College of Natural and Computational Science
Department of Mathematics

Solving Resistance Inductance Capacitance (RLC) series circuit Using Second Order Ordinary Differential Equation

Prepared by: Mohammed Rebato

Advisor: Mr Solomon Amsalu

A Project Submitted to the Department of Mathematics, Wolkite University in Partial Fulfillment of the Requirements of the Bachelor of Science Degree in Mathematics

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Contents

Wolkite University
Department of Mathematics

The undersigned hereby certify that they have read and recommend to the Department of Mathematics for acceptance of a project entitled **Solving Resistance Inductance Capacitance (RLC) series circuit Using Second Order Ordinary Differential Equation** by Mohammed Rebato in partial fulfillment of the requirements for the degree of Bachelor of Science.

Date: January, 2021

Advisor: ____________________

Advisor name

Examining committee: ____________________

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Abstract

This project deals about second order ordinary differential equation and its application in science and Engineering such as resistance inductance capacitance used in many electronic systems. It also deals about differential equation that describes the flow of series circuit.

Notations

y''	Second derivative of y with respect to x
y_h	General solution for homogeneous
y_p	Particular solution
$y(x)$	General solution for non homogeneous
$\int$	Integral
Ω	Ohmes
$I(t)$	Current
$q(t)$	Charge
L	Inductance
R	Resistance
C	Capacitance
$E(t)$	Electric potential

Introduction

Differential equation is an equation involving independent variable, dependent variable and the derivatives or differentials of one dependent variable with respect to one or more independent variables.

It is a mathematical tool invented by Isaac Newton 1676 and Gottfrid Leibniz[1993] Newton Leibniz years, the exact chronological origin and history to the subject of differential equation is abet of a murky subject for what seems to be a number of reasons; one being secretiveness, two being private publication issues [private works published only decades latter] and three being the nature of the battle of mathematical and scientific discovery, which is a type of intellectual "war"[in worlds of English polymath Thomas physicist Isaac Newton wrote his then unpublished]

The differential equation that has order two is called second order ordinary differential equation.it is an ordinary differential equation where the second derivative of the unknown function appears often occur in problems arising in the physical and engineering.

Many physical phenomena are modeled by second order ordinary differential equations some examples RLC series circuit.

Chapter 1

Preliminaries

1.1 Basic Terms

An equation containing the derivatives of one or more dependent variables with respect to one or more independent variable is said to be a differential equation. A differential equation is said to be ordinary differential equation if it contains only ordinary derivatives of one or more dependent variables with respect to a single independent variable. Independent variable is variables that designate values for which a function is defined are called independent variable. Dependent variable is variables which identify values of function are called dependent variables. The order of differential equation is the order of the highest derivative in the equation

1.1.1 Ordinary Differential Equation(ODE)

Definition 1.1.1. *ordinary differential equation is an equation which involves one independent variable and the derivative of dependent variable with respect to that independent variable.*

1.1.2 Basic Concept of Ordinary Differential Equation

ORDER:- is the order of differential equation is the order of the highest derivative in the equation.
DEGREE:- the degree of differential equation is determined by the power to which the highest derivative raised.
LINEARITY:-differential equation is linear only if the dependent variable and its derivative occurs the first degree.
NON LINEARITY:- differential equation is non linear the dependent variable and its derivative is more than first degree.
Forms of particular solution
a) If f(x) is a constant function, then $y_p = a$
b) If f(x) is a linear function, then $y = ax + b$
c) If f(x) is a quadratic function, then $y_p = ax^2 + bx + c$
d) If f(x) is exponential function, then $y_p = ae^x$

1.1.3 Homogeneous Equation and Non-Homogeneous Equation

Definition 1.1.2. *second order ordinary differential equation for the equation is*

$$y^{''} + p(t)y^{'} + q(t)y = r(t).$$

where p , q , r are given function on the interval I an element of R.

i. $y^{''} + p(t)y^{'} + q(t)y = r(t)$ is homogeneous if and only if r(t)=0 for all $t \in R$.

ii. $y^{''} + p(t)y^{'} + q(t) = r(t)$ has non homogeneous if and only if the source of r(t)$\neq 0$ for all $t \in R$.

A linear n^{th}-order differential Equation of the form

$$a_n x\frac{d^n y}{d^n y} + a_{n-1}\frac{d^{n-1}}{dx^{n-1}} + ... + a_1(x\frac{dy}{dx}) + a_0(x)y = 0$$

is saide to be homogeneous ,whereas an Equation

$$a_n x\frac{d^n y}{d^n y} + a_{n-1}\frac{d^{n-1}}{dx^{n-1}} + ... + a_1(x\frac{dy}{dx}) + a_0(x)y = g(x)$$

is saide to be non-homogeneous

Super-position principle-homogeneous equation

let $y_1(x), y_2(x) \dots y_n(x)$ by a solution of equation then the linear combination

$$y = c_1 y_1(x) + c_2 y_2(x) + \cdots + c_n y_n(x)$$

were $c_1, c_2 \dots$ are constants is a solution

Example 1.1.1. *verify whether the following differential equation are homogeneous or not?*

$$A.\ \ y^{''} - 5y = 0$$

$$B.\ \ y^{''} - 5y^{'} + 2y = 5x$$

solution:- *the equation is* $y^{''} + p(t)y^{'} + q(t)y = r(t)$.

then A is homogeneous then if and only if r(t)=0 for all $t \in R$.
B is not homogeneous then if and only if r(t)$\neq 0$ *for all* $t \in R$.

1.1.4 Linearly Dependent and Independent Functions

A set of functions is said to be linearly dependent if w=0
linearly independent if $w \neq 0$
wroskian :-suppose each of the function $f_1(x), f_2(x), f_3(x) \cdots f_n(x)$ posses at least n-1 derivatives the determinant.

$$W(f_1(x), f_2(x), f_3(x) \cdots f_n(x)) = \begin{vmatrix} f_1 & f_2 & f_3 \cdots f_n \\ f_1^{'} & f_2^{'} & f_3^{'} \cdots f_n^{'} \\ \vdots & & \\ f_1^{n-1} & f_2^{n-1} & f_3^{n-1} \cdots f_n^{n-1} \end{vmatrix}$$

Theorem 1.1.1. *The wronskian of two solution of differential equation is*

$$y^{''} + py^{'} + qy = 0$$

where p,q either constant or function of x alone is either identically zero or never zero.

Proof 1.1.1. *Let $y_1(x)$ and $y_2(x)$ be two solution of*

$$y^{''} + py^{'} + qy = 0 \tag{1.1}$$

Then we have

$$\Longrightarrow y_1^{''} + py_1^{'} + qy_1 = 0, and, y_2^{''} + py_2^{'} + qy_2 = 0$$
$$\Longrightarrow y_1^{''} = -(py_1^{'} + qy_1), and, y_2^{''} = -(py_2^{'} + qy_2) \tag{1.2}$$

Now the wronlskian W of y_1 and y_2 given by

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1^{'} & y_2^{'} \end{vmatrix} = y_1y_2^{'} - y_2y_1^{'} \tag{1.3}$$

$$\Longrightarrow W^{'} = y_1^{'}y_2^{'} + y_1y_2^{''} - [y_2^{'}y_1^{'} + y_2y_1^{''}]$$
$$= y_1y_2^{''} - y_2^{''}y_2$$

$$or\ W^{'} = -y_1(py_2^{'} + qy_2) + y_2(py_1^{'} + qy_1) using, equation (1.2)$$

$$or\ \ W^{'} = -p(y_1y_2^{'} - y_2y_1^{'}) = -pw\ using\ equation (1.3)$$

$$\Longrightarrow W^{'} + PW = 0$$

Showing that W is identically zero or never zero.

Example 1.1.2. *Show that*

$$y(x) = c_1e^x + c_2e^{-x}$$

is linearly independent solution of

$$y^{''} - y = 0?$$

solution: now to check whether the solution of

$$y(x) = c_1e^x + c_2e^{-x}$$

$$is\ the\ general\ solution\ of\ y^{''} - y = 0$$

$$we\ use\ the\ wroskian\ w(y_1, y_2) \neq 0$$

$$y_1 = e^x$$

$$y_2 = e^{-x}$$

$$y_1^{'} = e^x$$

$$y_2^{'} = -e^{-x}$$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1^{'} & y_2^{'} \end{vmatrix} = W(e^x, e^{-x}) = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$$

therefor $y(x) = c_1e^x + c_2e^{-x}$
is the general solution of $y^{''} - y = 0$
since $e^x * -e^{-x} - e^x * e^{-x} = -2 \neq 0$

1.1.5 Linear Second Order Homogeneous Ordinary Differential Equation with Constant Coefficient.

Auxiliary/Characteristic Equation
let begin considering second order differential equation

$$ay^{''}(x) + by^{'}(x) + cy(x) = 0 \tag{1.4}$$

where a , b, c are constant If we tray to find the solution of the form

$$y(x) = e^{mx}$$
$$y^{'}(x) = me^{mx}$$
$$y^{''}(x) = m^2e^{mx}$$

now substituting $y(x)$, $y^{'}(x)$, $y^{''}(x)$ the differential equation(1.4) yialds

$$am^2e^{mx} + bme^{mx} + ce^{mx} = 0 \tag{1.5}$$

$$(am^2 + bm + c)e^{mx} = 0$$
$$am^2 + bm + c = 0, since, e^{mx} = 0$$

Equation(1.5) is auxiliary Equation of the differential equation(1.4)

1.1.6 The General Solution of Homogeneous Second Order Ordinary Differential Equation

Definition 1.1.3. *let $y_1, y_2 \dots y_n$ be a function dominated set of solution of the homogeneous linear n^{th}-order ODE ,then the General solution is of the equation is $y = c_1y_1(x) + c_2y_2(x) + \cdots + c_ny_n(x)$ where $c_1, c_2 \dots c_n$*

$$ay^{''}(x) + by^{'}(x) + cy(x) = r(t) \text{ is homogeneous if and only if r(t)=0 for all } t \in R.$$

Step how to find the general solution for homogeneous.
$Step_1$:- write down the auxiliary equation $ay^{''}(x) + by^{'}(x) + cy(x) = 0$
$Step_2$:- calculate m_1andm_2 from $step_1$.
$Step_3$:- There will be 3 solution of $step_1$ corresponding to the 3 cases

1. **case**: two real distinct roots($b^2 - 4ac > 0$) then the general solution given by

$$y_x = c_1e^{m_1x} + c_2e^{m_2x}$$

2. **case** : one real root ($b^2 - 4ac = 0$) then the general solution given by

$$y_x = c_1e^{m_1x} + c_2xe^{m_1x}$$

3. **case** : complex conjugate roots $b^2 + -4ac < 0$. then the general solution given by

$$y_x = e^{\alpha(x)}(c_1cos\beta x + c_2sin\beta x)$$

$$where \;\; \alpha + i\beta = \frac{-b \pm \sqrt{4c - b^2i}}{2}$$

$$\alpha = \frac{-b}{2}$$

$$\beta = \frac{\sqrt{4c - b^2}}{2}$$

Example 1.1.3. *find the general solution of differential equation using auxiliary equation:*

a. $y^{''} - 5y^{'} + 6y = 0$

b. $y^{''} - 6y^{'} + 9y = 0$

c. $y^{''} - 4y^{'} + 13y = 0$

$a.\ y'' - 5y' + 6y = 0$
solution auxiliary equation :- $m^2 - 5m + 6 = 0$

$$(m-2)(m-3) = 0$$

$$m = 2 \& m = 3$$

$\therefore$ *the general solution is* $y(x) = c_1 e^{2x} + c_2 e^{3x}$

$b.\ y'' - 6y' + 9y = 0$
solution auxiliary equation :- $m^2 - 6m + 9 = 0$

$$(m-3)(m-3) = 0$$

$$m = 3 \& m = 3$$

$\therefore$ *the general solution is* $y(x) = c_1 e^{3x} + c_2 x e^{3x}$

$c.\ y'' - 4y' + 13y = 0$
solution auxiliary equation :- $m^2 - 4m + 13 = 0$

$$m = \frac{4 \pm \sqrt{4^2 - 4(13)}}{2}$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$m = \frac{4 \pm \sqrt{-36}}{2}$$

$$m = 4 \pm \frac{6i}{2}$$

$$\therefore m = 2 \pm 3i$$

$$\alpha = 2, \beta = 3$$

$\therefore$ *the general solution is* $y(x) = e^{2x}(c_1 cos3x + c_2 sin3x)$

1.1.7 Linear Second Order Non Homogeneous Ordinary Differential Equation With Constant Coefficient.

Definition 1.1.4. *let $y_p x$ be any particular solution of the Nan-homogeneous linear n^{th}-order (differential equation) & let $y_1, y_2 \dots y_n$ be a fundamental set of solution of the assoseted homogeneous (differential equation) then then General solution of the (differential equation)*

$$is \quad y = c_1 y_1(x) + c_2 y_2(x) + \cdots + c_n y_n(x)$$

$$y = y_c x + y_p x or y = y_n x + y_p x$$

$$any^n(x) + a_{n-1} y_{n-1}(x) + \cdots + a_1 y^{'}(x) + a_0 y(x) = g(x) \tag{1.6}$$

To solve (1.6) find $y_c(x)$ and $y_p(x)$ then the general solution of (1.6)

$$is \quad y(x) = y_c(x) + y_p(x)$$

$$y^{''} + p(t)y^{'} + q(t) = r(t)$$

Has non homogeneous if and only if the source of r(t)$\neq 0$ for all $t \in R$.

1.1.8 The General Solution of Non-homogeneous Second Order Ordinary Differential Equation

There are finding the general solution of the non homogeneous second order ordinary differential equations

there are method to find $y_p(x)$

1. The method of undetermined coefficient.
2. The method of variation of parameters.

1. **The Method of Undetermined Coefficient**

consider the following tables
$g(x)$..form $y_p(x)$.

1. any constant .... A
2. 5x + 7 .... Ax + B
3. $3x^2 - 2$.......................................$Ax^2 + Bx + C$
4. $x^3 - x + 1$.........................$Ax^3 + Bx^2 + Cx + D$
5. $sin4x$..................................$Acos4x + Bsin4x$
6. e^{ax}..Ae^{ax}

7. $(9x-2)e^{ax}$.............................$(Ax+B)e^{ax}$

8. x^2e^{5x}.....................$(Ax^2+Bx+C)e^{5x}$

9. $xe^{3x}cos4x$............$(Ax+B)e^{3x}cos4x+(cx+D)e^{3X}sin$ 4x

10. x^2sin4x..............$(Ax^2+Bx+C)cos4x+(Dx^2+Ex+F)sin4x$

11. $xe^{3x}cos4x$.....................$(AX+B)e^{3X}cos4x+(CX+D)e^{3X}sin4x$

If y_n is the general solution of the associated homogeneous equation

$$y^{''}+by^{'}+cy=0$$

and y_p is a particular solution of the non homogeneous equation

$$y^{''}+by^{'}+cy=r(x)$$

then y(x)=y_c+y_p is the general solution of the non homogeneous equation

$$y^{''}+by^{'}+cy=r(x)$$

the procedure for finding y_p is the method of undetermined coefficients the form of y_p to be chosen depend on $r(x)$

Example 1.1.4. *Using undetermined coefficients solves the following ordinary differential equation.*

$$y^{''}-3y^{'}+2y=8$$

solution :- *first homogeneous part using auxiliary equation:* $m^2-3m+2=0$

$$(m-1)(m-2)$$

$$m=1,2$$

$$y_c=c_1e^x+c_2e^{2x}$$

Second non homogeneous part:

$$Let Yp = a$$

$$y_p^{'} = 0$$

$$y_p^{''} = 0$$

$$\therefore becomes : y^{''} - 3y^{'} + 2y = 8$$

$$in\ to\ equation\ 0 - 3(0) + 2(a) = 8$$

$$2a = 8$$

$$a = 4$$

$$Y_p = 4$$

$\therefore the\ general\ solution\ is y(x) = c_1e^x + c_2e^{2x} + 4$

$$since y(n) = y_c + y_p$$

2. **The method of variation of parameters**

The linear second order differential equation in this standard form

$$y^{''} + p(x)y^{'}(x) + q(x)y(x) = f(x).... \tag{1.7}$$

and
$y_c(x) = c_1y_1(x) + c_2y_2(x)$ if we replace the parameters $c_1\ and\ c_2$
$u_1(x)\ and\ u_2(x)$

respect then
$y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$
$y_p^{'}(x) = u_1^{'}(x)y_1(x) + u_1(x)y_1^{'}(x) + u_2^{'}(x)y_2(x) + u_2(x)y_2^{'}$

$y_p^{''}(x) = u_2^{''}(x)y_1(x) + u_1^{'}(x)y_1^{'}(x) + u_1^{'}(x)y_1^{'}(x) + u_1^{'}(x)y_1^{'}(x) + u_1^{'}(x)y_1^{''}(x) + u_2^{''}(x)y_2(x) + u_2^{'}(x)y_2^{'}(x) + u_2^{'}(x)y_2^{'}(x) + u_2(x)y_2^{''}(x)$

now substiting $y_p(x), y_p^{'}(x), y_p^{''}(x)$ in to the differential equation

give: $u_1(y_1^{''} + p(x)y_1^{'}(x) + q(x)y_1(x)) + u_2(y_2^{''} + p(x)y^{'}x) + q(x)y_2(x) + u_1^{'}(x)y_1^{'}(x) + u_2^{'}(x)y_2^{'}(x) + u_2^{''}(x)y_2(x) + u_2^{'}(x)y_2^{'}(x) + u_2^{''}(x)y_2(x) + p(u_1^{'}(x)y_1(x) + u_2^{'}(x)y_2(x)) + u_1^{'}(x)y_1^{'}(x) + u_2^{'}(x)y_2^{'}(x) = f(x)$

then $\frac{d}{dx}(y_1u_1^{'} + y_2u_2^{'}) + p(y_1u_1^{'} + y_2u_2^{'}) + y_1^{'}u_1^{'} + y_2^{'}u_2^{'} = f(x)$

now let's furher suppetion

$$y_1u_1^{'} + y_2u_2^{'} = 0$$

$$y_1^{'}u_2^{'} + y_2^{'}u_2^{'} = f(x)$$

can be expressed in terms of determinants using cramer rule

$$u_1^{'} = \frac{W_1}{W} = -\frac{y_2f(x)}{W} \; and \; u_2^{'} = \frac{W_2}{W} = \frac{y_1f(x)}{W}$$

$$u_1(x) = \int U_1^{'}(x)$$

$$u_2(x) = \int U_2^{'}(x)$$

Example 1.1.5. *Using variation of parameters solve the following ordinary differential equation.* $y^{''} + 4y = sec2x$
solution :- *first homogeneous part by auxiliary equation:*

$$m^2 + 4 = 0$$

$$m = \pm 2i$$

$$\alpha = 0, \beta = 2$$

$$Implies \; Yc = c_1cos2x + c_2sin2x$$

$$Non-homogeneouspart:$$

$$y_1 = cos2x$$

$$y_2 = sin2x$$

$$f(x) = sec2x$$

$$W = \begin{vmatrix} cos2x & sin2x \\ -2sin2x & 2cos2x \end{vmatrix}$$

$$= 2cos^2 2x + 2sin^2 2x$$

$$= 2(cos^2 2x + sin^2 2x) = 2$$

$$because \;\; cos^2 2x + sin^2 2x = 1$$

$i)\ u_1 = -\int \frac{y_2 f(x)}{w} dx = -\int \frac{sin2x(sec2x)}{2} dx = -\frac{1}{2}\int \frac{sin2x}{cos2x} dx = \frac{1}{4} ln|cos2x|$

$ii)\ u_2 = \int \frac{y_1 f(x)}{w} dx = \int \frac{cos2x(sec2x)}{2} dx = \frac{1}{2}\int dx = \frac{x}{2}$

$this\ implies\ y_p = y_1 u_1 + y_2 u_2$

$y_p = cos2x(\frac{1}{4} ln|cos2x|) + sin2x(\frac{x}{2})$

$\therefore\ general\ solution\ is\ y(X) = y_c + y_p$

$y(X) = c_1 cos2x + c_2 sin2x + \frac{cos2x ln|cos2x|}{4} + \frac{x}{2} sin2x$

Chapter 2

Application of Second Order Ordinary Differential Equation

Second order ordinary differential equations have a variety of application in science and engineering. In this Resistance Inductance capacitance (RLC) series circuit is an electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor (C), connected in series or in parallel.

The name of the circuit is derived from the letters that are used to denote the constituent components of this circuit, where the sequence of the components may vary from RLC.

2.1 Resistance Inductance Capacitance (RLC) series circuit

2.1.1 Introduction to RLC series circuits.

The principal aim is to given a brief derivation of differential equation that describes the flow of current in a simple series circuit.

The electromotive force (whose units are volts) is represented by E and could typically be a battery or a generator, and the resistance (whose units are ohms) is represented by R.

If only the electromotive force(emf) and the resistance are present in the circuit, then ohms law assert that the instantaneous current I (units are amperes) is directly proportional to the emf(E)

Moreover, if R is the constant, Such that:
The sum of the voltage drops across resistors, indicators and capacitors is equal to the total electromotive force in a closed circuit

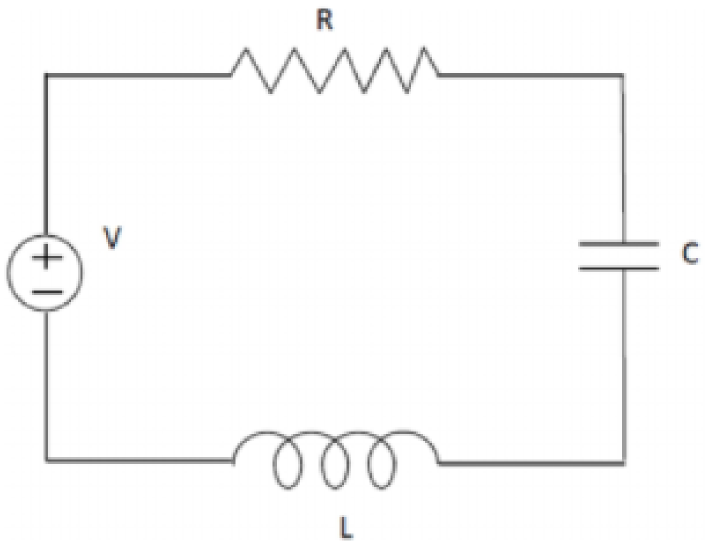


Fig 2.1 *Resistance Inductance Capacitance* (*RLC*) *series circuit*

Resistance electrical circuits often have an inductance (unit are henry) and a capacitance(units are farads) and inductor which is represented by L an electrical circuit is a path in which electrons from a voltage or current source flow. Electric circuit is a path way made of wires that electrons can flow through.

A battery or other power source gives the force (voltage) that makes the electrons move When the electrons get to a device like a light bulb, your computer, or refrigerator, they give it the power to make it work.

Capacitor is an element used to stores electric energy in the circuit Each of the elements in the circuit determines a voltage drop (or a potential drop).

Consider an electrical circuit containing a resistor, an inductor, and a capacitor, . Such a circuit is called an RLC series circuit. RLC circuits are used in many electronic systems, most notably as tuners in AM/FM radios. The tuning knob varies the capacitance of the capacitor, which in turn tunes the radio. Such circuits can be modeled by second-order, constant-coefficient differential equations.

Let
I(t).....denote the current in the RLC circuit and
q(t)......denote the charge on the capacitor.
L........denote inductance in henrys (H),
$R......denote\, resistance\, inohms(\Omega)$
C....... denote capacitance in farads (F).
E(t).....denote electric potential in volts (V).
Emf(E)...denote electromotive force in volts(v)

Kirchhoff's voltage rule states that the sum of the voltage drops around any closed loop must be zero. So, we need to consider the voltage drops across the inductor
(denoted EL),
the resistor (denoted ER), and

the capacitor (denoted EC). Because the RLC circuit includes a voltage source, E(t) , which adds voltage to the circuit, we have EL+ER+EC=E(t)

Using Faraday's law and Lenz's law, the voltage drop across an inductor can be shown to be proportional to the instantaneous rate of change of current, with proportionality constant L.

Thus $EL = L\frac{dI}{dt}$

Ohm's law, the voltage drop across a resistor is proportional to the current passing through the resistor, with proportionality constant R.
Therefore, ER=RI. Last, the voltage drop across a capacitor is proportional to the charge, q, on the capacitor, with proportionality constant $\frac{1}{c}$
Thus $EC = \frac{1}{c}q$
Adding these terms together, we get $L\frac{dI}{dt} + RI + \frac{1}{c}q$
Noting that $I = (\frac{dq}{dt})$,this becomes $\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{c}q = E(t)$

This drop in voltage (which can be measured with the use of a voltmeter) behaves according to the following physical laws.

a. The voltage drop across a resistance of R ohms equal to RI

b. The voltage drop across an inductance of L henry equals $LI^{'}$

c. The voltage drop across a capacitance of C farads equals Q/C

Applying Kirchhoff's laws as follow:

$$LI^{'} + RI + \frac{Q}{C} = E(t) \tag{2.1}$$

The current I equal the time rate of change of Q

$$I(t) = Q^{'}(t) \ and \ Q(t) = Q_0 + \int I(t)dt \tag{2.2}$$

Where, Q_0 is the change at time t_0.
since $I = Q^{'}$ and $I^{'} = Q^{''}$ then substitutes in equation (2.1) we get

$$\Rightarrow LQ^{''} = RQ^{'} + \frac{Q}{C} = E(t) \tag{2.3}$$

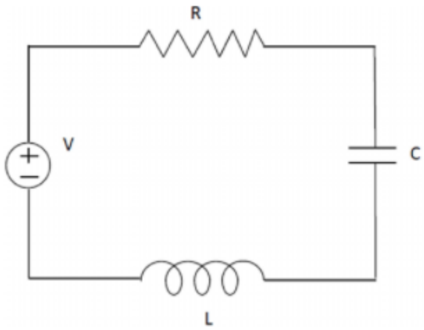
Differentiating both sides of equation (2.1) with respect to I and equation (2.2), we obtain

$$\Rightarrow L\frac{d}{dt}(\frac{dI}{dt}) + R(\frac{dI}{dt}) + \frac{1}{c}\frac{dQ}{dt} = \frac{d}{dt}(E(t))$$

$$\Rightarrow L\frac{I^2}{dt^2} + R\frac{dI}{dt} + \frac{1}{c}I = \frac{d}{dt}(E(t)) \tag{2.4}$$

Equation (2.3) and equation (2.4) is non homogeneous second order linear differential equation in Q and I with constant coefficients respectively

Example 2.1.1. *: find the charge and current at time t in the circuit. If R=400Ω , L=1H,* $C=16*10^{-4}F$*, E(t)=100cos10t*
And the switch closes than initial change and current are both zero



Solution : *with the given value of L,R,C and E(t) then*
$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{c}Q = E(t)$
Becomes $\frac{d^2Q}{dt^2} + 40\frac{dQ}{dt} + 625Q = 100COS10t$
The auxiliary equation is $r^2+40r+625=0$
with root: $r = \frac{-40\pm\sqrt{40^2-4(625)}}{2}$
$r = \frac{-40\pm\sqrt{1600-4(625)}}{2}$
$r = \frac{-40\pm\sqrt{1600-2500}}{2}$
$r = \frac{-40\pm\sqrt{-900}}{2}$
$r = -20 \pm 15i$
So the solution of the complementary equation is:
$Q_c(x) = e^{-20t}(c_1cos15t + c_2sin15t)$

For the method of undetermined coefficients we try the particular solution

$$Q_p(t) = Acos10t + Bsin10t$$

$$Q_p'(t) = -10Asin10t + 10Bcos10t$$

$$Q_p''(t) = -100Acos10t - 100Bsin10t$$

Substituting into equation (1) we get:

$$\Rightarrow \frac{d^2Q}{dt^2} + 40\frac{dQ}{dt} + 625Q = 100cos10t$$

$$\Rightarrow Q'' + 40Q' + 625Q = 100cos10t$$

$\Rightarrow (525A+400B)cos10t+(-400A+525B)sin10t=100cos10t$

$$\begin{cases} 525A+400B=100 \\ -400A+525B=0 \end{cases} \Rightarrow \begin{cases} 21A+16B=4 \\ -16A+21B=0 \end{cases}$$

$\therefore\ 16A=21B \Rightarrow A=\frac{21}{16}B$

$then \quad 21A+16B=4$

$\Rightarrow 21(\frac{21}{16}B)+16B=4$

$\Rightarrow \frac{441}{16}B+16B=4$

$\Rightarrow (\frac{441+256}{16})B=4$

$\Rightarrow (\frac{697}{16})B=4$

$\Rightarrow 697B=64$

$\Rightarrow B=\dfrac{64}{697}$

$then\ A=\dfrac{21}{16}B \quad = \dfrac{21}{16}(\dfrac{64}{697})=\frac{84}{697}$

,so a particular solution is $Q_p(t)=\frac{1}{697}(84cos10t+64sin10t)$

And the general solution is $Q(t)=Q_c(t)+Q_p(t)$
$Q(t)=e^{-20t}(c_1cos15t+c_2sin15t)+\frac{4}{697}(21cos10t+16sin10t)$

Imposing the initial condition Q(0)=0, we get

$Q(t)=c_1+\frac{84}{697}=0 \Rightarrow c_1=-\frac{84}{697}$

To impose the other initial condition we first differentiate to find the current
$I=\frac{dQ}{dt}=e^{-20t}[(20c_1+15c_2)cos15t+(-15c_1-20c_2)sin15t]+\dfrac{40}{697}(-21sin10t+16cos10t)$

$I(0)=-20c_1+15c_2+\frac{40}{697}=0,\ c_2=\frac{-464}{2091}$

Thus the formula for the change is
$I(t)=\frac{1}{2091}[e^{-20t}(-1920cos15t+13060sin15t)+120(-21sin10t+16cos10t)]$

Conclusion

Solve second order ordinary differential equation by using homogeneous second order ordinary differential equation and non homogeneous second order ordinary differential equation. $y^{''} + p(t)y^{'} + q(t)y = r(t)$ is homogeneous if and only if r(t)=0 for all $t \in R$. $y^{''} + p(t)y^{'} + q(t) = r(t)$ has non homogeneous if and only if the source of r(t)$\neq$ 0 for all $t \in R$ and also to solve Resistance Inductance capacitance (RLC) series circuit.

Resistance Inductance capacitance (RLC) series circuit is Consider an electrical circuit containing a resistor, an inductor, and a capacitor in second order ordinary differential equation with solve homogeneous and non homogeneous. (RLC) series circuit

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