Wolkite University
College of Natural and Computational Science
Department of Mathematics

LargeNon-Local Operator and Applications

A Project Submitted to Department of Mathematics in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mathematics.

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Approval

This Project has been examined and approved as meeting the requirements for the partial fulfillment of MSc in Mathematics

Examining board members

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Declaration

"I, declare that this project has been composed by me and that no part of the project has formed the basis for the award of any degree, diploma, associate ship, fellow ship or any other similar title to me".

Author's Signature

Permission

”This is certify that this project is compiled by ABERA DIJAGO in the department of mathematics, WOLKITE UNIVERSITY, Under my supervision.I here by also confirm that the project can be sumitted for evaluation by examiners and eventual defense”.

Advisor Signature

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Abstract

In this project we discuss on,the Laplace and Fourier transform that we have found, so useful for solving the integral transforms to a general class of a non-local operators that share a common set of properties. The so called linearities define a class of Laplace and Fourier transform which include many of the previous transform as special cases.The linearity of both transform helps to identify those assumptions that are needed to define Laplace and Fourier transform with the properties that we require a certain techniques to solve the function by using non-local operators.

Contents

1 Introduction

In Mathematics and its applications, a non-local operators and its applications are a real integral operator(transformer)of the form

$$Au(y) = \int_x U(x)K(x,y)dx$$

where k is some kernel function. it is necessary to know the value of U almost every where on the support of $k(\cdot, y)$ in order to compute the value of Au at y integration under x. Its application on image on denoising for an image

$$U(p) = \frac{1}{C(p)} \int_\pi V(p)f(p,q)dq$$

where π is the area of an image, p and q are two points with in the image, $U(p)$ is the filter value of the image at point $p, V(q)$ is unfiltered value of the images at point $q, f(p,q)$ is weighting function, $C(p)$ is a normalizing factor given by

$$C(p) = \int_\pi f(p,q)dq$$

and integral is evaluated $\forall q \in \pi$.

1.1 Statement of the Problem

This project will focus on establishing the existence of function is different on the different point of the given interval or boundary by using non-local operator and application.

1.2 Objective of the Project

1.2.1 General Objective

The general objective of this project was to establish certain techniques to solve the function by using non-local operator on the given function and its application and deduce to specific results of function on the given interval.

1.2.2 Specific Objectives

The specific objectives of the project is to:

- Define the non local-operators and it usage on the function.
- Use examples to support the main result.

1.3 Significance of the Project

The project of this problem will have the following importance;

- The out come of this project may contribute to activates a research on the project worked area.
- It may provide basic skill for an other project or research worker.

2 Preliminaries

2.1 Basic definitions and concepts

2.1.1 Local operator

Definition 2.1 *Local operator is the most important operator in image processing (i.e To calculate the values in one point in the out put image at local operator looks all values of that point in the input image).*

Let $N(x)$ be a neighborhood of point x.
A Local operator: $f \Rightarrow Tf$ considers all tuples $(y, f(y))$ for $y \in N(x)$.
We have

$$g(x) = (T(f(x)) = t(l(y, f(y)) \qquad y \in N(x)$$

An operator $A : X \Rightarrow Y$ is said to be local if for every $y \in Y$ there exist $x \in X$ such that $AU(y) = AV(y)$ for all functions U and V in $f(x)$ which are equivalent at x.For a local operator it is possible in principles to compute the value AU(h) using only knowledge of the values of U in arbitrary small neighborhood of a point x.
A local operator is one whole action only depends on the values of the function and its derivative at a single point.
For example Green function on the interval $[0, 1]$

$$\int_0^1 G(x)f(y)dy$$

it satisfies only on the two end values in the sense of local operator but not check all points between in the $[0, 1]$
Differential operator is an other example of a local operator.Assume that there is a map A from a function spaces f_1 to an other function spaces f_2 and a function $f \in f_2$ so that f is the image of $U \in f_1$ i.e $f = A(U)$.
A differential operator is represented as a linear combination,finitely generated

by U and its derivatives containing higher degree such as

$$P(X, D) = \sum_{|\alpha|} a_\alpha(x) D^\alpha$$

where the set of non-negative integers $\alpha = (\alpha_1, \alpha_2, ...)$is called multi index $|\alpha| = \alpha_1 + \alpha_2 + \alpha_3 + ... + \alpha_n$ called length.

$a_{\alpha(x)}$ are functions some open domain in n-dimensional space and $D^\alpha = D_1^{\alpha 1}(D_2^{\alpha 2})(D_3^{\alpha 3})(...D_n^{\alpha_n})$

Differential operator is linear operators.i.e Differentiation is linear so it has some properties

✓$D(f + g) = Df + Dg$

✓$D(af) = a(Df)$ where f and g are functions and a is a constant.

Any polynomial in D with function coefficients is also a differential operator we may also compose differential operators by the rule.

$(D_1 o D_2)f = D_1(D_2(f))$ in here it need some care.

Firstly any function coefficients in the operator of D_2 must be differentiable as many times as the application of D_1 requires.

In generally a local operators are relation between the values of an unknown functions and its derivatives or integration's of different order.In order to check whether the functions hold at a particular points one needs to known only the value of the function in an arbitrary small neighborhood, so that all derivatives can be computed.

The opposite of "local" in this context is "not global "but rather "non local".

3 Project Methodology

3.1 Project Design

In this project mathematical definition, prove and examples was create relating with the objective of the project.The mathematical expressions would be analytically designed to obtain the results.

3.2 Project Procedures

To attain the objective of the project, the procedure that would follow was accordance with non-local operators with functions. Some of procedures we use to project are;

- The standard procedures in Differential equation ”Laplace Transform” tutorial math educ Retrieved [2020].
- The procedure of Integral transform, encyclopedia of Mathematics, EMs press, 2001[1994].

4 Main Result and Discussion

4.1 Main Result

Under this section, we establish some usage of non-local operators and its applications.

4.1.1 Non-Local operator

Definition 4.1 *A non-local operator is a mapping which maps function on a metric space to functions in such away that the value of the out put function at a given point cannot be determined solely from the values of the in put function in any neighborhood of any point.*

For non-local operator it not possible to compute the value of $AU(y)$ using only knowledge of the value of U,so it can compute all small neighborhood of a point x.
i.e

$$AU(y) = \int_x U(x)K(x,y)dx$$

where k is some kernel function it is necessary to know the value of U almost every where on the support of K(.,y) in order to compute the value of AU at y.
A non-local is in order to check whether the functions holds at a point about the value of the function far from that points is needed.
Non-local operator is given by the integral transform such as Laplace transform and Fourier transform.

4.1.2 Integral Transform

Definition 4.2 *The Integral transform is function from its original function spaces into an other functions via integration (as a possible the same space).*

An integral transform is any Transform T of the Tf form

$$(Tf)(u) = \int_{t_1}^{t_2} f(t)k(t,u)dt.$$

The in put of this transform is a function f and the out put is an other function Tf.

4.1.2.1 Laplace Transform[1,2,3,6]

It is an integral transforms helps in solving the differential equations with bounder values without finding the general solution and values of arbitrary constants.

Definition 4.3 *Let $f(t)$ be functions defined for all positive values of t, then $F(s) = \int_0^\infty e^{-st} f(t)dt$ provided the integral exists, is called the Laplace transform of $f(t)$. It is denoted as $L[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)dt$.*

Its mostly encountered in differential equation in electrical engineering.For instance electrical circuits are represented as differential equations.

Laplace transform takes a function of real variable $f(t)$ defined for all $t \geq 0$ to a function of a complex variable $F(s)$ as follows

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s).$$

Some Examples of Laplace Transform

Constant functions

$f(t) = 1$, in this case, putting 1 in the transform yields $\frac{1}{s}$, which means that we want from a constant to a variable dependent functions. i.e.$L(1) = \frac{1}{s}$. *Proof:* $L(1) = \int_0^\infty 1.e^{-st}dt = [\frac{e^{-st}}{-s}]_0^\infty = \frac{-1}{s}[\frac{1}{e^{st}}]_0^\infty = \frac{-1}{s}[0-1] = \frac{1}{s}$. Hence $L(1) = \frac{1}{s}$ □

NB: After all we have seen $\frac{1}{x}$ integrating to $\ln x$ in calculus not a constant to variable situation of courses,but an expected transformation nonetheless.

Linear function

$f(t) = t$, after transformation it is turned into $\frac{1}{s^2}$ which means that we want from $1 \Rightarrow \frac{1}{s}$ and $t \Rightarrow \frac{1}{s^2}$. Now what about $f(t) = t^n$? which this simple power function we end up with $L\{t^n\} = \frac{n!}{s^{n+1}}$.
So there was a factorial in $L\{t\}$ all a long, hidden by the fact that $1! = 1$ what else is the transform hiding.

Common Laplace Transform

Would show that the emerging pattern cannot explain other function easily.

Diverging Functions of the Laplace Transform

For $f(t) = e^{at}$, The Laplace transform of the exponential e^{at} is actually deceptive simple $L\{e^{at}\}$=$\int_0^\infty e^{at}e^{-st}dt$ =$\int_0^\infty e^{(a-s)t}dt$.

Here , we see that so long as $Re(s) > a$, we would let that

$$\int_0^\infty e^{(a-s)t} = \frac{e^{(a-s)t\infty}}{(a-s)0}$$
$$= 0 - \frac{1}{a-s}$$
$$\frac{1}{s-a}$$

That is as long as $Re(s) > a$, the Laplace transform of e^{at} is simple $\frac{1}{(s-a)}$.
On the other hand , if we mix the exponential e^{at} with power function t^n,we would then we have
$L\{t^n e^{at}\}$=$\int_0^\infty t^n e^{(at)}e^{(-st)}dt$
which after a bit of recursion integration by part would became $\frac{n!}{(s-a)^{n+1}}$.
Here, notice how the transforms of exponential and power are both represented in the expression, with the factorial $n!$, then $\frac{1}{(s-a)}$, fraction and the $n + 1$ exponent.
In fact, it turns out that we can integrate any function with the Laplace

transform, as long as it does not diverge faster than the e^{at} exponential.

Looking inside the Laplace transform of sine

Let us un pack what happens to our sine function as we Laplace transform it we began by noticing that a sine function can be expressed as complex exponential.An indirect result of celebrated Euler's formula

$$e^{it} = cost + isint.$$

In fact ,a sine is often expressed in terms of exponential for case of calculation, so if we apply that to the function
$f(t) = sin(at)$,we would get

$$sin(at) = \frac{e^{iat} - e^{-iat}}{2i}.$$

Thus the Laplace transform of $sin(at)$ then becomes
$L\{sin(at)\}=\frac{1}{2i}\int_0^{\infty}(e^{iat} - e^{-iat})e^{-st}dt$
which means that we have a product exponential.
Distributing we get
$L\{sin(at)\}=\frac{1}{2i}\int_0^{\infty}(e^{iat} - e^{-iat-st})dt.$
Here,factoring the t in the exponents yields
$\{sin(at)\}=\frac{1}{2i}\int_0^{\infty} e^{(ia-s)t} - e^{(-ia-s)t}dt$ and $Re(s) > 0$.
By assumption, we can proceed with the integrating from 0 to ∞ as usual
$L\{sin(at)\}=\frac{e^{(ia-s)t}}{2(-a-is)}\Big|_0^{\infty}$ -$\frac{e(ia-s)t}{2(a-is)}\Big|_0^{\infty}$.
By evaluating that at the boundaries,we get
$L\{sin(at)\}=\frac{e(ia-s)\infty}{2(-a-is)}$-$\frac{e(ia-s)0}{2(-a-is)}$- $(\frac{e(-ia-s)\infty}{2(a-is)}$-$\frac{e(-ia-s)0}{2(a-is)})$

And because $Re(s) > 0$.
By assumption ,both $e^{(ia-s)\infty}$ and $e^{(-ia-s)\infty}$ oscillate to 0 (i.e vanish at ∞). After which we are then left
$L\{sin(at)\}=\frac{1}{-2(-a-is)}$ + $\frac{1}{2(a-is)}$, Once there merging the fractions together would yield

$$L\{sin(at)\} = \frac{2(a-is)-2(-a-is)}{-4(a-is)(-a-is)}$$
$$= \frac{2a-2is+2a+2is}{-4(a^2+isa-isa+s^2)}$$
$$= \frac{4a}{4(a^2+s^2)}$$
$$= \frac{a}{a^2+s^2}$$

which shows that after Laplace transform, sine is turned into a more tractable geometric function. By similar reasoning , the Laplace transform of cosine can be shown to be equal.

$L(\cos(at))$=$\frac{S}{a^2+s^2}$ $(Re(s)>0)$. So by the same manner we can Shaw $L(sinh(at)) = \frac{a}{s^2-a^2}$. Where $s^2 > a^2 and$
$L(cosh(at)) = \frac{s}{s^2-a^2}$ where $s^2 > a^2$.
Proof: $L(cosh(at)) = L[\frac{e^{at}+e^{-at}}{2}] = \frac{1}{2}L(e^{at}) + \frac{1}{2}L(e^{-at}) = \frac{1}{2}[\frac{1}{s-a} + \frac{1}{s+a}] = \frac{1}{2}[\frac{s+a+s-a}{s^2-a^2}] = \frac{1}{2}[\frac{2s}{s^2-a^2}] = \frac{s}{s^2-a^2}$.
Therefor $cosh(at) = \frac{e^{at}+e^{-at}}{2} = \frac{1}{2}[L(e^{at}) + L(e^{-at})] = \frac{1}{2}[\frac{1}{s-a} + \frac{1}{s+a}]$. □

Example 4.1 find the Laplace Transform of f(t) defined as

$$f(t) = \begin{cases} \frac{t}{k}, & \text{when } 0 < t < k, \\ 1, & \text{when } t > k \end{cases}$$

solution $L[f(t)] = \int_0^k \frac{1}{k}e^{-st}dt + \int_k^\infty 1e^{-st}dt = \frac{1}{k}[t\frac{e^{-st}}{-s}]_0^k - \int_0^k \frac{e^{-st}}{-s}dt + [\frac{e^{-st}}{-s}]_k^\infty = \frac{1}{k}[k\frac{e^{-ks}}{-s} - (\frac{e^{-st}}{s^2})_0^k] + \frac{e^{-ks}}{s} = \frac{1}{k}[\frac{ke^{-ks}}{-s} - \frac{e^{-ks}}{s^2} + \frac{1}{s^2}] + \frac{e^{-ks}}{s} = -\frac{e^{-sk}}{s} - \frac{1}{k}\frac{e^{-ks}}{s^2} + \frac{1}{k}\frac{1}{s^2} + \frac{e^{-ks}}{s} = \frac{1}{ks^2}[-e^{-ks}+1]$.

Summarizing as table form of the Laplace transform

Function	Laplace transform
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$

t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$Sin(at)$	$\frac{a}{a^2+s^2}$
$cos(at)$	$\frac{s}{a^2+s^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$sinh(at)$	$\frac{a}{s^2-a^2}$
$cosh(at)$	$\frac{s}{s^2-a^2}$

4.1.3 Fourier Transform

It is a Mathematics transform that decomposes functions depending on space or time into functions depending on spatial or temporal frequency,such as the expression of musical chord in terms of the volumes and frequencies of its constituent notes. The terms Fourier transform refers to both the frequency domain representation and mathematics peration that associates the frequency domain representation to a function of space or time.
The Fourier transform of a function of a time is a complex-valued function of frequency, whose magnitude (absolute value) represents the amount of that frequency present in the original function.
The frequency domain refers to the analysis of mathematical functions or signals with respect to frequency rather that times[5] put simply a time domain graphs show how signals changes over time,where as a frequency domain graph shows how much of the signal lies with in each given frequency.
Linear operations performed in one domain(time or frequency) have corresponding operations in the other domain which are some time easier to perform. The operation of differentiation in the time domain corresponding to multiplication by the frequency. So some differential equation are easier to analyze in the frequency domain.

Properties of the Fourier transform

Here we assume $f(x), g(x)$ and $h(x)$ are integrable function Lebesgue measurable on the real line satisfying.

$$\int_{-\infty}^{\infty} \mid f(x) \mid dx < \infty$$

we denote the Fourier transforms of these functions as $\hat{f}(\xi)$,$\hat{g}(\xi)$ and $\hat{h}(\xi)$ respectively.

Basic properties

The Fourier transform has the following basic properties [8]

1. Linearity

For any Complex numbers a and b , if $h(x) = af(x) + bg(x)$ then
$\hat{h}(\xi)$ =$a\hat{f}(\xi) + b\hat{g}(\xi)$.

2. Translation/ time shifting

For any real numbers x_0, if $h(x) = f(x - x_0)$,then $\hat{h}(\xi)$ =$e^{-2\pi i x_0}\xi f(\xi)$.

3. Modulation /frequency shifting

For any real number ξ_0,if $h(x) = e^{2\pi i x \xi_0} f(x)$, then $\hat{h}(\xi)$=$\hat{f}(\xi - \xi_0)$.

4.Time scalling

For a non-zero real number a ,if $h(x)$=$f(ax)$ then $\hat{h}(\xi)$=$\frac{1}{|a|}\hat{f}(\frac{\xi}{a})$,the case $a = -1$ leads to the time reversal property which states if $h(x)$=$f(-x)$, then $\hat{h}(\xi)$=$\hat{f}(-\xi)$.

5.Conjugation

If $h(x)$=$f(\bar{x})$ then $\hat{h}(\xi)$=$f(\bar{-}\xi)$.In particular, if f is real, then one has the reality condition $\hat{f}(-\xi)$= $\hat{f}(\xi)$,that is $\hat{h}$ is a Hermitial function and if f is purely imaginary, then $\hat{h}(\xi)$ =$f(\bar{\hat{\xi}})$.

Real and Imaginary part in time

If $h(x)$=$Re(f(x))$,then $\hat{h}(\xi)$=$\frac{1}{2}(\hat{f}(\xi)+\bar{f(-\xi)})$.
If $h(x)$=$Im(f(x))$,then $\hat{h}(\xi)$=$\frac{1}{2i}(\hat{f}(\xi)+\hat{\bar{f(-\xi)}})$.

6.Integration

Substituting $\epsilon=0$ in the definition we obtains $\hat{f(0)}$=$\int_{-\infty}^{\infty}f(x)dx$.
That is ,the evaluation of the Fourier transform at the origin ($\epsilon=0$) equals the integration of f over all its domain.

4.2 Application of non-local operator

Some examples of applications of non-local operators are:
✠ Image denoising using non-local means
✠ Analysis of dynamical system (using Laplace transformation)
✠ Time-series analysis (Fourier transformation)

4.2.1 Image denoising using non-local

The main idea of denoising mean noise reduction (i.e it is the process of removing noise from a signal. Noise reduction technique exists for audio and images.
Noise reduction algorithms tends to alter signals to a greater or lesser degree. The local signal and noise orthogonalization algorithm can be used to avoid (changes to the signals).
Noise reduction algorithms may disfort the signal to some degree.

4.2.1.1 Audio or Sound

In the case we an use by minimizing or maximizing the volume of the sound source or audio any mean of reducing the sound pressure with respect to a specific sound source and receiptor. The basic approach to qualifying of sound is increasing or decreasing of distance between source and receiver (i.e to reducing sound increasing the distance between source and receiver).

4.2.1.2 Images and Signal

Tend to alter signals to greater or lesser degree by near and far. If its problem is frequency dependent noise introduce by a dvice mechanism or signal process algorithms. (i.e by fulfill its correct frequency quality or image quality).
Non local means is an algorithm in image processing for image denoising. unlike "local mean" filters, which take the mean value of a group of pixels surrounding a target pixels . Pixels to smooth the image, non local means filtering takes a mean of all pixels in the image, weighted by low. similar, these pixels are to the target pixel.
This result in which greater post filtering clarity and less loss of details in the image compared with local means algorithm[].
Suppose π is the area of an image and p and q are two points with in the image . Then the algorithms[].

$$U(P) = \frac{1}{C(p)} \int_{\pi} V(q) f(p, q) dq$$

Where $U(p)$ is the filtered value of the image at point p, v(q) is un filtered value of the image at point q, f(p,q) is weighting function and the integral is evaluated $\forall_q \in \pi$.
$C(p)$ is a normalizing factor given by

$$C(P) = \int_{\pi} f(p, q) dq.$$

4.2.1.3 Discrete algorithm

For an image, π which discrete pixels. A discrete algorithms required

$$U(p) = \frac{1}{C(p)} = \sum_{q \in \pi} V(q) f(p, q)$$

where $C(p) = \sum_{q \in \pi} f(p, q)$.

4.2.2 Dynamical system

A dynamical system is in which a function describes the times dependence of point in a geometrical space. At any given time ,a dynamical system has a state given by a tuple of a real number (a vectors that can be represented by a point an appropriate state space (a geometrical manifold)). Examples include the mathematical models that describe the swining of a clock pendulum, the flow of water in pipe, and the number of fish each spring time in a lake.
The evolution rule of the dynamical system is a function that describes what future states follow from the current state often the function determines,that is for a given time interval only one future state follows from the current state[18],[19].
A dynamical is a manifold M called the phase(or state).state endowed with a family of smooth evolution function ϕ^t that for any element $t \in T$.The evolution function ϕ^t is often the solution of a differential equation of motion
$X = V(x) = Ax + b$.
The equation gives the time derivatives represented by the dot or trajectory $X(t)$ on the phase space Starting at some point x_0.
The vector field $V(x)$ is a smooth function that every point on the phase space M provides the velocity vector of the dynamical system at that point. The vector are not vectors in the phase space M, but in the tangent space $T_x M$ of the point x.

4.2.2.1 Linear dynamical system

Linear dynamical system can be solved in terms of simple function and the behavior of all objects classified. In a linear system the phase space is the N-dimensional Euclidean space.

Flows

For a flow the vector field $V(x)$ is an affine function of the position in the phase space that is $X = V(x) = Ax + b$ with A a matrix b a vector of numbers and x the position vector.
The solution to this system can be found by using the superposition principle (Linearity). The case $b \neq 0$ with $A = 0$ is just a straight line in the direction of b.

$$\phi^t(x_1) = x_1 + bt$$

where b is zero and $A \neq 0$ the origin is an equilibrium (or singular) point of the flow, that is , if $x_0 = 0$ the the orbit remains therefore other initial conditions , the equation of motion is given by the exponential of a matrix for an initial point x_0

$$\phi^t(x_0) = e^{tA}x_0$$

where $b = 0$, the eigen values of A determine the structure of the phase space.From the eigen values and eigen vectors of A it is possible to determine if an initial point will converges or diverges to the equilibrium point at the origin.

4.2.3 Time Series Analysis

A Time series is a series of data points indexed (or listed or graphed) in the time order. Most commonly a time series is a sequence taken at a successive equally spaced points in the times.
In statistical graphics "a data point "may be an individual item with a statistical display ,such points may related to either a single member or population to a summary statistic calculate for a given sub population.
In general Time Series are used for [12,13,14].

- Signal processing

- Pattern recognition
- In-statistics
- Mathematical finance
- Weather forecasting
- Earth quake prediction
- Control engineering
- Astronomy
- Communication engineering

4.2.3.1 In Statistics

It is the science of collecting ,displaying and analyzing data it concerns the collection ,organization, analysis, interpretation and presentation of data [12,13]. In applying statistics to a scientific, industrial or social problem it is conventional to begin with a statistical population or statistic as model to be studied. Population can be divides groups of people or subjects such as "all people living in country" or "every atom composing a crystal". Statistical deals with every aspect of data, including the planning of data collection in terms of the design of surveys and experiments[15].

4.2.3.2 Signal processing

Signal processing is an electrical engineering sub field that focuses on analyzing, modifying and synthesizing signals such as sound,image and scientific measurements[12]. Signal processing technique can be used to improved transmission storage efficiently and subjective quality and to also emphasize or detect components of interest in measured signal[17].

Conclusion

The project is in generally focus on establishing the existence of function on the different point of neighborhood can get different value of function

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