

NUMERICAL SOLOUTION OF SYSTEM OF LINEAR EQUATION By LU DECOMPOSITION AND JACOBI'S METHOD

PROJECT SUBMITTED TO DEPARTMENT OF MATHEMATICS IN PARTIAL FULFILLMENT OF THE REQUIREMENT BACHELOR SCIENCE IN MATHEMATICS

BY: FIRDOSE ABDU

APRIL, 2019

WOLKITE, UNIVERSITY

COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCE
DEPARTMENT OF MATHEMATICS

NUMERICAL SOLOUTION OF SYSTEM OF LINEAR EQUATION

BY: FIRDOSE ABDU
ADVISOR: Mr.NURI A. (M.Sc.)

PROJECT SUBMITTED TO DEPARTMENT OF MATHEMATICS IN PARTIAL FULFILLMENT OF THE REQUIREMENT BACHELOR SCIENCE IN MATHEMATICS

WOLKITE, ETHIOPIA
APRIL, 2019

Contents

Acknowledgement **i**

Abstract **ii**

Notations **1**

1 Introduction **2**

1.1 Back ground of the study 2

1.2 statement of the problem 3

1.3 Objective 3

1.3.1 General objective 3

1.3.2 Specific objective 3

2 Mathematical Preliminary **4**

2.1 Systems of Linear Equations 4

2.2 Review Numerical methods to find the solution for system of linear equation 6

2.2.1 Gauss Elimination Method 6

2.2.2 Matrix inverse method 7

2.2.3 Difficulty when we apply Gauss Elimination method and Matrix inversion method 10

3 LU Decomposition and Jacobi's Iteration Method **11**

3.1 Definition 11

3.2 LU Decomposition 12

3.3 Definition of jacobi's method 20

3.4 Compare LU decomposition method and Jacobi's method Using Examples 28

4 Conclusion and Summary 29

4.1 Conclusion 29

I here by certify that I have read and recommend this project titled"Numerical solution of system of linear equation"prepared under guidence by Firdose Abdu. I recommend that it be submitted as fulfilling the project requirement.

Advisor: _____ signature: _____ date: _____

Examining committee: _____

_____

March, 2019

Acknowledgement

First and foremost I would like to thanks my creature Allah who is a most merciful and beneficiary. Next I would like to present my excellent gratitude and grate for respect to my family for their continues in all situations. Also I would like to thanks my respectful Advisor Mr.Nuri A. for these supports ,constructive comment and sharing ideas on my project.

Abstract

The study of numerical solution of system of linear equation in numerical method is solved by LU decomposition and Jacobi's iteration method by applying different steps and iteration.We compare direct method and iterative method through example which shows that the iterative method is economical for computer memory efficient and feasible

Notations

A	Coefficient matrix.
B	Constant number.
det	determinant
L	Lower triangular matrix.
U	upper triangular matrix.
Δ	Delta

Chapter 1

Introduction

1.1 Back ground of the study

Linear systems of equations are associated with many problems in engineering and science, as well as with applications of mathematics to the social sciences and quantitative study of business and economic problems.A system of algebraic equations has the form

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n &= b_3 \\
\vdots \quad \vdots \quad \vdots \quad \ddots \quad \vdots \quad \vdots & \\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n &= b_m
\end{aligned}$$

where the coefficients a_{ij} and the constants b_j are known and x_i represents the unknowns. In matrix notation,the equations are written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix} X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{1.1}$$

or simply $Ax = b$

If the coefficient matrix is singular,the equation may have infinite number of solution, or no solution at all,depending on the constant vector. liner algebraic equation occur in almost all branches of engineering.

The most important application in engineering is in the analysis of linear systems or any system whose response is proportional to the input is deemed to be linear.Hence,it is desirable to have an equation solving algorithm that can handle any number of constant vector with minimal computational effort.

1.2 statement of the problem

A solution to a system of linear equation is a set of numbers that,when we substitute numbers for specified variables in the system .A system of equation of two algebraic expression which involves one or more literals (variables) is called an equation . The set of values of variable which makes the open sentence true is called the solution set.

1.3 Objective

1.3.1 General objective

The general objective of this project is computing system of linear equation by LU decomposition and Jacobi's iteration method.

1.3.2 Specific objective

The specific objective of this project are;

- Define LU decomposition and jacobi's iteration method

- Explain LU decomposition and jacobi's iteration method

Chapter 2

Mathematical Preliminary

2.1 Systems of Linear Equations

Important field of application for numerical methods is numerical linear algebra that deals with solving problems of linear algebra numerically (matrix-vector product, finding eigenvalues, solving systems of linear equations).
Methods for solving systems of linear equations.are of two types, direct and iterative.

- Direct methods go through a definite procedure to find the solutions. In the absence of roundoff error such methods would yield the exact solution within a finite number of steps.such as Gauss Elimination,Gauss Elimination with Partial Pivoting, creamer's rule and LU decomposition are the major approaches.

- Iterative methods, which is similar to solving non-linear equations, find the solution iteratively.These are methods that are useful for problems involving special, very large matrices. such as Jacobi's and Gauss Seidel.

A system of m linear equations in n unknowns is a set of m equations, numbered from 1 to m going down, each in n variables x_i which are multiplied by coefficients $a_{ij} \in F$, whose sum equals some $b_j \in R$:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned}$$

where the coefficients a_{ij} and the constants b_j are known and x_i represents the unknowns. In matrix notation,the equations are written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{2.1}$$

If we condense this to matrix notation by writing $x = (x_1, ..., x_n), b = (b1, ..., bm)$ and $A \in R^{m \times n}$ the coefficient matrix of the system, the matrix whose elements are the coefficients a_{ij} of the variables in (S), then we can write (S) as

$m \times n$

The Gaussian elimination procedure decomposes A into a product of a unit lower triangular matrix L and an upper triangular matrix U. This is the famous LU decomposition. Together with the ensuing backward substitution the entire solution algorithm for $Ax = b$ can therefore be described in three steps:

- Decomposition:$A = LU$
- Forward substitution: solve $Lv = b$
- Backward substitution: solve $Ux = v$.

2.2 Review Numerical methods to find the solution for system of linear equation

We had to solve two simultaneous linear equation in order to find the break even point and the equilibrium point.These are two example of real world problem that call for the solution of a system of linear equation in two or more variables. The modeling of linear system invariably gives rise to equations of the form $AX = b$,where b is the input and x represents the respons of the system

2.2.1 Gauss Elimination Method

Definition:Gauss elimination is a popular technique for solving simulate linear algebraic equation it reduces the coefficient matrix in to an upper triangular matrix through a sequence of operation carried out on the matrix.Consider the following system of linear simultaneous equation

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$$

Example 1

Using Gauss elimination method solve the equation

$x_1 + 2_{x2} + 3_{x3}$=10

$2_{x1} + 3_{x2} - 3_{x3} = 1$

$2_{x1} - x_2 + 2_{x3} = 7$

solution :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ 2 & -1 & 2 \end{bmatrix} X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = B = \begin{bmatrix} 10 \\ 1 \\ 7 \end{bmatrix}$$

$$R_2 - 2R_1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -9 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -19 \\ 7 \end{bmatrix}$$

$$R_3 - 2R_1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -9 \\ 0 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -19 \\ -13 \end{bmatrix}$$

$$R_3 - 5R_2 \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -9 \\ 0 & 0 & 41 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -19 \\ 82 \end{bmatrix}$$

Thus we have.

$41_{X3} = 82 \Rrightarrow X_3 = 2$

$-X_2 - 9_{X3} = -19 \Rrightarrow -X_2 = -19 + 18 = 1$

$X_1 + 2_{X2} + 3_{X3} = 10$

$x_1 + 8 = 10 \Rrightarrow X_1 = 10 - 8 = 2$

Hence $x_1 = 2$,$x_2 = 1$,$x_3 = 2$

2.2.2 Matrix inverse method

Definition :where A and B are matrix of proper size to do this we need the matrix equivalent of the inverses of real number such matrix when every it exists is called the inverse of matrix. system of linear equation arise frequently and if n equation n

unknowns are given we write

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$$

The set of number x_1,x_2,$\ldots x_n$ which reduces to an identity is called the solution set of the system if we denote the matrix coefficient by

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix} X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{2.2}$$

Then the system can be compactly written in the form of a matrix equation.

$AX = B$

If the matrix A is non-singular that is if

$$A = \Delta = \begin{bmatrix} a_{11} & a_{12} \ldots & a_{1n} \\ a_{21} & a_{22} \ldots & a_{2n} \\ a_{31} & a_{32} \ldots & a_{3n} \\ \vdots & & \\ a_{n1} & a_{n2} \ldots & a_{nn} \end{bmatrix} \neq 0$$

Then has a unique solution.

If det$A \neq 0$,there is an inverse

Example 2

Obtain the solution of the following liner simultaneous equation by the matrix inversion method.

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 4 & 2 & -1 \\ 1 & 3 & 1 \end{bmatrix} X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = B = \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix} \tag{2.3}$$

solution :

$$|A| = \begin{bmatrix} 1 & -1 & 3 \\ 4 & 2 & -1 \\ 1 & 3 & 1 \end{bmatrix} = 40$$

The matrix of cofactors is given by

$$C = \begin{bmatrix} 5 & -5 & 10 \\ 10 & -2 & -4 \\ -5 & 13 & 6 \end{bmatrix}$$

The transpose 0f C is the adjoin t of A or

$$AdjA = C^T = \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix}$$

$$A^{-1} = AdjA\,|A|\,\tfrac{1}{40} = \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix}$$

Therefor, $X = A^{-1}Y = \frac{1}{40}\begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

or

$x_1 = 0$,$x_2 = 1$,$x_3 = 2$

2.2.3 Difficulty when we apply Gauss Elimination method and Matrix inversion method

- Gauss Elimination Method

It is warranty or Guaranty Method but if the size of matrix is very large to reduce the given system of equation in to reduced echelon form it takes a lot of time so,it is time consuming.

If the pivoting elem is zero,it is difficult so it should be ill conditioned or well posed condition to do with Gaussian Elimination

- Matrix inversion method

If the size of the matrix is very large or there are more variables to find inverse it is difficult.

If the coefficient of the given system of the equation is not singular (i'e det (A) or $|A|=0$)

Chapter 3

LU Decomposition and Jacobi's Iteration Method

3.1 Definition

A system of m linear equations in these n variables can be written as

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n &= b_3 \\
\vdots \quad \vdots \quad \vdots \quad \ddots \quad \vdots \quad \vdots & \\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n &= b_m
\end{aligned}$$

where a_{ij} and b_i are all real numbers.

Such a linear system is called a homogeneous linear system if

$$b_1 = b_2 = \ldots = b_m = 0.$$

A system of linear equations in n unknowns has a unique solution, provided that the determinant of the coefficient matrix is non-singular i.e., if $|A| \neq 0$. The rows and columns of a non-singular matrix are linearly independent in the sense that no row (or column) is a linear combination of the other rows (or columns). If the coefficient matrix is singular, the equations may have infinite number of solutions, or no solutions at all, depending on the constant vector. Linear algebraic equations occur in almost all branches of engineering. The most important application in engineering is in the analysis of linear systems (any system whose response is proportional to the input

is deemed to be linear). Linear systems include structures, elastic solids, heat flow, seepage of fluids, electromagnetic fields and electric circuits i.e., most topics taught in an engineering curriculum. If the system is discrete, such as a truss or an electric circuit, then its analysis leads directly to linear algebraic equations. Summarising, the modelling of linear systems invariably gives rise to equations of the form $Ax = b$, where b is the input and x represents the response of the system. The coefficient matrix A, which reflects the characteristics of the system, is independent of the input. In other words, if the input is changed, the equations have to be solved again with a different b, but the same A. Hence, it is desirable to have an equation solving algorithm that can handle any number of constant vectors with minimal computational effort,or more linear equation involving the same set of variables,say,x_1,x_2,. . . ,x_n. The system of a linare equation may be Homogeneous or Non-homogeneous linear system A homogeneous system of linear equation is a systsm of simultaneous linear equations in which each constant term is zero .A system containing at least one non-zero constant term is a non- homogeneous system.

$x_1 - 2_{x2} + 3_{x3} = 0$

$x_1 + x_2 - x3 = 0$

$3_{x1} - x_2 + 2_{x3} = 0$

Homogeneous

$x_1 + x_2 - x_3 = 1$

$2_{x1} - x_2 - 3_{x3} = 0$

$-3_{x1} + x_2 - x_3 = 2$

Heterogeneous.

A solution of a linear system is an assignment of values to the variables ,x_1,x_2,. . . ,$x_n$such that each of the equations is satisfied. the set of all possible solutions is called the solution set.

3.2 LU Decomposition

This method is based on the square matrix A and can be expressed as the product of lower triangular matrix and an upper triangular matrix. suppose we have the system of

equation $AX = B$. The motivation for LU decomposition is based on the observation that systems of equations involving triangular coefficient matrices are easier to deal with. In deed, the whole point of Gaussian elimination is to replace the coefficient matrix with one that is triangular. The LU decomposition is another approach designed to exploit triangular systems. suppose that $A = LU$, where, L is a lower triangular matrix and U is an upper triangular matrix.

Do matrices always have an LU decomposition?

No, Sometimes it is impossible to write a matrix in the form lower triangular $\times$ upper triangular. An invertible matrix A has an LU decomposition provided that all its leading sub matrices have non-zero determinants. The k^{th} leading sub matrix of A is denoted A_k and is the $k \times k$ matrix found by looking only at the top k rows and left most k columns. if the decomposition fails the matrix is not positive definite.

Consider the system of linear equation.

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n = b_3$

$\vdots$

$a_{n1}x_2 + a_{n2}x_n + \ldots + a_{nn}x_n = bn$

The above system can be written as $AX = B$ where

$$A = \begin{bmatrix} a_{11} & a_{12}\ldots & a_{1n} \\ a_{21} & a_{22}\ldots & a_{2n} \\ a_{31} & a_{32}\ldots & a_{3n} \\ \vdots & & \\ a_{n1} & a_{n2}\ldots & a_{nn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{3.1}$$

Let $A = LU$

$$A = \begin{bmatrix} a_{11} & a_{12}\dots & a_{1n} \\ a_{21} & a_{22}\dots & a_{2n} \\ a_{31} & a_{32}\dots & a_{3n} \\ \vdots & & \\ a_{n1} & a_{n2}\dots & a_{nn} \end{bmatrix} = L = \begin{bmatrix} 1 & 0 & 0\dots & 0 \\ L_{21} & 1 & 0\dots & 0 \\ L_{31} & L_{32} & 1\dots & 0 \\ \vdots & & & \\ L_{n1} & L_{n2} & L_{n3}\dots & 1 \end{bmatrix} U = \begin{bmatrix} U_{11} & U_{12} & U_{13}\dots & U_{1n} \\ 0 & U_{22} & U_{23}\dots & U_{2n} \\ 0 & 0 & U_{33}\dots & U_{3n} \\ \vdots & & & \\ 0 & 0 & 0\dots & U_{nn} \end{bmatrix}$$

Multiplying out LU and setting the answer equal to A gives

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a32 & a_{33} \end{bmatrix} = LU \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12}+U_{22} & L_{21}U_{13}+U_{23} \\ L_{31}U_{11} & L_{31}U_{12}+L_{32}U_{22} & L_{31}U_{13}+L_{32}U_{23}+U_{33} \end{bmatrix}$$

Now we use this to find the entries in L and U. Fortunately this is not nearly as hard as it might at first seem. We begin by running along the top row to see that

$U_{11} = a_{11}, U_{12} = a_{12}, U_{13} = a_{13}$

Now consider the second row

$L_{21}U_{11} = a_{21}, L_{21} = a_{21}/a_{11},\ L_{31}U_{11} = a_{31}, L_{31} = \frac{a_{31}}{a_{11}}$
$L_{21}U_{12} + U_{22} = a_{22}, U_{22} = a_{22} - \frac{a_{21}}{a_{11}}a_{12}\ L_{21}U_{13} + U_{23} = a_{21}, U_{23} = a_{23} - \frac{a_{21}}{a_{11}}a_{13}\ L_{31}U_{12} + L_{32}U_{22} = a_{32}, L_{32} = \frac{1}{U_{22}}[a_{32} - \frac{a_{31}}{a_{11}}a_{12}]$ and $L_{31}U_{13} + L_{32}U_{23} + U_{33} = a_{11}$ $LV = B$ is equivalent to the system
LV=B

$$L = \begin{bmatrix} 1 & 0 & 0\dots & 0 \\ L_{21} & 1 & 0\dots & 0 \\ L_{31} & L_{32} & 1\dots & 0 \\ \vdots & & & \\ L_{n1} & L_{n2} & L_{n3}\dots & 1 \end{bmatrix} V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ v_n \end{bmatrix} = B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}$$

$v_1 = b_1$

$L_{21}v_1 + v_2 = b_2$

$L_{31}v_1 + L_{32}v_2 + v_3 = b_3$

$L_{n1}v_1 + L_{n2}v_2 + L_{n3}v_3 + vn = b_n$

The above system can be solved to find the values of v_1, v_2, v_3, $\ldots v_n$ which give us the matrix V.

UX=V

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13}\ldots & U_{1n} \\ 0 & U_{22} & U_{23}\ldots & U_{2n} \\ 0 & 0 & U_{33}\ldots & U_{3n} \\ \vdots & & & \\ 0 & 0 & 0\ldots & U_{nn} \end{bmatrix} X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ x_n \end{bmatrix} = V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ v_n \end{bmatrix}$$

This gives

$U_{11}X_1 + U_{12}X_2 + U_{13} + U_{1n}X_n = V_1$

$U_{22}X_2 + U_{23}X_3 + U_{2n}X_n = V_2$

$U_{33}X_3 + U_{3n}X_n = V_3$

$U_{nn}X_n = V_n$

which can be solved for x_3, $x_2$and x_1 by the backward substitution process.

Example 3

$5x_1 + 2x_2 + x_3 = 12$

$x_1 + 4x_2 + 2x_3 = 15$

$x_1 + 2x_2 + 5x_3 = 20$

solution :

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 5 \end{bmatrix}, B = \begin{bmatrix} 12 \\ 15 \\ 20 \end{bmatrix}$$

$A = LU$

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 5 \end{bmatrix} = L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 0 \end{bmatrix} U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 5 \end{bmatrix} = LU = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{bmatrix}$$

and comparing elements row by row we see that

$U_{11} = 5$ $U_{12} = 2$,$U_{13} = 1$
$L_{21} = \frac{1}{5}$ $U_{22} = \frac{18}{5}$ $U_{23} = \frac{9}{5}$
$L_{31} = \frac{1}{5}, L_{32} = \frac{4}{9}, U_{33} = 4$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & 1 & 0 \\ \frac{1}{5} & \frac{4}{9} & 1 \end{bmatrix} U = \begin{bmatrix} 5 & 2 & 1 \\ 0 & \frac{18}{5} & \frac{9}{5} \\ 0 & 0 & 4 \end{bmatrix}$$

LV=B

$$\mathrm{L} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & 1 & 0 \\ \frac{1}{5} & \frac{4}{9} & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = B = \begin{bmatrix} 12 \\ 15 \\ 20 \end{bmatrix}$$

$v_1 = 12$,
$v_2 = \frac{63}{5}$,

$v_3 = 12$

UX=V

$$U = \begin{bmatrix} 5 & 2 & 1 \\ 0 & \frac{18}{5} & \frac{9}{5} \\ 0 & 0 & 4 \end{bmatrix} X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = V = \begin{bmatrix} 12 \\ \frac{63}{5} \\ 12 \end{bmatrix}$$

from this we get

$x_3 = 3$

$x_2 = 2$

$x_1 = 1$

Example 4

$20x_1 + x_2 - 2x_3 = 17$

$3x_1 + 20x_2 - x_3 = -18$

$2x_1 - 3x_2 + 20x_3 = 25$

solution :

$$A = \begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix} B = \begin{bmatrix} 17 \\ \text{-18} \\ 25 \end{bmatrix}$$

A=LU

$$A = \begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix} = L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 0 \end{bmatrix} U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix} = LU \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{bmatrix}$$

$U_{11} = 20$ $U_{12} = 1$,$U_{13} = -2$
$L_{21} = \frac{3}{20}$ $U_{22} = \frac{397}{20}$ $U_{23} = \frac{-7}{10}$
$L_{31} = \frac{1}{10}, L_{32} = \frac{-62}{397}, U_{33} = \frac{101}{5}$

$$\mathrm{L} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{20} & 1 & 0 \\ \frac{1}{10} & \frac{-62}{397} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 20 & 1 & -2 \\ 0 & \frac{397}{20} & \frac{-7}{10} \\ 0 & 0 & 20.02 \end{bmatrix}$$

LV=B

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{20} & 1 & 0 \\ \frac{1}{10} & \frac{-62}{397} & 1 \end{bmatrix} V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = B = \begin{bmatrix} 17 \\ -18 \\ 25 \end{bmatrix}$$

$V_1 = 17$ $V_2 = \frac{-411}{20}$
$v_3 = 20.59$
$UX = V$

$$U = \begin{bmatrix} 20 & 1 & -2 \\ 0 & \frac{397}{20} & \frac{-7}{10} \\ 0 & 0 & 20.091 \end{bmatrix} X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = B = \begin{bmatrix} 17 \\ \frac{-411}{20} \\ 20.59 \end{bmatrix}$$

from this we get

$x_3 = 1.025$

$x_2 = -0.999$

$x_1 = 1$

3.3 Definition of jacobi's method

Definition 3.3.1. *A matrix said to be diagonally dominant if the absolute value of each main diagonal element is greater than the sum of the absolute value of each element in the same row. these are methods which compute a sequence of progressively accurate iterats to approximate the solution of $AX = B$. We need such methods for solving many large linear systems sometime the matrix is to large to be stored in the computer memory making a direct method too difficult to use. this method is also known as the method of simultaneous displacemets consider the system of linear equation.*

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n &= b_2 \\ \vdots \qquad\quad \vdots \quad \vdots \qquad\quad \vdots \\ a_{31}x_1 + a_{32}x_2 + \ldots a_{nn}x_n &= b_n \end{aligned}$$

we assume that the coefficients a_{11}, a_{22} and a_{33} are the largest cofficients in the respective equations so that.

$|a_{11}| > |a_{12}| + \ldots + |a_{1n}|$

$|a_{22}| > |a_{21}| + \ldots + |a_{2n}|$

$\vdots$

$|a_{nn}| > |a_{n1}| + \ldots + |a_{nn-1}|$

Jacobi's iteration method is applicable only if the conditions given above are satisfied. Now, we can write

$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2 - \ldots a_{1n}x_n)$

$x_2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1 - \ldots a_{2n}x_n)$

$\vdots$

$x_n = \frac{1}{a_{nn}}(b_n - a_{n1}x_1 - \ldots a_{nn}x_n)$

Let the initial approximations be x_1^0, $x_2^0 \ldots x_n^0$ respectively

The following iterations are then carried out : The first improvements are found as

$x_1^1 = \frac{1}{a_{11}}(b_1 - a_{12}x_3^0 - \ldots a_{1n}x_n^0)$

$x_2^1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^0 - \ldots a_{2n}x_n^0)$

$x_3^1 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^0 - \ldots a_{32}x_n^0)$

$Iteration2$

$x_1^2 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^1 - \ldots a_{1n}x_n^1)$

$x_2^2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^1 - \ldots a_{2n}x_n^1)$

$\vdots$

$x_n^2 = \frac{1}{a_{nn}}(b_n - a_{n1}x_1^1 - \ldots a_{nn}x_n^1)$

$(n+1)^{th}iterationn$

$x_1^n + 1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^n - \ldots a_{1n}x_n^n)$

$x_2^n + 1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^n - \ldots a_{2n}x_n^n)$

$\vdots$

$x_n^n + 1 = \frac{1}{a_{nn}}(b_n - a_{n1}x_1^n - \ldots a_{nn}x_n^n)$

the above iteration process is continued until the values of x_1, x_2 , x_3 and x_n are found to a pre-assigned degree of accuracy. That is, the procedure is continued until the relative error between two consecutive vector norm is satisfactorily small. In Jacobi's method, it is a general practice to assume $x_1^0 = x_2^0 = x_3^0 = \ldots = x_n^0 = 0$ The method can be extended to a system of n linear simultaneous equations in n unknowns

Example 5

solve by jacobi's iteration method

$5x_1 + 2x_2 + x_3 = 12$

$x_1 + 4x_2 + 2x_3 = 15$

$x_1 + 2x_2 + 5x_3 = 20$

solution :

The above system is diagonally dominant in each equation the absolute value of the largest coefficient is greater than the sum of the remaining coefficients.

The the equation can be written as

1^{st} iteration

$x_1^1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^0 - a_{13}x_3^0)$

$x_1^1 = \frac{1}{5}(12 - 0 - 0)$

$x_1 = 2.40$

$x_2^1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^0 - a_{23}x_3^0)$

$x_2^1 = \frac{1}{4}(15 - 0 - 0)$

$x_2^1 = 3.75$

$x_3^1 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^0 - a_{32}x_2^0)$

$x_3^1 = \frac{1}{5}(20)$

$x_3^1 = 4$

$2^{nd} iteration$

$x_1^2 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^1 - a_{13}x_3^1)$

$= \frac{1}{5}(12 - 2(3.75 - 4)$

$x_1^2 = 0.10$

$x_2^2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^1 - a_{23}x_3^1)$

$= \frac{1}{4}(15 - 2.40 - 2(4))$

$x_2^2 = 1.15$

$x_3^2 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^1 - a_{32}x_2^1)$

$x_3^2 = \frac{1}{5}(20 - 2.40 - 2(3.75))$

$x_3^2 = 2.02$

3^{rd} iteration

$x_1^3 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^2 - a_{13}x_3^2)$

$= \frac{1}{5}(12 - 2(1.15) - 2.02)$

$x_1^3 = 1.54$

$x_2^3 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^2 - a_{23}x_3^2)$

$= \frac{1}{4}(15 - 0.10 - 2(2.02))$

$x_2^3 = 1.72$

$x_3^3 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^2 - a_{32}x_2^2)$

$x_3^3 = \frac{1}{5}(20 - 0.10 - 2(1.15))$

$x_3^3 = 3.57$

4^{th} iteration

$x_1^4 = \frac{1}{a_{11}}(b_1 - a_{12}x_1^3 - a_{13}x_3^3)$

$= \frac{1}{5}(12 - 2(1.72) - 3.75)$

$x_1^4 = 0.61$

$x_2^4 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^3 - a_{23}x_3^3)$

$= \frac{1}{5}(15 - 1.54 - 2(3.57))$

$x_2^4 = 1.17$

$x_4^3 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^3 - a_{32}x_2^3)$

$= \frac{1}{5}(20 - 1.54 - 2)(1.72))$

$x_4^3 = 2.60$

5^{th} iteration

$x_1^5 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^4 - a_{13}x_3^4)$

$x_1^5 = \frac{1}{5}(12 - 2)(1.17) - 2.06)$

$x_1^5 = 1.41$

$x_2^5 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^4 - a_{23}x_3^4)$

$= \frac{1}{4}(15 - 0.61 - 2)(1.17))$

$x_2^5 = 2.29$

$x_3^5 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^4 - a_{32}x_2^4)$

$= \frac{1}{5}(20 - 0.61 - 2(1.17))$

$x_3^5 = 3.41$

6^{th} iteration

$x_1^6 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^5 - a_{13}x_3^5)$

$x_1^6 = \frac{1}{5}(12 - 2(2.29) - 3.41)$

$x_1^6 = 0.80$

$x_2^6 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^5 - a_{23}x_3^5)$

$x_2^6 = \frac{1}{4}(15 - 1.41 - 2)(3.41))$

$x_2^6 = 1.69$

$x_3^6 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^5 - a_{32}x_2^5)$

$x_3^6 = \frac{1}{5}(20 - 1.41 - 2(2.29))$

$x_3^6 = 3.20$

7^{th} iteration

$x_1^7 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^6 - a_{13}x_3^6)$

$x_1^7 = \frac{1}{5}(12 - 2(2.69) - 3.20)$

$x_1^7 = 1.08$

$x_2^7 = \frac{1}{4}(15 - 0.80 - 2(1.69) - 3.20)$

$x_2^7 = 1.95$

$x_3^7 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^6 - a_{32}x_26)$

$x_3^7 = \frac{1}{5}(20 - 0.80 - 2(1.69))$

$x_3^7 = 3.16$

8^{th} iteration

$x_1^8 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^7 - a_{13}x_3^7)$

$x_1^8 = \frac{1}{5}(12 - 2(1.95) - 3.16)$

$x_1^8 = 1.084$

$x_2^8 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^7 - a_{23}x_3^7)$

$x_2^8 = \frac{1}{4}(15 - 1.08 - 2(3.16))$

$x_2^8 = 1.95$

$x_3^8 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^7 - a_{32}x_27)$

$x_3^8 = \frac{1}{5}(20 - 1.08 - 2(1.95))$

$x_3^8 = 3.164$

The value of x_1 ,x_2 and x_3 at the end of the 8^{th} iteration are

$x_1 = 1.084$

$x_2 = 1.95$

$x_3 = 3.164$

Example 6

Solve the system by using jacobi's iteration method

$20x_1 + x_2 - 2x_3 = 17$

$3x_1 + 20x_2 - x_3 = -18$

$2x_1 - 3x_2 + 20x_3 = 25$

solution :

$|a_{11}| > |a_{12}| + |a_{13}|$

$|20| > |1| + |2| = 3$

$|a_{22}| > |a_{21}| + |a_{23}|$

$|20| > |3| + |1| = 4$

$|a_{33}| > |a_{31}| + |a_{32}|$

$|20| > |2| + |3| = 5$

Therefore the coefficient of the system is diagonal dominant

assume $x_1^0 = 0$, $x_2^0 = 0$, $x_3^0 = 0$

1^{st} iteration

$x_1^1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^0 - a_{13}x_3^0)$

$x_1^1 = \frac{1}{20}(17 - 0 - 0)$

$x_1^1 = 0.85$

$x_2^1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^0 - a_{23}x_3^0)$

$x_2^1 = \frac{-18}{20}$

$x_2^1 = -0.9$

$x_3^1 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^0 - a_{32}x_2^0)$

$x_3^1 = \frac{25}{20}$

$x_3^1 = 1.25$

2^{nd} iteration

$x_1^2 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^1 - a_{13}x_3^1)$

$x_1^2 = \frac{1}{20}(17 + 0.9 + 2(1.2))$

$x_1^2 = 1.02$

$x_2^2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^1 - a_{23}x_3^1)$

$x_2^2 = \frac{1}{20}(-18 - 3)(0.85) + 1.25)$

$x_2^2 = -0.965$

$x_3^1 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^1 - a_{32}x_2^1)$

$x_3^1 = \frac{1}{20}(25 - (2)(0.85) - (3)(1.25))$

$x_3^2 = 1.03$

3^{rd} iteration

$x_1^3 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^2 - a_{13}x_3^2)$

$x_1^3 = \frac{1}{20}((17 + 0.965) + 2(1.03))$

$x_1^3 = 1.00125$

$x_2^3 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^1 - a_{23}x_3^1)$

$x_2^3 = \frac{1}{20}(-18 - 3(1.02) + 1.00125)$

$x_2^3 = -1.0015$

$x_3^3 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^2 - a_{32}x_2^2)$

$x_3^3 = \frac{1}{20}(25 - 2(1.02) - 3(0.965))$

$x_3^3 = 1.00325$

4^{th} iteration

$x_1^4 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^3 - a_{13}x_3^3)$

$x_1^4 = \frac{1}{20}(17 + 1.0015 - 2(1.00325))$

$x_1^4 = 1.004$

$x_2^4 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^3 - a_{23}x_3^3)$

$x_2^4 = \frac{1}{20}(-18 - 3(1.00125) + 1.00325)$

$x_2^4 = -1.000025$

$x_3^4 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^3 - a_{32}x_2^3$

$x_3^4 = \frac{1}{20}(25 - 2(1.00125) - 3(1.0015))$

$x_3^4 = 0.99965$

$5th$ iteration

$x_1^5 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^4 - a_{13}x_3^4)$

$x_1^5 = \frac{1}{20}(17 + 1.000025 + 2(0.99965))$

$x_1^5 = 0.999966$

$x_2^5 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^4 - a_{23}x_3^4)$

$x_2^5 = \frac{1}{20}(-18 - 3(1.004) + 0.99965)$

$x_2^5 = -1.000078$

$x_3^5 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^4 - a_{32}x_2^4)$

$x_3^5 = \frac{1}{20}(25 - 2(1.004) + 3(-1.000025))$

$x_3^5 = 0.99956$

$6th$ iteration

$x_1^6 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^5 - a_{13}x_3^5)$

$x_1^6 = \frac{1}{20}(17 + 1.000078 + (2)(0.99956))$

$x_1^6 = 1.0000$

$x_2^6 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^5 - a_{23}x_3^5)$

$x_3^6 = \frac{1}{20}(-18 - (3)(0.999966) + 0.99956)$

$x_2^6 = -0.99999$

$x_3^6 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^5 - a_{32}x_2^5)$

$x_3^6 = \frac{1}{20}(25 - (2)(0.999966) + (3)(-1.000078))$

$x_3^6 = 0.9999$

Therefore;the values of x become very small and small

Hence the values of x is approximately

$x_1 = 1,\ x_2 = -1,\ x_3 = 1$

3.4 Compare LU decomposition method and Jacobi's method Using Examples

LU decomposition Method	jacob's Iteration method
$1^{st} EXAMPLE$	$1^{st} EXAMPLE$
$x_1 = 1$ $x_2 = 2$ $x_3 = 3$	$x_1 = 1.084$ $X_2 = 1.95$ $x_3 = 3.164$
$2^{nd} Example$	$2^{nd} Example$
$X_1 = 1.025$ $X_2 = -0.999$ $X_3 = 1$	$X_1 = 1$ $X_2 = -1$ $X_3 = 1$

Chapter 4

Conclusion and Summary

4.1 Conclusion

In this project we have presented an overview of the method of system equation specifically the Numerical method .These projects provided the relevant and useful elements of matrix analysis for the solution of linear simultaneous algebraic equation.The solution of n linear simultaneous algebraic equation in n unknowns is presented.There are two classes of method of solving system of linear equation Direct and Indirect method . Direct methods transform the original equation on to equivalent equation that can be solved more easily.

Indirect method start with a guess of the solution x and then repeatedly refine the solution until a certain convergence criterion is reached.

Three direct method (Gauss Elimination method ,Matrix inversion and LU Decomposition)are presented. One indirect method (Jacob's iterative method) are presented.This method give comparable results and is easy to compute further more .Iterative method is more applicable and better convergent if the size of matrix is very large or more variables.

Bibliography

[1] Aimin YANG and Yiming CHEN, MPI Parallel Programming Environment and Programming Research, 3rd Journal in 2005 of Academy Newspaper of Heibei Polytechnic University, 2005,11:41-44

[2] CAI Zhao-quan, Wei-wenhong, WANG Gao Cai, Zong-hui, LU Qing-wu. A method based on the network structure Bruijn De parallel matrix multiply algorithm [J]. Computer applications .2009 (03).

[3] Chunfeng LIU, Aimin YANG. The Parallel Algorithm of QR Decomposition of Matrix in Cluster System. World Academic Press. 2006:107-111.

[4] (France) PGCiarlet forward. Matrix numerical analysis and optimization [M]. Beijing: Higher Education Press. 1990.

[5] Stewart, G.W. (1998). Matrix Algorithms, Vol 1. Basic Decompositions, Siam, Philadelphia.

[6] TanLong, LI Jian-zhong. Based on Mobile Agent-matrix parallel computing research [J]. Journal of Harbin business college.2001,9 (17).

[7] Yiming Chen. Network Parallel BEM for Band Precision Rolling, COMPUTATIONAL MECHANICS, WCCM VI in conjunction with APCOM'04, Beijing, China. Tsinghua University Press and Springer-Verlag, 2004,9: 5-1