College of Natural and Computational Science

Department of Mathematics

Project On: Numerical Solution of Ordinary Diffirential Equatin Using COlloction Method

Prepared by:

Misire Lelisa

Advisor:

Mr.Desta sodano(M.Sc.)

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The undersigned hereby certify that they have read and recommend to the Department of Mathematics for acceptance of a project entitled **Numerical Solution Of Ordinary Diffirential equaton Using Collocation Method** by Misire Lelisa in partial fulfillment of the requirements for the degree of Bachelor of Science(B.Sc.)

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Advisor: ______________________

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Abstract

In this project we have tried to express Numerical solution of ordinary differential equation in the overall part of the project the main issue was to find out some application of the collocation method. Collocation method provides continuous approximation to the solution.We discuss about one step collocation method and its order as well as perturbed and discontinuous collocation method and multi step collocation method.

Notation

IRK....................implicit Runge-Kutta

IVP.....................initial value problem

ODE.....................ordinary differential equation

Chapter 1

INTRODUCTION

1.1 Back ground of the study

The subject of ordinary differential equation is an essential tool for modeling many physical situations:spring mass systems ,resistor-capacitor,inductance circuits,bending of beams,chemical reaction,pendulems,the motion of rotating mass around another body,and so forth. These equations have also demonstrated their usefulness in ecolgy and economics. The predator-prey problems has become a classic example of differential equation. The solution to a differential equation is the function that satisfies the differential equation and that also satisfies certain initial conditions on the function.

In solving differential equations analytically ,we usually find a general solution containing arbitrary constants and evaluate the arbitrary constants so that the expression agrees with the initial condition.

For nth order differential equation n-independent initial conditions must usually be known.

The analytical methods are limited to certain special forms of equations;but numerical methods have no such limitations to only standard forms. we obtain the solution as a tabulation of the values on the function at various values of the independent variable.

There are different numerical methods for solving ordinary differential equations such as Euler's method, Taylor's method, Multi step method, Runge-Kutta method, collocation method and shooting method.

The focus of this seminar is on collocation method which is applied in the construction of numerical solution of ordinary differential equations.

Collocation method is based on the idea of approximation the exact solution of a given functional equation with a suitable approximant belonging to a chosen finite dimensional space a piece wise algebraic polynomial ,which exactly satisfies the equation on a certain sub set of the integration interval (i.e the set of the so called collocation points).

The systematic study of collocation methods for initial value problems in ordinary differential equation has been originated in the late 1960's.

The idea of multi step collocation method was first introduced by Lie and Norsett(1989).

Multi step collocation method depends on more parameters than classical ones with out any significant increase in computational cost ,by regarding them as special case of multi step Runge-Kutta method therefore there are much more degrees freedom to be sent in order to obtain strong stability and higher order and stage order of convergence.This technique,when applied to problems based on function equation,allows the derivation of methods having many desirable properties.In fact, collocation method provide approximation over the entire integration interval to the solution of the equation,more over collocation function can be expressed as a linear combination of function

1.2 Statement of the problem

This project deals on the problem of the general form given the ordinary differential equation

$y' = f(x, y)$ on the interval $x \in [x_0, x]$ with the initial value $y(x_0) = y_0$,on each interval$[x_n, x_{n+1}]$ Euler type method use the two end point (and possibly a combination)to approximate the function f on this interval.multi step method use the previous already computed Approximats to get a better hand of f n the interval $[x_n, x_{n+1}]$,the collocation idea a special case of Runge-kutta methods wich we discuss in this seminer is to pick several points in the interval $[x_n, x_{n+1}]$ and use a polynomial px,whose derivative satisfies the ordinary differential equation these points as an Approximant.

$$\begin{cases} P'(x) = F(x, P(x)) \\ P(x_0) = y_0 \end{cases}$$

1.3 Objectives

1.3.1 General objective

The main objective of this Project is to present a review of Collocation method for ordinary differential equation.

1.3.2 Specific objective

1.To discuss one step collocation method and its order

2.To explain perturbed and discontinuous collocation method

3.To introduce multi step collocation method

1.4 preliminaries

1.4.1 polynomial Interpolation

let p_s,denote the space of real polynomials of degree $\leq$s,Given and set of s distinct abscissa points $c_1 \leq c_2$and corresponding date $g_1, g_2....g_s$ there exist a unique polynomials the $p(x) \in p_{s-1}$ satisfying$p(ci) = gi,\ i = 1, 2...., s$ this polynomials the interpolating polynomial. The Lagrange interpolating polynomials l_i = 1,...,s for a set of abscissa are defined by the i^th Lagrange interpolating polynomiall_i is the interpolating polynomial corresponding to the data the set of Lagrange interpolating polynomials from a basis for P_s. In this basis the interpolating polynomial p(x) assumes the simple form. $P(x) = \sum_{i=1}^{s} g_i l_i(x)$...1.3

Lagrange polynomial can be stated concisely as $f_n(x) = \sum_{i=0}^{n} L_i(x) f(xi)$

$L_i(x) = \prod_{j=0}^{n} \frac{x-x_j}{x_i-x_j}$

For Example n=1

$f_1(x) = \frac{x-x_1}{x_0-x_1} f(x_0) + \frac{x-x_0}{x_1-x_0} f(x_1)$

n=2

$f_1(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$

1.5 Numerical Quadrature;

A numerical method to approximate a definite integral of one independent variable is known as a numerical quadrature rule.

Consider a smooth function g on the real line we can approximate the definite integral of g on the interval [0,1] by exactly integrating the interpolating polynomial of order $s-1$ based on s points $0 \leq c_1 \leq c1 \leq ... \leq c_s \leq 1$.

The points c_i are then known as quadrature points. the data are the values of g at the quadrature points $g_i = g(c_i)$,i=1,2...s.

Defining the weights

$b_i = \int_0^1 l_i(x)dx$

The quadrature formula becomes

$\int_0^1 g(x)dx \approx \int_0^1 p(x)dx = \sum_{i=0}^{s} b_i g(c_i)$.....................1.4

To approximate the integral $\int_{t_0}^{t_0+1} g(t)dt$, we define a function $t(x) = t_0 + hx$. Then dt=hdx and(1.4)becomes

$\int_{t_0}^{t_0+h} g(t)dt = \int_0^1 g(t_0+hx)hdx \approx h\sum_{i=1}^{s} b_i g(t_0+hc_i)$............................1.5

This by construction,a quadrature formula using s distinct abscissa point will exactly integrate any polynomial in$p_s - 1$ Quadrature refers to any method for numerically approximating the value of a denite integral $\int_a^b f(x)dx$. The goal is to attain a given level of precision with the fewest possible function evaluations.The crucial factors that control the diculty of a numerical integration problem are the dimension of the argument x and the smoothness of the integrand f.

Chapter 2

Collocation Method

A collocation method approximates the solution the ordinary differential equation over the interval $[t_0, T]$ by a function piece wise polynomial over each sub interval $[t_i, t_i + 1]$.

This function satisfy the ordinary differential equation at sellacted collocation points with in each interval and take take values equal to an approximation y_i at each of the interval.

One of the advantage of collocation method is that it provides a continuous approximation to the solution where as many other numerical methods produce only a table of values of the approximant solution at discrete points.

Definition 2.0.1. *let* $c_1....c_2$ *be distnict real number (usually* $0 \leq c_i \leq 1$*) The collocation polynomial p(t) is a polynomial degree s satisfying*

$p(t_0) = y_0$ $p^{(}t_0+c_ih) = f(t_o+c_ih), p(t_0+c_ih), i = 1, 2..., 5.$*and the numerical solution of the collocation method is defined by* $y_1 = p(t_0 + th)$

Suppose that the ODE $Y'(t) = f(t, y(t)), y(t_0) = y_0$, is to be solved over the interval $[t_0, t_0 + C_kh]$. Choose C_K from $0 \leq c_1 \leq c_2 < ... < c_n \leq 1$. The corresponding (polynomial)collocation method approximates the solution y by the polynomial p of degree n which satisfies the initial condition $p(t_0) = y_0$, and the differential equation $P'(t_k) = f(t_k, P(t_k))$ at all collocation points $t_k = t_0 + c_khfork = 1, ..., n$. This gives n+1 conditions, which matches the n+1 parameters needed to specify a polynomial of degree n. All these collocation methods are in fact implicit Runge-Kutta methods. The coefficientsC_k in the Butcher tableau of Runge-Kutta method are the collocation points

,How ever not all implicit Runge-Kutta methods are collocation methods.

Example 2.0.1. *for The trapezoidal rule*

Pick as an example the two collocatio points $C_1 = 0 and c_2 = 1$ *(so* $n = 1$*).*

The collocation conditions are

$P(t_0) = y_0$

$P'(t_0) = f(t_0),$

$P'(t_0 + h) = f(t_0 + h, P(t_0 + h))$

there are three conditions so P should be a polynomial of degree 2.write p in the form $P(t) = \alpha(t - t_0)^2 + \beta(t - t_0) + \gamma$

to simplify the computations,then the collocation conditions can be solved to give the coefficients

$\alpha = \frac{1}{2h}(f(t_0 + h, P(t_0 + h)) - f(t_0, P(t_0))$

$\beta = f(t_0, P(t_0))$

$\gamma = y_0$

The collocation method is now given (implicitly)by

$Y_1 = p(t_0 + h) = Y_0 + \frac{1}{2}h(f(t_0 + h, Y_1) + f(t_0, Y_0))$ *where* $Y_1 = P(t_0 + h)$ *is the approximate solution at* $t = t_0 + h$ *this method is known as the trapezoidal rule differential equations.*

2.1 One Step Collocation Method

let the initial value problem on the first order ODES

$$\begin{cases} y'(t) = f(t, y_(t)) \\ y(t_0) = y_0 \text{..} 2.1 \end{cases}$$

take the nodes $t_{0,1}, ...t_n = T$ with step size h on has interval $[t_0, T]$

let $0 \leq c_1 \leq c_2 \leq .... \leq c_s = 1$ and consider the interval $[t_0, t_1]$

i,e $[t_0, t_0 + h]$

Aim;to approximate y(t)that i is to find y, by finding a polynomial p of degree $\leq s$ such that

$$\begin{cases} p(t_0) = y_0 \\ p(t_0 + c_ih) = f(t_0 + c_ih, p(t_0 + c_ih))...2.2 \end{cases}$$

i=1,2.....s The $y_1 = p_t$

let f be a polynomial of degree s-1 and $f(c) = p^(t_o + ch)$(As p is a polynomial F is a polynomial in c) let $f_j = f(c_i) = p^(t_0 + c_j + h)$

j=1,2....,s, that pis a polynomial of at most degree s-1 through the s date points (c_j, f_j) As the lagrange polynomial through these points degree s-1,

$f(c) = \sum_{j=1}^{s} F_jL_j(c)$

by letting $f = t_0 + ch$

$p(t) = f(c) = F(c) = \sum_{j=1}^{s} F_jL_j\ (\frac{t-t_0}{h})$..2.3

note that $t = t_0c = 0$ and $t = t_0 + c_ihc = c_i$ and $d_t = hdc$

for i=1,2,...s now integrating (2.3) from t_0 to $t_0 + c_ih$,

we have

$\int_{t_0}^{t_0+c_ih} p'(t)dt = \sum_{j=1}^{s} \int_0^{c_i} F_jL_j(c)hdc$

$p(t_0 + c_ih) - p(t_0) = h$

$p(t_0 + c_ih) = y_0 + h$

let $a_{ij} = \int_0^{c_i} L_j(x)dxandb_j = \int_0^1 L_j(x)dxi, j = 1, 2, .., s$ Then $p(t_0 + c_i) = y_0 + h$

By substituting (2.5 in 2.2),we get $f_i = f(c_i) = p^(t_0 + c_ih) = f(t_0 + c_ih, y_0 + h)$

This equation gives s unknowns (f_i) in s equations hence we can obtain solutions for thef_is

Also integrating(2.3)from $t_0tot_1 = t_0 + h$

$P(t_1)P(t_0) = \int_{t_0}^{t_1} \sum_{j=1}^{s} F_jL_j\ (\frac{t-t_0}{h})dt$

$= h\sum_{j=1}^{s} F_j \int_0^1 L_i(x)dx$

$= h\sum_{j=1}^{s} F_jb_j$ i.e $p(t_1) = p(t_0) + h\sum_{j=1}^{s} F_jb_j$

$y_1 = y_0 + h\sum_{j=1}^{s} F_jb_j$ To summarize the one step collocation method

1. determine h

2. select collocation points

Find the fi's using the equation $F_i = f(t_0+c_1h, y_0+h\sum_{j=1}^{s} f_ja_{ij})$ where $a_{ij's}$ are obtained by evaluating $\int_0^1 L_i(x)dx$ Then $y_1 = y_0 + h\sum_{i=1}^{s} F_ib_i$ where the bi's are obtained by evaluating $\int_0^1 l_i(x)dx$

To find y_2,use t_1 as an initial point and the sub interval $[t_1, t_2]$ and the collocation point.

In general, the collocation method is written

$F_i = f(y_n + h\sum_{j=1}^{s} a_{ij}F_j)$, i=1,2,......,s

$y_{n+1} = t_n + h\sum_{i=1}^{s} b_iF_i$

The following example illustrates the method.

Example 1.1 Use the collocation method to find a formula which approximates y_1 for the case s=2 (take $c_1 = 0, c_2 = 1$),then use use this formula to approximate the solution of the differential equation

$$\begin{cases} y'(t) = y \\ y(0) = 1, t \in [0, 2] \end{cases}$$

solution $L_i(x) = \prod_{i=1}^{s} \frac{x-c_i}{c_j-c_i}$

$L_j(x) = \prod_{i=1}^{2} \frac{x-c_i}{c_j-c_i}$,j=1,2

$L_2(x) = \frac{x-c_1}{c_2-c_1} = x$

$a_{i1} = \int_0^{c_i} l_1(x)dx = \int_0^{c_i}(1-x)dx = x - \frac{x^2}{2} = c_i - \frac{C_i^2}{2}$

$a_{11} = C_1 - \frac{C_i^2}{2} = 0$ $a_{21} = C_1 - \frac{C_2^2}{2} = 1 - \frac{1}{2} = \frac{1}{2}$ $a_{i2} = \int_0^{c_i} L_2(x)dx = \int_0^{c_i} xdx = \frac{x^2}{2} = \frac{C_i^2}{2}$,then

$a_{12} = \frac{c_1^2}{2} = 0$

$a_{22} = \frac{C_2^2}{2} = \frac{1}{2}$

$b_j = \int_0^1 L_j(x)dx$,then

$b_1 = \int_0^1 L_1(x)dx = \int_0^1(1-x)dx = x - \frac{x^2}{2} = \frac{1}{2}$

$b_2 \int_0^1 L_2(x)dx = \int_0^1 xdx = \frac{x^2}{2} = \frac{1}{1}$

The collocation method solution can be given as

$y_1 = y_0 + h\sum_{i=1}^{2} b_iF_i$

$F_i = f(t_0 + C_ih, Y_0 + h\sum_{j=1}^{2} F_ja_{ij})$,i=1,2....

$y_1 = y_0 + h(b_1F_1 + b_2F_2) \implies y_1 - y_0 = \frac{h}{2}(F_1 + F_2)$......................................2.1

$F_i = f(t_0 + c_ih, y_0h\sum_{j=1}^{2} F_ja_{ij})$, i=1,2

$F_i = f(t_0 + c_i h, y_0 h(F_1 a_{i1} + F_2 a_{i2}))$

$F_1 = f(t_0 + c_1 h, y_0 + h(F_1 a_{11} + F_2 a_{12}))$

$F_1 = f(t_0, y_0), since c_1 = 0, a_{11} = 0, a_{12} = 0$

$F_2 = f(t_0 + c_0 h, y_0 + h(F_1 a_{21} + F_2 a_{22})) = f(t_0 + h, y_0 + (\frac{h}{2} F_1 + F_2))$...2.2

Substituting (2.1)in(2.2),we have

$F_2 = f(t_0 + h, y_0 + y_1 - y_0) = f(t_0 + h, y_1)$

There fore $y_1 = y_0 + h \sum_{i=1}^{2} b_i F_i$

$y_1 = y_0 + \frac{h}{2}(F_1 + F_2), Since b_1 = b_2 = \frac{1}{2}$

$y_1 = y_0 + \frac{h}{2}[f(t_0, y_0) + f(t_1, y_1)]$

This results in the trapezoidal rule numerical method.

SOLUTION: given the differential equation $y'(t) = y$

$y(0) = 1$, $t \in [0, 2]$

Take the nodes 0,1,2 then we have sub interval [0,2]and [1,2]

We know $y'(t) = f(t, y(t))y, and t_0, t_1 = 1 then h = t_1 - t_0 = 1$

Then using $y_1 = y_0 + \frac{h}{2}[f(t_0, y_0) + f(t_1, y_1)]$

$$y_1 = 1 + \frac{1}{2}[f(0, 1) + f(1, y_1)]$$

$$y_1 = 1 + \frac{1}{2}[1 + y_1]$$

$$\frac{1}{2} = \frac{3}{2} \implies y_1 = 3$$

Theorem 2.1 The collocation method of (2.2)is equivalent to the s-stage Runge-Kutta method with coefficients $a_{ij} = \int_0^{c_i} L_j(w)dw, b_j = \int_0^1 L_j(w)dw$,i,j=1,2,...,s where $L_j(w)$is the Lagrange polynomial $L_j(w) = \prod_{i=1}^{s} \frac{w-c_i}{c_j-c_i}$

PROOF:let p(x)be the collocation polynomial and define $p'(x_0+wh) = \sum_{j=1}^{s} K_j L_j(w)dw$,and by integration we get $p(x_0 + c_i h) = y_0 + h \sum_{j=1}^{s} K_j \int_0^{c_i} L_j(w)dw$

Inserted to the formula of collocation polynomial $p'(x_0 + c_i h) = f(x_0 + c_i h, p(x_0 + c_i h))$,it gives the frist formula of the Runge-Kutta equations

$K_i = f(x_0 + c_i h, y_0 + h \sum_{j=1}^{s} a_{ij} K_j)$

Integration from 0 to 1 yields $y_1 = y_0 + h \sum_{j=1}^{s} b_j k_j$ This shows that one step collocation methods are certainly constituded by implicit Runge- Kutta (IRK).

One step collocation methods from a sub set of implicit Runge -Kutta method.

$y_{n+1} = y_n + h \sum_{i=1}^{s} b_i f(x_n + c_i h, y_i)$

$y_i = y_n + h \sum_{j=1}^{s} a_{ij} f(x_n + c_i h, y_i)$ i=1,2,.......s where $a_{ij} = \int_0^{c_i} L_j(w)dw, b_j = \int_0^1 L_j(w)dw, i, j = 1, 2..., s$ and $L_j(w)$,j=1,2,......,s are lagrange fundamental polynomials.

2.1.1 Order Of collocation Method

lemma 2.1:The polynomial p(t) is an approximation of order s to the exact solution of

$$\begin{cases} y'(t) = f(t, y_(t)) \\ y(t_0) = y_0 \end{cases}$$

on the whole interval i.e

$||p(t) - y(t)|| \leq ch^s + 1$ for t$\in$ $[t_0, t_0 + h]$ and for sufficiently small h.

Proof: The collocation collocation polynomial satisfies

$$p'(t_0 + wh) = \sum_{j=1}^{s} f(t_0 + c_i h, p(t_0 + c_i h))l_j(w)$$

while the exact solution of (2.1) satisfies

$$y'(t_0 + wh) = \sum_{j=1}^{s} f(t_0 + c_i h, y(t_0 + c_i h))l_j(w) + h^E(w, h)$$

where the interpolation error E(w,h) is bounded i.e $||E(w, h)|| \in 2, t \in [t_0, t_0 + h] \frac{||y^s + 1(t||)}{s!}$ Integrating the difference of the above two equation gives $y(t_0 + wh) - p(t_0 + wh) = h \sum_{i=1}^{s} \delta fi \int_0^z l_i(\sigma)d\sigma = h^s + 1 \int_0^z E(\sigma, h)d\sigma$ with

$\delta f_i = f(t_0 + c_i h, y(t_0 + c_i h)) - f(t_0 = c_i h, p(t_0 + c_i h))$

Using a lipschitz condition for f(x,y) on the integral

$$y(t_0 + wh) - p(t_0 + wh) = h \sum_{i=1}^{s} \delta fi \int_0^z l_i(\delta)d\sigma + h^s + 1 \int_0^z E(\sigma, h)d\sigma$$

Yields

$t \in [t_0, t_0 + h]^| |y(t) - p(t)||t \leq [t_0, t_0 + h]^| |y(t) - p(t)|| + consth^s + 1$

Implying $||p(t)_y(t)|| \leq ch^s + 1$ for sufficiently small $h > 0$

Example 1.2 what is the order of the method n example 1.1? Solution: since the formula is a trapezoidal rule it has order 2.

or else by using the above lemma it has order 2.

2.2 Perturbed Collocation Method

Only some implicit Runge -Kutta methods are of one step collocation type methods.

An extension of collocation idea the so called perturbed collocation is applied to all implicit Runge-Kutta methods. We denote by π_m the linear space of polynomials of degree at most m and consider the polynomial $N_j \in \pi_m$

$$N_j x = \frac{1}{j}\sum_{i=0}^{m}(p_{ij-\sigma_{ij}})x^i$$

j=1,2.....,m....2,6

whereσ_{ij} is the usual kronecker delta, we next define the parturbation operator

$$px_{0h} = \pi_m \implies \pi_m by(px_0, h^u)(x) = u(x) + \sum_{i=1}^{m} N_j \frac{(x-x_0)}{h} u^j(x_0)h^j ....2,7$$

2.3 Discontinuous collocation Method

Perturbed collocation has been considered as a modification of the one step collocation techniques,in such a way that much more Rung-Kutta methods could regarded as perturbed collocation based methods,rather than one step collocation based. There are possible extension of collocation idea which apply to wider classes of Rung-kutta methods such as the so called discontinuous collocation .

Definition 2.3.1. *let* $c_1, c_3..., c_m - 1$ *be distinct real numbers (usually between 0 and 1),and let b1,bm be two arbitrary real numbers. The corresponding discontinuous method is then defined via polynomial of degree m-2 satisfying*

$U(x_0 = y_0 - hb_1(U'(x_0 - f(x_0, U(x_0))),$

$U'(x_0 + c_ih) + f(x_0 + c_ih, U(X_0 + C_ih)), i = 2, 3....m - 1$

$y_1 = U(X_1 - hbm(U'(x_1 - f(x_1, U(x_1)))$

THEOREM 2.2 The discontinuous collocation method given in definition is equivalent to m-stage Runge-Kutta method with coefficient determine by $c_1 = 0, c_m = 1$ *and* $a_{i1} = b_1, a_{im} = 0$,*i=1,2,....,m*

PROOF: As in the proof of theorem(2.1)we put $K_i = U'(x_0 + c_ih)$*(this time for i=2,3,...m-1)that*

$U'(x_0+wh)=\sum_{j=2}^{m-1}K_jL_j(w)$ by lagrange interpolating formula here $L_J(w)$ corresponds to $c_2, c_3, ..., c_{m-1}$ and is a polynomial of degree m-3.

By integration and using the definition of $U(x_0)$we get

$U(x_0+c_ih)=U(x_0)+h\sum_{j=2}^{m-1}K_j\int_0^{c_i}L_j(w)dw$

$=y_0+hb_1k_1+\sum_{j=2}^{m-1}K_j(\int_0^{c_i}L_j(w)dw-b_1L_j(0))$ *with* $k_1=f(y_0)$

Inserted in $U'(x_0+c_ih)=f(x_0+c_ih, U(X_0+C_ih))$

This gives the first formula of the Runge-Kutta equation with

$a_{ij}=\int_0^{c_i}L_j(w)dw-b_1L_j(0)$

Note that if $b_1=0=b_m$, and then the discontinuous collocation method in definition is equivalent to the m-2 one step collocation method based on $c_2, c_3..., c_{m-1}$.

2.4 Multi step Collocation Method

The idea be hind multi step collocation is to let the collocation polynomial use information form the previous point in the integration consider the construction of multi step collocation methods for constant step size h and give expression fr the coefficient for arbitrary values of the number of steps and the numbers of collocation point .These expressions will be seen to be natural generalizations of the corresponding coefficients in the one step case, let point be approximations to $y_{n+i}=y(x_{n+i})$.
i=0,1,2,....k-1,$c_1...., c_m$ district real numbers. Then a k-step multi step collocation method with m collocation point is constricted as follows

Find $p\in\pi_{m+k1}$ is a linear space of polynomials ,such that $p(x_{n+i})=p_{n+i,}$, i=0,1,2,......,k
Find $p\in\pi_{m+k-1}$ is a linear space of polynomials, such that

$p(x_{n+i})=p_{n+i}$, i=0,1,2,........,k-1

$p'(x_n+k-1+c_jh)=f(x_n+k_1+c_jh, p(x_n+k-1+c_jh))$, j=1,2,...,m

Then an approximation to y_{n+k} we take $p_{n+k}=p(x_{n+k})$

Chapter 3

Application of collocation method

In mathematics , a collocation method is a method for the numerical solution of ODE,PDE and integral equation . The idea is to choose a finite dimensional space of candidate solutions (usually polynomials up to a certain degree)and a number of points in the domain (called collocation point)and to select that solution which satisfies the given equation at the collocation points.

Suppose that the ODE $y'(t) = f(t, y(t)), y(t_0) = y_0$is to be solved over the interval $[t_0, t_0+c_{kh}]$. chose C_k from $0 \leq C_1 < C_2 < ... < C_n \leq 1$ The corresponding (polynomial) collocation method approximates the solution y by the polynomials p of degree n which satisfies the initial condition $p(t_0) = y_0$, and the differential equation $p'(t_k) = f(t_k, p(t_k))$ at all collocation points. $t_k = t_0 + c_k h$for k=1,.....,n.

This gives n+1 conditions which matches the n+1 parameters needed to specify a polynomial of degree n. All these collocation methods are in fact implicit Runge-Kutta methods.

CONCLUSION

Many commonly used implicit Runge-Kutta methods are based on quadrature methods,and those can be divided in to several classes.

Implicit Runge-Kutta method can also be categorized according to whether they are collocation methods.

The basic principle behind collocation method is to chose a function (usually polynomial) from a simple space and a set of collocation points. We then require the function to satisfy the given problem equations at the collocation points.

Under the topic of collocation methods the following conclusions could be made

1. Collocation methods are subset of implicit Runge-Kutta methods.
2. Collocation polynomials of degree s lead to collocation methods f order s or better.
3. Collocation methods have the useful feature that we obtain a continuous approximation of the solution $p(x)$ on each interval $[x_n, x_{n+1}]$
4. The accuracy of collocation method is attained by taking small step size.
5. collocation methods with polynomials of degree s are equivalent to s- stage Runge-Kutta methods.

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