1}{90}$ $\Delta^6 y_-$3+..........]

at x=x_n and p=0

Similarly ($\frac{d^3y}{dx^3}$)x$\sim x_0$=$\frac{1}{h^3}$ [$\frac{1}{2}$($\Delta^3 y_-$1+$\Delta^3 y_-$2)+...........]

Chapter 3

Methodology Of The Study

The project would be developed through a number of resources and materials which are quite relevant papers such as; reference books, Internets and different website.

All information for this project would be obtained from documentary sources or secondary sources particularly mathematical areas such as; reference books, Internets. The collected information should be arranged from the beginning to the end. The collected information would be analysis and process using different procedures such as; some theorems should be prove, problem s should be solved, some application would be tested
The collected information would be examined details.
Finally, the conclusion would be made.

3.1 Statement of the problem

The reason why the title was selected is that the necessary techniques which were helpful for the complex problem and short cut for complex calculating and solved the problem directly.And also initial value problems are solved without first determining a general solution. Similarly non homogeneous equations are solved without first solving the corresponding homogeneous equations.The study would provide detail information by answering the following questions.

3.2 Objective Of The Study

3.2.1 General objective

The main purpose of this project is to discuss how and when the numerical differentiation solve the differential equation and corresponding initial value problem.

3.2.2 Specific objective

The specific objectives of those studies were

- To list some properties of numerical differentiation
- To prove some important theorems and solve related problems
- explain the definitions of forward, backward, and center divided methods for numerical differentiation

Chapter 4

Numerical Differentiation And Its Application

Definition:: Numerical differentiation: Numerical differentiation is the process by which we can find the derivative or derivatives of a function at some values of the independent variable when we are given a set of values of that function.

4.1 Derivatives based on Newton forward interpolation formula

Suppose the function y = f (x) is known at (n + 1) equispaced points x_0, x_1, ..., x_n and they are y_0, y_1 ...., y_n respectively i.e., y_i = f (x_i), i = 0, 1, ..., n. Let $x_i = x_0$ + ih and u=$\frac{x-x_0}{h}$where h is spacing. The Newtons forward interpolation is

y=f(x)=y_0+uΔy_0+$\frac{u(u-1)}{2!}\Delta^2 y_0$+...+$\frac{u(u-1)...(u-n-1)}{n!}\Delta^n y_0$

=y_0+uΔy_0+$\frac{u^2-u}{2!}\Delta^2 y_0$+$\frac{u^3-3u^2+2u}{3!}\Delta^3 y_0$+$\frac{u^4-6u^3+11u^2-6u}{4!}\Delta^4 y_0$ +$\frac{u^5-10u^4+35u^3-50u^2+24u}{5!}\Delta^5 y_0$+...

Differentiation Eq(4.1)w.r.t.x we get

f'(x)=$\frac{1}{h}$[Δy_0+$\frac{2u-1}{2!}\Delta^2 y_0$+$\frac{3u^2-6u+2}{3!}\Delta^3 y_0$+$\frac{4u^3-18u^2+22u-6}{4!}\Delta^4 y_0$+ $\frac{5u^4-40u^3+105u^2-100u+24}{5!}\Delta^4 y_0$ Note here that $\frac{du}{dx}$=$\frac{1}{h}$

Differentiation Eq(4.2)w.r.t.x we obtain

f"(x)=$\frac{1}{h^2}$ [$\Delta^2 y_0$+$\frac{6u-6}{3!}\Delta^3 y_0$+$\frac{12u^2-36u+22}{4!}\Delta^4 y_0$+$\frac{20u^3-120u^2+120u-100}{5!}$ $\Delta^5 y_0$+..........] and so

Equations(4.2)and(4.3) give the approximate derivatives of f(x)at arbitrary point x=x_0+uh.

When x=x_0 u=0 Eqs(4.2)and(4.3)become

f'(x_0)=$\frac{1}{h}$ [Δy_0-$\frac{1}{2}\Delta^2 y_0$+$\frac{1}{3}\Delta^3 y_0\frac{1}{4}\Delta^4 y_0$+$\frac{1}{5}\Delta^5 y_0$-...]

f"(x_0)=$\frac{1}{h^2}$ [$\Delta^2 y_0$-$\Delta^3 y_0$+$\frac{11}{12}\Delta^4 y_0$-$\frac{5}{6}\Delta^5 y_0$+...]and so on

Example 4.1.1. *From the following table find value of*$\frac{dy}{dx}$ *and*$\frac{d^2y}{dx^2}$

x	1	1.1	1.2	1.3	1.4	1.5
y	5.4680	5.6665	5.9264	6.2551	6.6601	7.1488

Solution:
The forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1.0	5.4680			
		0.1985		
1.1	5.6665		0.0688	
		0.2599		0.0074
1.2	5.9264		0.0763	
		0,3287		0.0074
1.3	6.2551		0.0763	
		0.4050		0.0074
1.4	6.6601		0.0837	
		0.4887		
1.5	7.1488			

Here x_0 = 1.0 and h = 0.1 then u=0 and hence

$\frac{dy}{dx}$=y'(1,0)=$\frac{1}{h}$[Δy_0-$\frac{1}{2}\Delta^2 y_0$+$\frac{1}{3}\Delta^3 y_0$............]=$\frac{1}{0.1}$[0.1985-$\frac{1}{2}$(0.0614)+$\frac{1}{3}$(0.0074)]

=1.7020

$\frac{d^2y}{dx^2}$=y"(1.0)=$\frac{1}{h^2}$[Δy_0-$\Delta^3 y_0$+.....]=$\frac{1}{(0.1)^2}$[0.0614-0.00074]=5.4040

Example 4.1.2. *Obtain the first and second derivatives of the function tabulated below at the points x=1.1 and 1.2*

x	1	1.2	1.4	1.6	1.8	2.0
y	0	0.128	0.544	1.298	2.440	4.02

Solution:
We first construct the forward difference table as shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.0	0				
		0.128			
1.2	0.128		0.288		
		0.416		0.05	
1.4	0.544		0.338		0
		0.754		0.05	
1.6	1.298		0.388		0
		1.142		0.05	
1.8	2.440		0.438		
		1.580			
2.0	4.02				

since x=1.1 is a non-tabulated point near the beginning of the table we take x_0=1.0 and compute

p=$\frac{x-x_0}{h}$=$\frac{1.1-1.0}{0.2}$=0.5

Hence $\frac{dy}{dx}$=$\frac{1}{h}$[Δy_0+$\frac{2p-2}{2}\Delta^2 y_0$+$\frac{3p^2-6p+2}{2}\Delta^3 y_0$]

=$\frac{1}{0.2}$[0.128+0+$\frac{3(0.5)^2-6(0.5)+2}{6}$(0.05)]=0.62958

$\frac{d^2y}{dx^2}$=[$\Delta^2 y_0$+(p-1)$\Delta^3 y_0$]=$\frac{1}{(0.2)^2}$[0.288+(0.5-1)0.05]=6.575

now x=1.2 is a tabulated point near the beginning of the table for x=x_0=1.2 p=0 and

$$\frac{dy}{dx}=\frac{1}{h}[\Delta y_0\text{-}\frac{1}{2}\Delta^2 y_0+\frac{1}{3}\Delta^3 y_0]=\frac{1}{0.2}[0.416\text{-}\frac{1}{2}(0.338)+\frac{1}{3}(0.05)]=1.31833$$

$$\frac{d^2y}{dx^2}=\frac{1}{h^2}\ [\Delta^2 y_0\text{-}\Delta^3 y_0]=\frac{1}{h^2}\ [0.338\text{-}0.05]=7.2$$

4.2 Derivatives Based On Newtons Backward Interpolation Formula

Here we assume the function y=f(x)is known at (n+1) points $x_0 x_1$...xn i.e y_1=f(x_i)i=0,1,2...n are known let x_1=x_0+ih=0,1,2...n and v=$\frac{x-x_n}{h}$

Then the Newtons backward interpolation formula given by

$$f(x)=y_n+\nabla y_n+\frac{v(v+1)}{2!}\nabla^2 y_n+\frac{v(v+1)(v+2)}{3!}\nabla^3 y_n+\nabla^3 y_n+\frac{v(v+1)(v+2)(v+3)}{4!}$$

$$\nabla^4 y_n\frac{v(v+1)(v+2)(v+3)(v+4)}{5!}\nabla^5 y_n+...$$

When the Eq(4.6) is differentiated w.r.t.x successively we obtained

$$f'(x)=\frac{1}{h}\ [\nabla y_n+\frac{2v+1}{2!}\nabla^2 y_n+\frac{3v^2+6v+2}{3!}\nabla^3 y_n+\frac{4y^3+18v^2+22v+6}{4!}\nabla^4 y_n$$

$$+\frac{5v^4+40v^3+105v^2+24}{5!}\Delta^5 y_n+...]$$

$$f''(x)=\frac{1}{h^2}\ [\Delta^2 y_n+\frac{6v+6}{3!}\Delta^3 y_n+\frac{12v^2+36v+22}{4!}\Delta^4 y_n+\frac{20v^3+120v^2+210v+100}{5!}$$

$\Delta^5 y_n$+...] and so on

Equation(4.7)can be used to determined the approximated differentiation of first second etc order at any point x

Where x=x_n+vn if x=x_n then v=0

Equations(4.7)and(4.8)become

$$f'(x_n)=\frac{1}{h}\ [\nabla y_n+\frac{1}{2}\nabla^2 y_n+\frac{1}{3}\nabla^3 y_n+\frac{1}{4}\nabla^4 y_n+\frac{1}{5}\nabla^5 y_n+...]\text{ and}$$

$$f''(x_n)=\frac{1}{h^2}\ [\nabla y_n+\nabla^3 y_n+\frac{11}{12}\nabla^4 y_n+\frac{5}{6}\nabla^5 y_n+...]$$

Example 4.2.1. *A slider ina machine moves along a fixed straight rod its distance x(m) along the rod are given in the following table for various of the time t(second)*

t sec	*1*	*2*	*3*	*4*	*5*	*6*
x(m)	*0.0201*	*0.0844*	*0.3444*	*1.0100*	*2.3660*	*4.7719*

Find the velocity and acceleration of the slider at time t=6sec

Solution:
The backward difference table is

t	x	∇x	$\nabla^2 x$	$\nabla^3 x$	$\nabla^4 x$	$\nabla^5 x$
1.0	*0.0201*					
2.0	*0.08444*	*0.0643*				
3.0	*0.3444*	*0.260*	*0.1957*			
4.0	*1.0100*	*0.6656*	*0.4056*	*0.2100*		
5.0	*2.366*	*1.3560*	*0.6904*	*0.2847*	*0.0748*	
6.0	*4.7719*	*2.4059*	*1.0499*	*0.3595*	*0.0748*	*0.000*

Here h=1.0

$$\frac{dx}{dt}=\frac{1}{h}\ [\nabla \text{x}+\frac{1}{2}\nabla^2 x+\frac{1}{3}\nabla^3 x+\frac{1}{4}\nabla^4 x+\frac{1}{5}\nabla^5 x+...]$$

$$\frac{1}{1.0}[2.4059+\frac{1}{2}(1.0499)+\frac{1}{3}(0.3595)+\frac{1}{4}(0.0748)+\frac{1}{5}(0.0)]=3.0694$$

$$\frac{d^2x}{dt^2}=\frac{1}{h^2}[\Delta^2 x+\Delta^3 x+\frac{11}{12}\Delta^5 x+...] = \frac{1}{(1.0)^2}[1.0499+0.3595+\frac{11}{12}(0.0748)+\frac{5}{6}(0)]$$

=1.4780

4.3 Derivative Based On Strings Interpolation Formula

suppose y±i=f(x±i),i=0,1,........n are given for 2n+1 equi spaced points x_0,x±1,x±2,....x±n where x±i=x_0±ih,i=0,1,...n

The stirrings interpolation polynomial is given by

f(x)=$y_0+\frac{u}{1!}[\frac{\Delta y\_1+\Delta y_0}{2}]+\frac{u^2}{2!}\Delta^2 y\_1+\frac{u^3-u}{3!}[\frac{\Delta^3 y\_2+\Delta^3 y\_1}{2}+\frac{u^4-u^2}{4!}\Delta^4 y\_2+$
$\frac{u^5-5u^3+4u}{5!}[\frac{\Delta^5 y\_3+\Delta^5 y\_2}{2}]+...$

Where u=$\frac{x-x0}{h}$

When Eq (4.11)is differentiated with respect to x successively we obtain

f'(x)=$\frac{1}{2}[\frac{\Delta y\_1+\Delta y_0}{2}$+u$\Delta^2 y_1+\frac{3u^2-1}{6}[\frac{\Delta^3 y_2+\Delta^3 y_1}{2}+\frac{2u^3-u}{12}\Delta^4 y_2+\frac{5u^4-15u^2+4}{120}\Delta^5 y_3+$
$\Delta^5 y\_2]+$ ....(4.12)*and*

$f''(x)=\frac{1}{h^2}[\Delta^2 y\_1+\frac{\Delta^3 y\_2+\Delta^3-1}{2}+\frac{6u^2-1}{12}\Delta^4 y\_2+\frac{2u^3-3u}{12}(\frac{\Delta^5 y\_3+\Delta^5 y\_2}{2})+........](4.13)$

At x=x_0 u=0 and Eqs(4.12)and(4.13) become

f'(x_0)=$\frac{1}{h}[\frac{\Delta y_0+\Delta y\_1}{2}$-$\frac{1}{6}(\frac{\Delta^3 y\_1+\Delta^3 y\_2}{2})+\frac{1}{30}(\frac{\Delta^5 y\_2+\Delta^5 y\_3}{2})+......](4.14)$

f"(x_0)=$\frac{1}{h^2}[\Delta^2 y\_1-\frac{1}{12}\Delta^4 y\_2+.....]$

Example 4.3.1. *Find* $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ *for x=0.2 for the data given in following*

x	*0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
y	*0*	*0.10017*	*0.20134*	*0.30452*	*0,41076*	*0.52115*

Solution:

construct the following difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	*0*				
		0.10017			
0.1	*0.10017*		*0.001*		
		0.10017		*0.00105*	
0.2	*0.20134*		*0.00201*		*0.00004*
				0.00105	
0.3	*0.30452*		*0.00306*		*0.000004*
		0.10624		*0.00109*	
0.4	*0.41076*		*0.00415*		
		0.11039			
0.5	*0.52115*				

Here we use stirrings formula Hence for x=0.2 we have

$\frac{dy}{dx}=\frac{1}{2}[\frac{\Delta y_1+\Delta y_0}{2}-\frac{1}{6}][\frac{\Delta^3 y_2+\Delta^3 y_1}{2}]$

$=\frac{1}{0.1}[(\frac{0.10117+0.10318}{2})-\frac{1}{12}(0.00101+0.00105)]$

$=1.020033$

$\frac{d^2y}{dx^2}=\frac{1}{h^2}[\Delta^2 y_-1-\frac{1}{12}\Delta^4 y_-2]=\frac{1}{(0.1)^2}[0.00201-\frac{1}{12}(0.00004)]$

$=0.200666$

Example 4.3.2. *compute the values of f'(3.1)and f'(3.2) using the following table*

x	1	2	3	4	5
f(x)	0	1.4	3.3	5.6	8.1

Solution:The central difference table is

x	y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_-2=1$	0				
		1.4			
$x_-1=2$	1.4		0.5		
$x_0=3$	3.3		0.4		-0.1
		2.3		-0.2	
$x_1=4$	5.6		0.2		
		2,5			
$x_2=5$	8.1				

$x_0=3, h=1, u=\frac{3.1-3}{1}=0.1$

$f'(3.1)=\frac{1}{h}[\frac{\Delta y_-1+\Delta y_0}{2}+u\Delta^2 y_-1+\frac{3u^2-1}{6}(\frac{\Delta^3 y_-2+\Delta^3 y_-1}{2})+\frac{2u^3-u}{12}\Delta^4 y_-2+....]$

$=\frac{1}{1}[\frac{1.9+2.3}{2}+0.1(0.4)+\frac{3(0.1)^2-1}{6}(\frac{-0.1-0.2}{2})+\frac{2(0.1)^3-0.1}{12}(-0.1)]$

$=[2.1+0.04+0.02425+0.00082]=2.16507$

$f'(3.1)=\frac{1}{h^2}[\Delta^2 y_-1+(\frac{\Delta^2 y_-2+\Delta^3 y_-1}{2})+\frac{6u^2-1}{12}\Delta^4 y_-2+.....]$

$=\frac{1}{1^2}[0.4=0.1(\frac{-0.1-0.2}{2})+\frac{6(0.1)^2-1}{12}(-0.1)]=[0.4-0.015+0.00783]=0.39283$

4.4 Application of Numerical Differentiation

• Advantages of numerical differentiation:

Sometimes differential equations are very difficult to solve analytically or models are needed for computer simulations. In these cases finite-difference methods are used to solve the equations instead of analytical ones.
Finite-difference methods are numerical methods for solving differential equations by approximating them with difference equations, in which finite differences approximate the derivatives

• Disadvantage of numerical differentiation:

There are several ways to treat the numerical differentiation problem. The most common and simplest way, which is used by the engineers, is finite difference. One of the disadvantages of this method is that, in the case that the data contains errors, even if we have many data, we cannot use all data

4.4.1 Maximum and Minimum Value Tabulate Form

From calculus, we know that if a function is differentiable, then the maximum and minimum value of that function can be determined by equating the first derivative to zero and solving for the variable.
Given a set of data points (x_i, y_i) , i = 0,1,2,....., n, we can get the interpolating polynomial of degree n. Now we wish to estimate the value of x at which the curve is maximum or minimum. We know that the maximum and minimum values of a function can be determined by equating the first derivative to zero and solving for the variable.
The same procedure can be applied to find the maxima and minima of a tabulated function. Assume that the points are equally spaced with a step size of h.

This method is extendable for the tabulated function.

Now, consider the Newton's forward difference formula given in Eq.(4.1).

Differentiating Eq.(4.1) w.r.t. u, we obtain

$\frac{dy}{du}$=Δy_0+$\frac{2u-1}{2}\Delta^2 y_0$+$\frac{3u^2-3u+2}{6}\Delta^3 y_0$+........(4.16)

For maximum or minimum $\frac{dy}{du}$=0 neglecting the term after the third difference to obtain a quadratic equation in u

Hence Δy_0+(u-$\frac{1}{2}$)$\Delta^2 y_0$+[$\frac{u^2}{2}$-$\frac{u}{2}$+$\frac{1}{3}$]$\Delta^3 y_0$=0...........(4.17)

or$\frac{\Delta^3 y_0}{2}u^2$+[$\Delta^2 y_0$-$\frac{1}{2}\Delta^2 y_0$]u+[$\Delta t_0$-$\frac{1}{2}\Delta^2 y_0$+$\frac{1}{3}\Delta^3 y_0$]=0

or $a_0 u^2$+$a_1 u$+a_2

Which gives the values of u

Here a_0=$\frac{1}{2}\Delta^3 y_0$

a_1=$\Delta^2 y_0$-$\frac{1}{2}\Delta^3 y_0$

a_2=Δy_0-$\frac{1}{2}\Delta^2 y_0$+$\frac{1}{3}\Delta^3 y_0$

The values of x will then the n be obtained from x=x_0+uh

Example 4.4.1. *Find x correct to four decimal places for which y is maximum from the following data given in tabular form Find also the values of y*

x	*1*	*1.2*	*1.4*	*1.6*	*1.8*
y	*0*	*0.128*	*0.544*	*1.298*	*2.44*

Solution:We first construct the forward difference tables as shown below

x	y	Δy	Δ^2y	Δ^3y
1.0	0			
1.2	0.128	0.288		
1.4	0.544	0.416	0.338	0.05
1.6	1.298	0.754	0.388	0.05
1.8	2.44	1.142		

Let x_0=1.0

Here a_0=$\frac{1}{2}$(0.05)=0.025

a_1=0.288-$\frac{1}{2}$(0.05)=0.2635

a_2=0.288-$\frac{1}{2}$(0.288)+$\frac{1}{3}$(0.05)=0.128-0.144+0.0166=0.000666

Hence a_0u^2+a_1u+a_2=0 which gives the values of u

,or 0.025u^2+0.263u+0.000666=0

$u_{1.2}$=$\frac{-0.263 \pm \sqrt{(0.263)^2 - 4(0.025)(0.000666)}}{2(0.025)}$=(0,-10.5175)

Hence u=0 or u=-10.5175

There fore x=1.0 and x=1.0-10.575(0.2)=-1.1035

At x=1.0,y=0 and at x-1.1035 we apply the newtons forward interpolation formula

y=y_0+uΔy_0+$\frac{u(u-1)}{2!}\Delta^2 y_0$+$\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$+...

=0+(-10.5175)(0.128)+$\frac{(-10.5175)(-11.5175)}{2}$(0.288)

+$\frac{(-10.5175)(11.5175)(-12.5175)}{(3)(2)(1)}$(0.05)

=3.46132(maximum values)

conclusion

Numerical differentiation is not particularly accurate process due to a conflict between round off errors and errors inherent in interpolation.
Hence a derivative of function can never be computed with the same precision as the function it self
Differentiation is one of the important concepts inc calculus,which has been used almost everywhere in many fields of mathematics and applied mathematics.it is natural that numerical differentiation should be an important technique for the engineers.However,since it is ill-posed in handamard's sense,which means that any small error in the measurements will be enlarged,it is very difficult for the engineers to use this unique.in this article,we propose anew simple numerical method to reconstruct the original function and its derivatives from scattered input data and show that our method is effective and can be realized easily .
since many of the equations arising in real life application can not be solved analytically or we can say that their analytical solution does not exist.for such type of problems certain numerical methods exists in the literature.

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