

College of Natural and Computational Sciences

Department of physics

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Title: Lorentz transformation Equation in Modern Physics.

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Symbols and acronyms

v	Relative velocity between frames
c	Speed of light,
S’	Transformation of inertial frame
S	Inertial frame

Abstract

Sometimes it becomes a matter of natural choice for an observer (A) that he prefers a coordinate system of two-dimensional spatial x-y coordinates from which he observes another observer (B) who is moving at a uniform speed along a line of motion, which is not collinear with as chosen x- or y-axis. It becomes necessary in such cases to develop Lorentz transformations where the line of motion is not aligned with either the x- or the y-axis. In this paper we develop these transformations and show that under such transformations, two orthogonal systems (in their respective frames) appear non-orthogonal to each other. We also illustrated the usefulness of the transformation by applying it to three problems including the rod-slot problem. We show that the Lorentz transformations for the space-time coordinates of the same event are a direct consequence of the principle of relativity and of Einsteins distant clocks synchronization procedure.

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Chapter 1

Introduction

1.1 Background of study

In physics, the Lorentz transformation (or transformations) is named after the Dutch physicist Hendrix Lorentz. It was the result of attempts by Lorentz and others to explain how the speed of light was observed to be independent of the reference frame, and to understand the symmetries of the laws of electromagnetism. The Lorentz transformation is in accordance with special relativity, but was derived before special relativity. The transformations describe how measurements of space and time by two observers are related. They reflect the fact that observers moving at different velocities may measure different distances, elapsed times, and even different orderings of events. The Galilean transformation is a good approximation only at relative speeds much smaller than the speed of light. The Lorentz transformation is a linear transformation. It may include a rotation of space; a rotation-free Lorentz transformation is called a Lorentz boost. In Murkowski space, the Lorentz transformations preserve the space-time interval between any two events. They describe only the transformations in which the space-time event at the origin is left fixed, so they can be considered as a hyperbolic rotation of Murkowski space. The more general set of transformations that also includes translations is known as the Poincare group[1-2].

1.2 Statement of the problem

Many researchers were shown that Lorentz transformation equation .Would have been conducted the study in order to create a good understanding of the topic to anyone who are interested on this title and present the topic in understandable and simple manner. This research project work would have been expected to answer the following question at the end of the research project work.

1. What is the Lorentz transformation equation?
2. How about the derivation of Lorentz transformation equation?
3. How calculate the mathematical properties of the Lorentz transformation equation?
4. How length contraction and time dilation from Lorentz transformation equation?
5. How calculate the Matrix representation of the Lorentz transformation equation?

1.3 Objective of the study

1.3.1 General objective

To understand Lorentz transformation equation

1.3.2 Specific objective

The specific objective of this study was:

- To explain the Lorentz transformation equation.
- To derive the derivation of Lorentz transformation equation.
- To calculate the mathematical properties of the Lorentz transformation equation.
- To show the length contraction and time dilation from Lorentz transformation equation.
- To show the matrix representation of the Lorentz transformation equation.

1.3.3 Significance of the study

The aim of this work is to develop a better understanding about Lorentz transformation equation. It serves as a reference for researcher doing on the same topic in the future. Another significance of this paper is for students who take this project to get information for their of improving their knowledge. This project is also expected to propose some inspirations for overcoming current defects or limitations and give a brief outlook for the future work. This study is being to give the significant listed below. Motivating other project to involve on Lorentz transformation equation .It helped for student in physics to use as a base further study. To share experience and knowledge about Lorentz transformation equation.

1.3.4 The scope of the study

Our studies have been focused on Lorentz transformation equation. Physics of Lorentz transformation equation is presented by using: all necessary theories, assumptions, experimental and mathematical calculations.

1.3.5 The Limitation of Study

On the process of carrying out some activity challenges was happened.There are some limitations while doing this project work

- Lack of internet accessibility
- Lack of more time to do this project
- Lack of budget

Chapter 2

Literature Review

Now that we have seen the main consequences of the postulates of Special Relativity, i.e. the relativity of simultaneity, time dilation, and length contraction it is clear that the Galileo transformation, with its absolute time, is incorrect. These important physical phenomena can be seen as direct consequences of the correct transformation relating inertial frames, the Lorentz transformation. This transformation is the key for the formulation of special relativity , which we will see in the next chapter. Here we study the Lorentz transformation and its properties and derive length contraction and time dilation directly from it, in addition to the transformation property of velocities. We must emphasize that, although the Lorentz transformation was discovered by studying the Maxwell equations, its validity is more general. The Lorentz transformation relates inertial frames without reference to the kind of physics studied in them. Lorentz-invariance is a general requirement for any physical theory, not just for electromagnetism[3-4]. The inverse Lorentz transformation is obtained by the exchange $x \longleftrightarrow x'$, and $v \longleftrightarrow -v'$ according to the Principle of Relativity.

$$x = \frac{x' + vt'}{\sqrt{(1 - v^2/c^2)}} \tag{2.1}$$

$$y = y'. \tag{2.2}$$

Lorentz was also trying to explain the null result of the Michelson-Morley experiment by a physical contraction of the apparatus in the direction of motion. His interpretation, however, is rather misleading: the Lorentz transformation relates measurements performed in two different inertial systems[5]. A priori, the constant c appearing in

the Lorentz transformation is a fundamental velocity which needs not coincide with the speed of electromagnetic waves in vacuum, and is only later identified with it. This is not the historical route, in which the Lorentz transformation was derived from the Maxwell equations. There are many facets to the quantity c in various areas of physics[6].

$$z = z' \tag{2.3}$$

$$t = \frac{t' + v\frac{x'}{c'}}{\sqrt{(1 - v^2/c^2)}} \tag{2.4}$$

Although the Lorentz transformation was obtained before the formulation of Special Relativity as the correct transformation between inertial frames which respects the Maxwell equations, its meaning was not grasped until Einsteins 1905 paper[7-10].

2.1 Lorentz transformation for frames in standard configuration

Consider two observers O and O', each using their own Cartesian coordinates system to measure space and time intervals. O uses (t, x, y, z) and O uses (t', x', y', z'). Assume further that the coordinate systems are oriented so that, in 3 dimensions, the x-axis and the x'-axis are collinear, the y-axis is parallel to the y'-axis, and the z-axis parallel to the z'-axis. The relative velocity between the two observers is v along the common x-axis; O measures O to move at velocity v along the coincident xx' axes, while O' measures O to move at velocity -v along the coincident xx' axes. Also assume that the origins of both coordinate systems are the same, that is, coincident times and positions. If all these hold, then the coordinate systems are said to be in standard configuration. The inverse of a Lorentz transformation relates the coordinates the other way round; from the coordinates O' measures (t', x', y', z') to the coordinates O measures (t, x, y, z), so t, x, y, z are in terms of t', x', y', z'. The mathematical form is nearly identical to the original transformation; the only difference is the negation of the uniform relative velocity (from v to v), and exchange of primed and unprimed quantities, because O' moves at velocity v relative to O, and equivalently, O moves at velocity v relative to O'. This symmetry makes it effortless to find the inverse transformation (carrying out the exchange and

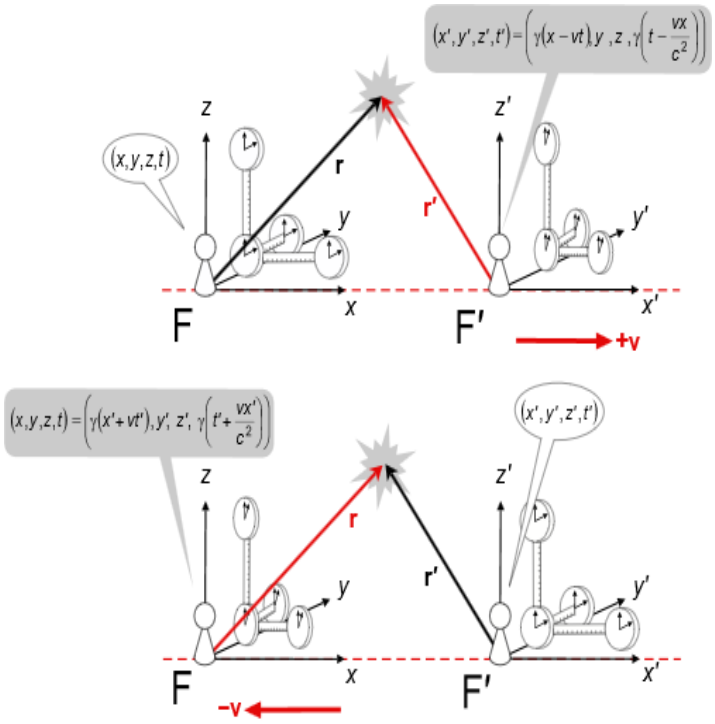


Figure 2.1: Lorentz transformation

negation saves a lot of rote algebra), although more fundamentally; it highlights that all physical laws should remain unchanged under a Lorentz transformation. In the x-direction The frame O moves at velocity v along the x-axis of frame O is given.

$$t' = \gamma(t - \frac{vx}{c^2}) \tag{2.5}$$

$$x' = (x - vt) \tag{2.6}$$

$$y' = y \tag{2.7}$$

$$z' = z \tag{2.8}$$

Where:

- v is the relative velocity between frames in the x-direction,
- c is the speed of light,
- $\gamma = \frac{1}{\sqrt{(1-\beta^2)}}$ is the Lorentz factor (Greek lowercase gamma),
- $\beta = \frac{v}{c}$ is the velocity coefficient (Greek lowercase beta), again for the x-direction.The use of β and γ is standard throughout the literature[11].

For the y-direction:

$$t' = \gamma(t - \frac{vx}{c^2}) \tag{2.9}$$

$$x' = x \tag{2.10}$$

$$y' = \gamma(y - vt) \tag{2.11}$$

$$z' = z \tag{2.12}$$

For the z-direction:

$$t' = \gamma(t - \frac{vx}{c^2}) \tag{2.13}$$

$$x' = x \tag{2.14}$$

$$y' = y \tag{2.15}$$

$$z' = \gamma(z - vt) \tag{2.16}$$

According to the principle of relativity, there is no privileged frame of reference, so the inverse transformations frame O' to frame O must be given by simply negating v:

$$t' = \gamma(t - \frac{vx}{c^2}) \tag{2.17}$$

$$x' = \gamma(x - vt) \tag{2.18}$$

$$y' = y \tag{2.19}$$

$$z' = z \tag{2.20}$$

Where the value of remains unchanged.

Chapter 3

Methodology

3.1 Methodology of study

The information for this study would have been gathered in different way. The source of information is only secondary source of data. The source information is published article, internet, and text books.

3.2 Project design

To write this senior project, we used previous written source and gather extra information from internet. This project was working of the study by reviewing Lorentz transformation equation. Based on the published articles, the necessary information for the research work has been gathered starting from January to March 2019.

Chapter 4

Lorentz transformation

4.1 Derivation of the Lorentz Transformation

The Lorentz transformation can be derived on the basis of the two postulates of Special Relativity. First, due to in the second postulate, we can orient the spatial axes of an inertial frame S' with those of another inertial frame S and limit ourselves to considering motion of the two frames in standard configuration. As a starting point for deducing the transformation relating two inertial frames $S = \{t, x, y, z\{$ and $S' = \{t', x', y', z'\}$ in relative motion with velocity v in standard configuration, it is reasonable to assume that the transformation is linear

$$x = G(x - vt), \tag{4.1}$$

where G is the dimensioless constant that depends only on v/c
There is no relative motion in the y and z directions, hence these coordinates must not be affected by the transformation,

$$y' = y, z' = z \tag{4.2}$$

Consider now a spherical pulse of electromagnetic radiation emitted at the origin of S at $t = 0$. It is received at a point on the x-axis and, during its propagation,

$$x = ct. \tag{4.3}$$

The same law must be true in S' due to the constancy of the speed of light,

$$x' = ct'. \tag{4.4}$$

or $ct' = G(ct - vt)$ from Eq. (4.1), which leads to

$$t' = G(t - \frac{vt}{c}) \tag{4.5}$$

while

$$ct = G(ct' + vt') = G(c + v)t' \tag{4.6}$$

By substituting Eq. (4.2) into Eq. (4.5), one obtains

$$ct = G(c + v)G(t - \frac{vt}{c}) = \frac{G^2}{c}(c + v)(c - v)t,$$

and

$$G^2 = \frac{c^2}{(c + v)(c - v)}$$

Therefore, we have

$$G = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma$$

the Lorentz factor, and

$$x' = \gamma(x - vt). \tag{4.7}$$

Use then $x = \gamma(x' + vt')$ and $x' = \gamma(x - vt)$ and substitute x' into x to obtain

$$x = \gamma[\gamma(x - vt) + vt']$$
$$x = \gamma^2 x - \gamma^2 vt + \gamma vt',$$

from which we obtain

$$t' = \frac{x - \gamma^2 vt + \gamma^2 vt}{\gamma v}$$

and

$$t' = \frac{x}{\gamma v}(1 - \gamma^2) + \gamma t$$

Since

$$1 - \gamma^2 = 1 - \frac{1}{1 - \frac{v^2}{c^2}} = \frac{1 - \frac{v^2}{c^2} - 1}{1 - \frac{v^2}{c^2}} = \frac{-\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}}$$

we have

$$\frac{1-\gamma^2}{\gamma} = \sqrt{1-\frac{v^2}{c^2}}\frac{-\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = \frac{-\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \equiv -\gamma\frac{v^2}{c^2}$$

$$t' = \frac{1-\gamma^2}{\gamma}\frac{x}{v} + \gamma t = -\gamma\frac{v^2}{c^2}\frac{x}{v} + \gamma t$$

and, finally,

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \tag{4.8}$$

which completes the derivation. Equations (4.7), (4.2), and (4.8) constitute the Lorentz transformation.

4.2 Mathematical Properties of the Lorentz Transformation

Let us examine the properties of the Lorentz transformation.

- Qualitatively, the Lorentz transformation mixes t and x, therefore 3-dimensional lengths and time intervals cannot be left invariant. Quantitatively, length contraction and time dilation can be derived from the Lorentz transformation as a consequence, which will be done in the following sections.
  The quantity

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

  ('Minkowski line element') is invariant under Lorentz transformations. This invariance is easy to see: by using the Lorentz transformation we have

$$\begin{aligned}(ds')^2 &\equiv -c^2(dt')^2 + (dx')^2 + (dy')^2 + (dz')^2 \\ &= -c^2\gamma^2\left(dt - \frac{v}{c^2}dx\right)^2 + \gamma^2(dx - vdt)^2 + (dy')^2 + (dz')^2 \\ &= -c^2\gamma^2\left(-\frac{v^2}{c^2}\right)dt^2 + 2c^2\gamma^2\frac{v^2}{c^2}dtdx + \gamma^2dx^2\left(1-\frac{v^2}{c^2}\right) - 2\gamma 2vdtdx + dy^2 + dz^2 \\ &= c^2dt^2 + dx^2 + dy^2 + dz^2 \equiv ds^2.\end{aligned}$$

- The Galilei transformation can some how be recovered from the Lorentz transformation in the limit of small velocities $|v|/c << 1$, although the derivation is a bit finicky. First, expand the Lorentz factor γ to first order in v/c:

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1 + \frac{v^2}{2c^2} + \ldots \approx 1$$

and

$$x' \approx xvt,$$
$$y' = y,$$
$$z' = z.$$

Strictly speaking, the transformation of the time coordinate gives, to first order in v/c,

$$t' = t - \frac{v}{c^2}x$$

not $t' = t$ as in the Galilei transformation: the relativity of simultaneity persists to first order (it is a first order effect in v/c). If the Lorentz transformation reduced to the Galilei transformation to first order, then infinitesimal Lorentz transformations and infinitesimal Galilei transformations would coincide, which is not the case. Although the Lorentz transformation does not quite reduce to the Galilei transformation, which is recovered only in the limit $v/c \longleftrightarrow 0$, Newtonian mechanics and the Galilei transformation turn out to be adequate in the limit $|v| << c$.

- Since the Lorentz transformation is linear and homogeneous, finite coordinate differences transform in the same way as infinitesimal coordinate differences:

4.3 Length Contraction from the Lorentz Transformation

Length contraction and time delay can be derived directly from the Lorentz transformation.

Consider a rod at rest in the inertial frame $S' = t', x', y', z'$ and moving with speed v with respect to another inertial frame $S = t, x, y, z$. The endpoints of the rod are x_A and x_B with $l_0 \equiv x_B - x_A$ the rest length of the rod. According to the Lorentz transformation, it is

$$x'_A = \frac{x_A - vt_A}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad x'_B = \frac{x_B - vt_B}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4.9}$$

In order to measure the rod, we must find the coordinates of the endpoints at the same time $t_A = t_B \equiv t$ (the two endpoints are observed simultaneously). We have

$$l_0 = x'_B - x'_A = \frac{((x_B - vt)) - (x_A - vt_A)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{x_B - x_A}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or

$$l_0 = \frac{l}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}} \tag{4.10}$$

the Lorentz-FitzGerald formula for length contraction.

There is no contraction in the directions transversal to the motion. As a result of length contraction, a moving rod can be fit momentarily in a space in which it would not fit at rest (which originates the car-in-the-garage paradox). However, nothing has happened to the rod. If you measure it again in its rest frame, it still has the same rest length l_0.

4.4 Time Dilation from the Lorentz Transformation

Let us now derive time dilation directly from the Lorentz transformation.

Let a clock at rest at x'in the inertial frame S'record two events happening at the same location $x'$and separated by the proper time interval $\Delta\tau$. The two events have coordinates $(t'_1, x', 0, 0)$ and $(t'_2, x', 0, 0) \equiv (t'_1 + \Delta\tau, x', 0, 0)$. What is the time interval measured by a clock in the inertial frame S which is moving with constant speed v with respect to S'? The inverse Lorentz transformation gives, for these two events,

$$t_1 = \frac{t'_1 + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad t_2 = \frac{t'_2 + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and the time interval in S is

$$\Delta t = t_2 - t_1 = \frac{(t'_2 + \frac{v}{c^2}x') - (t'_1 + \frac{v}{c^2}x')}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t'_2 - t'_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or

$$\Delta t = \frac{\Delta \tau}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma \Delta t \tag{4.11}$$

A moving clock runs slower than a static one by the Lorentz factor γ.

An ideal clock is defined as one that is not affected by acceleration. The finite interval of proper time recorded by an (ideal) clock between proper instants t_0 and t is

$$\Delta t = \int_{t_0}^{t} dt \sqrt{1 - \frac{v^2}{c^2}} \tag{4.12}$$

4.5 Matrix Representation of the Lorentz Transformation

The Lorentz transformation

$$\hat{L_v} : \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \longrightarrow \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$$

where 'ct' is space time coordinet

is a linear homogeneous coordinate transformation in the space (ct, x, y, z) and can be represented by a symmetric 4×4 matrix $\hat{L_v}$ with components given by

$$\underline{\hat{L_v}} = \left(L_(v)^{\alpha}_{\beta}\right) = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4.13}$$

where $\beta \equiv v/c$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.

This is a real symmetric 44matrix parametrized by the parameter v. To check that this representation is correct, take the product

Matrix Representation of the Lorentz Transformation

$$\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ct - \gamma\beta x \\ -\gamma\beta ct + \gamma x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma(ct - \frac{v}{c}x) \\ \gamma(x - vt) \\ y \\ z \end{pmatrix}$$

therefore,

$$\begin{aligned} ct' &= \gamma(ct - \frac{v}{c}x) \\ x' &= \gamma(x - vt), \\ y' &= y \\ z' &= z \end{aligned}$$

which is the Lorentz transformation.

The matrix $\hat{L_v}$ of the Lorentz transformation has unit determinant:

$$\begin{aligned} Det\left(\underline{\hat{L_v}}\right) &= Det \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ &= \gamma Det \begin{pmatrix} \gamma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - (-\gamma\beta) Det \begin{pmatrix} -\gamma\beta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \gamma\gamma + \gamma\beta(-\gamma\beta) = \gamma^2(1 - \beta^2) \\ &= \left(\frac{1}{\sqrt{1 - \beta^2}}\right)\left(1 - \beta^2\right) \\ &= 1. \end{aligned}$$

The inverse of the matrix $\hat{L_v}$ is thematrix $\hat{L}_{(-v)}$ corresponding to the inverse Lorentz

transformation,

$$\hat{L}_{(-v)} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4.14}$$

In fact, we have

$$\hat{L}_v\hat{L}_{(-v)} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \hat{L}_{(-v)}\hat{L}_v$$

and

$$\hat{L}_v\hat{L}_{(-v)} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \hat{L}_{(-v)}\hat{L}_v$$

The matrix corresponding to $v = 0$ and $\gamma = 1$ is obviously the identity matrix.

Chapter 5

Conclusion

The theoretical discovery of the Lorentz transformation was an important step of the learning process leading to Special Relativity, but its deep meaning was not understood before Einstein. In our presentation we have made it clear that the Lorentz transformation can be derived from the two postulates of Special Relativity,which are physically more transparent than what, at first sight, appears only as a mathematical transformation. From the physical point of view it is more satisfactory to construct the theory beginning from two very clear ideas rather than from a telling, but less transparent, mathematical symmetry.

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