College Of Natural and Computational Science
Department Of Mathematics

project on

System Of Linear Equation and it's application

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Contents

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The undersigned hereby certify that she has read and recommend to the Department of Mathematics for acceptance of a project entitled system of linear equation and its application by Student Aziza Kedir in partial fulfillment of the requirements for the degree of Bachelor of Science in Mathematics.

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Abstract

The project an over view of the theory of Numerical system of linear equation and its applications. It specially focuses on Numerical method to solve system of linear equations. System of linear equations has been applied in mathematical Engineering and various field of since the rapid increasing its application has led to formulating and upgraded of several existing methods and new approaches. we compare direct method and iterative method through examples which shows that the iterative method is economical for computer memory efficient and feasible.

Notation

$detA$..determinant matrix of A

$adjA$.....................................adjoint matrix A

M_{ij}....................................minors of a matrix

Chapter 1

Introduction

In this chapter we will present the solution of n linear simultaneous Algebraic Equations in n unknowns. Linear systems of equation are associated with many problems in engineering and science, as well as with applications of mathematics to the social sciences and quantitative study of business and economic problems.

A finite set of linear equations is called a system of linear equations or a linear system. The variables in a linear system are called the unknowns. A system of algebraic equations has the form

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

where the coefficients a_{ij} and the constants b_j are known x_i Represents the unknowns. In the matrix notation the equations are written as

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{1.1}$$

or simply $Ax = b$

A system of linear equation in n unknowns has a unique solution, provided that the determinant of the coefficient matrix is non singular. i.e if $|A| \neq 0$.The rows and columns of a non singular matrix are linearly Independent in the sense that no row(column) is a linear combination of the other rows(columns).If the coefficient matrix is singular, the equations may have Infinite number of solutions, or no solutions at all, depending on the constant vector.

Summarizing the modeling of linear systems Invariably gives rise to equations of the form $Ax = b$, where b is the input and x represents the response of the system.The coefficient matrix A, which reflects the characteristics of the system, is independent of the input. In other words, if the input is changed,the equations have to be solved again with a different b, but the same A. Hence, it is desirable to have an equation solving algorithm that can handle any number of constant vectors with minimal computational effort.

1.1 Objective of the project

1.1.1 General objective

The general objective of this project is to understand the system of linear equation and its application.

1.1.2 Specific objective

- To compute system of linear equation by using direct and indirect methods.
- To identify methods of solving system of linear equations.
- To identify the difference of Gauss elimination method and Gauss Jordan method.
- To show how to fit a polynomial function to a set of data points in the plane.

Chapter 2

Systems of Linear Equations

Important field of application for numerical methods is numerical linear algebra that deals with solving problems of linear algebra numerically (matrix-vector product, finding eigenvalues, solving systems of linear equations). A system of n linear equations in n unknowns is a set of n equations, numbered from $1 ton$ going down, each in variables x_i which are multiplied by coefficients $a_{ij} \in F$,whose sum equals some $b_j \in R$:

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$
$a_{21}x_1 + a_{22}x_2 + \ldots + a_{1n}x_n = b_2$
$\vdots$
$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

Methods Of Solution

There are two methods for solving system of linear algebraic equations: direct and indirect(iterative) method.

2.1 Direct Method

The common characteristics of direct methods are that they transform the original equation in to equivalent equations(equations that have the same solution) that can be solved more easily.

In this section we will present three types of direct methods. they are

1. **Matrix Inverse Method**

2. **Gauss Elimination Method**

3. **Gauss Jordan Method**

2.1.1 Matrix Inverse Method

If A and B are $m \times n$ matrices such that

$AB = BA = I$

then B is said to be the inverse of A and is denoted by

$B = A^{-1}$

In order to find the inverse A^{-1}, provided the matrix A is given, let us consider the product,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \times \begin{bmatrix} |m_{11}| & -|m_{21}| & \cdots & (-1)^{1+n}|m_{n1}| \\ -|m_{12}| & |m_{22}| & \cdots & (-1)^{2+n}|m_{n2}| \\ \vdots & \vdots & & \vdots \\ (-1)^{1+n}|m_{1n}| & (-1)^{2+n}|m_{2n}| & \cdots & |m_{nn}| \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} \\ (-1)^{i+j} a_{kj} \\ |m_{ij}| \end{bmatrix} \tag{2.1}$$

An element of the matrix on the right side of equation (2.1) has the value

$$\sum_{j=1}^{n} (-1)^{i+j} a_{kj} |m_{ij}| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix} = |a| if i = k \tag{2.2}$$

If $i \neq k$ the determinant possesses two identical rows, since the determinant corresponding to $i \neq k$ is obtained from the matrix $[a]$ by replacing the ith row by the kth row and keeping the kth row intact. Therefore, if $i \neq k$ the value of the element is zero.

Equation (2.1) can be written as:

$A adj A = |A| I$ ....(1)

multiplying equation (1) throughout by A^{-1} and dividing the result by $|A|$, we get
$A^{-1} = \frac{adjA}{detA}$
so that the Inverse of Matrix A is obtained by dividing its adjoint matrix by its determinant $|A|$
If $detA$ is equal to zero,then the elements of A^{-1} approach Infinity (or are In determinant at best), in which case the Inverse A^{-1} is said not to exist, and the matrix A is said to be singular. The Inverse of a matrix exists only if determinant is not zero, that is the matrix must be non-singular.

Example

$$A = \begin{bmatrix} -1 & 4 \\ 3 & 1 \end{bmatrix} \tag{2.3}$$

$C_{11} = (-1)^{1+1}|-1| = -1$
$C_{12} = (-1)^{1+2}|4| = -4$
$C_{21} = (-1)^{2+1}|3| = -3$
$C_{22} = (-1)^{2+2}|1| = 1$

$$C = \begin{bmatrix} -1 & -4 \\ -3 & 1 \end{bmatrix} \tag{2.4}$$

$$C^T = \begin{bmatrix} -1 & -3 \\ -4 & 1 \end{bmatrix} \tag{2.5}$$

$|A| = (1 \times (-1)) - (4 \times 3) = -13$

$$A^{-1} = \frac{C^T}{|A|} = \frac{-1}{13} \begin{bmatrix} -1 & -3 \\ -4 & 1 \end{bmatrix} \tag{2.6}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{13} & \frac{3}{13} \\ \frac{4}{13} & \frac{-1}{13} \end{bmatrix} \tag{2.7}$$

2.1.2 Gauss Elimination Method

Definition: Gauss elimination is a popular technique for solving simultaneous linear algebraic equations. it reduces the coefficients matrix into an upper triangular matrix through a sequence of operations carried out on the matrix. the vector b is also modified in the process.Two linear systems $Ax = b$ and $A'x = b'$ equations are said to be equivalent if any solution of one is a solution of the other. Also, let$Ax = b$ is a linear non-homogeneous system of n equations. suppose we subject this system to the system of following operations.

- Multiplication of one equation by a non-zero constants
- Addition of a multiple of one equation to another equation
- Interchange of two equations

In Gauss elimination method ,the unknowns are eliminated such that the elimination process leads to un upper triangular system and the unknowns are obtained by back substitution.It is assumed $a_{11} \neq 0$.The method can be described by the following steps.

step 1:eliminate x_1 from the second and third equation.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \ldots (4)$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \ldots (5)$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \ldots (6)$

$(5) - (\frac{a_{21}}{a_{11}})(4) and (6) - (\frac{a_{31}}{a_{11}})(4)$

gives

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \ldots (7)$

$a'_{22}x_2 + a'_{23}x_3 = b'_2 \ldots (8)$

$a'_{32}x_2 + a'_{33}x_3 = b'_3 \ldots (9)$

step 2:eliminate x_2 from equation (9) using equation (8) by assuming $a'_{22} \neq 0$,We perform the following operation:

$(9) - (\frac{a'32}{a'22})(8)$

to obtain

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \ldots (10)$

$a'_{22}x_2 + a'_{23}x_3 = b'_2 \ldots (11)$

$a''_{33}x_3 = b''_3 \ldots (12)$

step 3:to find x_1, x_2 and x_3 we apply back substitution starting from equation (12) giving x_3, then x_2 from equation (11) and x_1 from equation (10)

Example

solve the following equations by gauss elimination method

$2x + 4y - 6z = -4$

$x + 5y + 3z = 10$

$x + 3y + 2z = 5$

Solution

multiplying row 2 and 3 by -2

$2x + 4y - 6z = -4$

$-2x - 10y - 6z = -20$

$-2x - 6y - 4z = -10$

by adding row 1 and 2,we get

$-6y - 12z = -24 \ldots (1)$

by adding row 1 and 3 we get

$-2y - 10z = -14 \ldots (2)$

To eliminate z using equation 1 and 2

$-6y - 12z = -24$

$-2y - 10z = -4$

multiplying row 2 by -3

$-6y - 12z = -24$

$6y + 30z = 42$

by adding row 1 and 2 we get

$18z = 18$

$z = 1$

$-6y - 12z = -24$

$-6y - 12(1) = -24$

$-6y = -12$

$y = 2$

$$2x + 4y - 6z = -4$$
$$2x + 4(2) - 6(1) = -4$$
$$2x + 8 - 6 = -4$$
$$2x = -6$$
$$x = -3$$

2.1.3 Gauss Jordan Method

Definition: Gauss Jordan Method is an extension of the Gauss elimination method.The set of equations $Ax = b$ is reduced to a diagonal set $Ix = b'$, where I is a unit matrix. This is equivalent to $x = b'$. The solution vector is therefore obtained directly from b'.The Gauss Jordan Method implements the same series of operations as Implemented by Gauss elimination process.
Gauss Jordan Method is a modification of Gauss elimination method. The series of operations performed are quite similar to the Gauss elimination method. In the Gauss elimination method,an upper triangular matrix is derived while in the Gauss Jordan method an identify matrix is derived. Hence,back substitutions are not required.

Remark 2.1.1. *Gauss elimination helps to put a matrix in row echelon form,while Gauss Jordan puts a matrix in reduced row echelon form. for small systems, it is usually more convenient to use Gauss Jordan elimination and explicit solve for each variable represented in the matrix system.*

Example: solve the following system of linear equations using the Gauss Jordan method

$2x - y = -4$

$-3y + 2z = -6$

$4x - z = -5$

Solution

The augmented matrix is

$$\begin{bmatrix} 2 & -1 & 0 & | & -4 \\ 0 & -3 & 2 & | & -6 \\ 4 & 0 & -1 & | & -5 \end{bmatrix}$$

Multiplying first row by-2 and adding the result to the 3rd row.

$$\begin{bmatrix} 2 & -1 & 0 & | & -4 \\ 0 & -3 & 2 & | & -6 \\ 0 & 2 & -1 & | & 3 \end{bmatrix}$$

Multiplying the second row by$\dfrac{-1}{3}$

$$\begin{bmatrix} 2 & -1 & 0 & | & -4 \\ 0 & 1 & \frac{-2}{3} & | & 2 \\ 0 & 2 & -1 & | & 3 \end{bmatrix}$$

Multiplying the second row by -2 and adding the result to the 3rd row.

$$\begin{bmatrix} 2 & -1 & 0 & | & -4 \\ 0 & 1 & \frac{-2}{3} & | & 2 \\ 0 & 0 & \frac{1}{3} & | & -1 \end{bmatrix}$$

Multiplying the 3rd row by 3

$$\begin{bmatrix} 2 & -1 & 0 & | & -4 \\ 0 & 1 & \frac{-2}{3} & | & 2 \\ 0 & 0 & 1 & | & -3 \end{bmatrix}$$

Multiplying the 3rd row by $\dfrac{2}{3}$ and adding the result to the 2nd row

$$\begin{bmatrix} 2 & -1 & 0 & | & -4 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -3 \end{bmatrix}$$

Multiplying the first row by $\frac{1}{2}$

$$\begin{bmatrix} 1 & \frac{-1}{2} & 0 & | & -2 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -3 \end{bmatrix}$$

Multiplying the second row by $\frac{1}{2}$ and adding the result to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -3 \end{bmatrix}$$

so we get

$x = -2$

$y = 0$

$z = -3$

2.2 Indirect(Iterative) Method

Iterative(Indirect) methods start with a guess of the solution x, and then repeatedly refine the solution until a certain convergence criterion is reached. Iterative methods are generally less efficient than direct methods due to the large number of operations or Iterations Required.

In this section we will present two indirect(iterative) methods.
There are

1. Jacobi's Iteration Method

2. Gauss Seidal Iteration Method

2.2.1 Jacobi's Iteration Method

The method is also known as the method of simultaneous displacements.
consider the system of linear equations

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

Here we assume that the coefficients a_{11}, a_{12} and a_{13} are the largest coefficients in the respective equations.so that

$|a_{11}| > |a_{12}| + |a_{13}|$

$|a_{22}| > |a_{21}| + |a_{23}|$

$|a_{33}| > |a_{31}| + |a_{32}|$

Jacobi's iteration method is applicable only if the conditions given in equation (2) are satisfied.now we can write the first equation;

$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3)$

$x_2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}x_3)$

$x_3 = \frac{1}{a_{33}}(b_3 - a_{31}x_1 - a_{32}x_2)$

Let the initial approximations be x_1^0,x_2^0 and x_3^0 respectively.The following iterations are then carried out.

Iteration 1:The first Improvements are found as

$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3)$

$x_2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}x_3)$

$x_3 = \frac{1}{a_{33}}(b_3 - a_{31}x_1 - a_{32}x_2)$

Iteration 2:The second Improvements are obtained as

$x_{12} = \frac{1}{a_{11}}(b_1 - a_{12}x_{21} - a_{13}x_{31})$

$x_{22} = \frac{1}{a_{22}}(b_2 - a_{21}x_{11} - a_{23}x_{31})$

$x_{32} = \frac{1}{a_{33}}(b_3 - a_{31}x_{11} - a_{32}x_{21})$

Example solve the following equations by Jacobi's method

$15x + 3y - 2z = 85$

$2x + 10y + z = 51$

$x - 2y + 8z = 5$

solution

$|15| > |3| + |-2|$

$|10| > |2| + |1|$

$|8| > |1| + |-2|$

then Jacobi's method applicable.we rewrite the given equations as follows

$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z) = \frac{1}{15}(85 - 3y + 2z)$

$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z) = \frac{1}{10}(51 - 2x - z)$

$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y) = \frac{1}{8}(5 - x + 2y)$

let the initial approximations be:

$X^0 = Y^0 = Z^0 = 0$

Iteration 1:

$X_1 = \frac{d_1}{a_1} = \frac{85}{15} = \frac{17}{3}$

$Y_2 = \frac{d_2}{b_2} = \frac{51}{10}$

$Z_3 = \frac{d_3}{c_3} = \frac{5}{8}$

Iteration 2:

$x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1) = \frac{1}{15}(85 - 3 \times \frac{51}{10} - (-2) \times \frac{5}{8})$

$x_2 = 4.73$

$y_2 = \frac{1}{b_2}(d_2 - a_2 x_1 - c_2 z_1) = \frac{1}{10}(51 - 2 \times \frac{17}{3} - 1 \times \frac{5}{8})$

$y_2 = 3.904$

$z_2 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1) = \frac{1}{8}(5 - 1 \times \frac{17}{3} - (-2) \times \frac{51}{10})$

$z_2 = 1.192$

Iteration 3:

$x_3 = \frac{1}{15}(85 - 3 \times 3.904 + 2 \times 1.192) = 5.045$

$y_3 = \frac{1}{10}(51 - 2 \times 4.73 - 1 \times 1.192) = 4.035$

$z_3 = \frac{1}{8}(5 - 1 \times 4.173 + 2 \times 3.904) = 1.010$

Iteration 4:

$x_4 = \frac{1}{15}(85 - 3 \times 4.035 + 2 \times 1.010) = 4.994$

$y_4 = \frac{1}{10}(51 - 2 \times 5.045 - 1 \times 1.010) = 3.99$

$z_4 = \frac{1}{8}(5 - 1 \times 4.173 + 2 \times 3.904) = 1.010$

Iteration 5:

$x_5 = \frac{1}{15}(85 - 3 \times 3.99 + 2 \times 1.003) = 5.002$

$y_5 = \frac{1}{10}(51 - 2 \times 4.994 - 1 \times 1.003) = 4.001$

$z_5 = \frac{1}{8}(5 - 1 \times 4.994 + 2 \times 3.99) = 0.998$

Iteration 6:

$x_6 = \frac{1}{15}(85 - 3 \times 4.001 + 2 \times 0.998) = 5.0$

$y_6 = \frac{1}{10}(51 - 2 \times 5.002 - 1 \times 0.998) = 4.0$

$z_6 = \frac{1}{8}(5 - 1 \times 5.002 + 2 \times 4.001) = 1.0$

Iteration 7:

$x_7 = \frac{1}{15}(85 - 3 \times 4 + 2 \times 1) = 5.0$

$y_7 = \frac{1}{10}(51 - 2 \times 5 - 1 \times 1) = 4.0$

$z_7 = \frac{1}{8}(5 - 1 \times 5 + 2 \times 4) = 1.0$

2.2.2 Gauss Seidal Iteration Method

Definition: The Gauss Seidal Method is applicable to predominantly diagonal systems. a predominantly diagonal system has large diagonal elements. The absolute value of the diagonal element in each case is larger than the sum of the absolute values of the other elements in that row of the matrix A. for such predominantly diagonal systems,the Gauss seidal method always converges to the correct solution ,irrespective of the choice of the initial estimates. since the most recent approximations of the variables are used while proceeding to the next step,the convergence of the gauss method is twice as fast as in Jacobi's method.

convergence is assured in the Gauss seidel method if the matrix A is diagonally dominant and possible definite.If it is not in a diagonally form,it should be connected to a diagonally dominant form by row exchanger, before starting the Gauss seidal iterative schame.

Gauss seidal method is an iterative solution procedure which is an improved version of Jacobi's method.The method is also known as the method of successive approximations. consider the system of linear simultaneous equations;

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

If the absolute value of the largest coefficient in each equation is greater than the sum of the absolute values of all the remaining coefficients,then the Gauss-seidal iteration method will converge.
If this condition is not satisfied,the gauss seidal method is not applicable.

We can write the first equation as;

$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3)$

$x_2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}x_3)$

$x_1 = \frac{1}{a_{33}}(b_3 - a_{31}x_1 - a_{32}x_2)$

Let the initial approximations be x_1^0, x_2^0 and x_3^0 respectively.The following iterations are then carried out.

Iteration 1:The Improvements of x_1, x_2 and x_3 are obtained as;

$x_{11} = \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3^0)$

$x_{21} = \frac{1}{a_{22}}(b_2 - a_{21}x_{11} - a_{23}x_3^0)$

$x_{31} = \frac{1}{a_{33}}(b_3 - a_{31}x_{11} - a_{32}x_{21})$

Iteration 2:The second Improvements of x_1, x_2 and x_3 are obtained as;

$x_{12} = \frac{1}{a_{11}}(b_1 - a_{12}x_{11} - a_{13}x_{31})$

$x_{22} = \frac{1}{a_{22}}(b_2 - a_{21}x_{12} - a_{23}x_{31})$

$x_{33} = \frac{1}{a_{33}}(b_3 - a_{31}x_{12} - a_{32}x_{22})$

The above iteration process is continued until the values ofx_1, x_2andx_3 are obtained to a pre-assigned or desired degree of accuracy.

Example

solve the following equations by Gauss-seidal method

$8x + 2y - 2z = 8$

$x - 8y + 3z = -4$

$2x + y + 9z = 12$

solution

In the above equations:

$|8| > |2| + |-2|$

$|-8| > |1| + |3|$

$|9| > |2| + |1|$

$x_1 = \frac{1}{a_1}(d_1 - b_1 y^0 - c_1 z^0)$

$y_1 = \frac{1}{b_2}(d_2 - a_2 x_1 - c_2 z^0)$

$z_1 = \frac{1}{c_3}(d_3 - a_3 - x_1 - b_3 y_1)$

let the initial approximations be:

$X^0 = Y^0 = Z^0 = 0$

Iteration1:

$x_1 = \frac{d_1}{a_1} = \frac{8}{8} = 1$

$y_1 = \frac{1}{b_1}(d_2 - a_2 x_1) = \frac{1}{-8}(-4 - 1 \times 1) = 0.625$

$z_1 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1) = \frac{1}{9}(12 - 2) = 2 \times 1 - 1 \times 0.625 = 1.042$

Iteration2:

$x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1) = \frac{1}{8}(8 - 2 \times 0.625 - (-2) \times 1.042) = 1.104$

$y_2 = \frac{1}{b_2}(d_2 - a_2 x_2 - c_2 z_1) = \frac{1}{-8}(-4 - 1 \times 1.104 - 3 \times 1.042) = 1.029$

$z_2 = \frac{1}{c_3}(d_3 - a_3 x_2 - b_3 y_2) = \frac{1}{9}(2 - 2 \times 1.104 - 1 \times 1.029) = 0.974$

Iteration3:

$x_3 = \frac{1}{a_1}(d_1 - b_1 y_2 - c_1 z_2) = \frac{1}{8}(8 - 2 \times 1.029 - (-2) \times 0.974) = 0.986$

$y_3 = \frac{1}{b_2}(d_2 - a_2 x_3 - c_2 z_2) = \frac{1}{-8}(-4 - 1 \times 0.986 - 3 \times 0.974) = 0.989$

$z_3 = \frac{1}{c_3}(d_3 - a_3 x_3 - b_3 y_3) = \frac{1}{9}(12 - 2 \times 0.986 - 1 \times 0.989) = 1.004$

Iteration4:

$x_4 = \frac{1}{8}(8 - 2 \times 0.989 - (-2) \times 1.004) = 1.004$

$y_4 = \frac{1}{-8}(-4 - 1 \times 1.004 - 3 \times 1.004) = 1.002$

$z_4 = \frac{1}{9}(12 - 2 \times 1.004 - 1 \times 1.002) = 0.999$

Iteration5:

$x_5 = \frac{1}{8}(8 - 2 \times 1.002 - (-2) \times 0.999) = 0.999$

$y_5 = \frac{1}{-8}(-4 - 1 \times 0.999 - 3 \times 0.999) = 1$

$z_5 = \frac{1}{9}(12 - 2 \times 0.999 - 1 \times 1) = 1$

Iteration6:

$x_6 = \frac{1}{8}(8 - 2 \times 1 + 2 \times 1) = 1$

$y_6 = \frac{1}{-8}(-4 - 1 \times 1 - 3 \times 1) = 1$

$z_6 = \frac{1}{9}(12 - 2 \times 1 - 1 \times 1) = 1$

Chapter 3

Application Of System Of Linear Equation

System of linear equations arise in a wide variety of applications and are one of the central themes in linear algebra. In this section I will show two applications. The first application shows how to fit a polynomial function to a set of data points in the plane. The second application focuses on networks.

3.1 Polynomial Curve Fitting

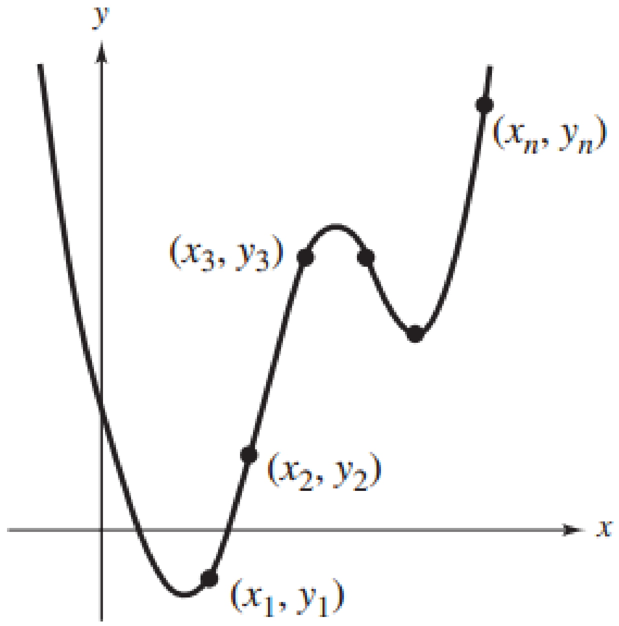

figure 3.1.1

suppose a collection of data is represented by n points in the xy-plane
$(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$
and to find a polynomial function of degree $n-1$
$p(x) = a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1}$
whose graph passes through the specified points. This procedure is called **polynomial curve fitting**. If all x-coordinates of the points are distinct, then the n points,as shown in the figure.
To solve the n coefficients of $p(x)$,substitute each of the n points into the polynomial function and obtain n linear equations in n variables $a_0, a_1, a_2 \ldots a_{n-1}$.

$$a_0 + a_1x_1 + a_2x_1^2 + \ldots + a_{n-1}x_1^{n-1} = y_1$$

$$a_0 + a_1x_2 + a_2x_2^2 + \ldots + a_{n-1}x_2^{n-1} = y_2$$

$$\vdots$$

$$a_0 + a_1x_n + a_2x_n^2 + \ldots + a_{n-1}x_n^{n-1} = y_n$$

Example

Determine the polynomial $p(x) = a_0 + a_1x + a_2x_2$ whose graph passes through the points $(1, 4), (2, 0)$ and $(3, 12)$

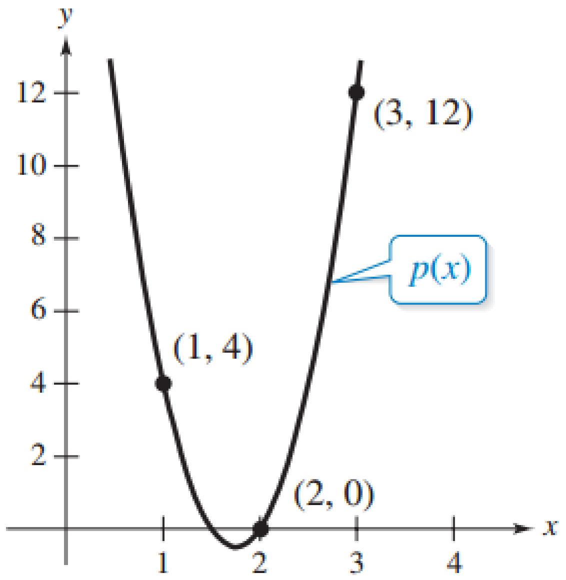

figure 3.1.2

solution:

substituting $x = 1$, 2 and 3 in to $p(x)$and equation the results to the respective y-values produces the system of linear equation in the variables $a_0, a_1$and a_2.

$p(1) = a_0 + a_1(1) + a_2(2)^2 = a_0 + a_1 + a_2 = 4$

$p(2) = a_0 + a_1(2) + a_2(2)^2 = a_0 + 2a_1 + 4a_2 = 0$

$p(3) = a_0 + a_1(3) + a_2(3)^2 = a_0 + 3a_1 + 9a_2 = 12$

The solution of this system is

$a_0 = 16$

$a_1 = -18$

$a_2 = 6$

so the polynomial function is $p(x) = 6x^2 - 18x + 16$

3.2 Network Analysis

Networks composed of branches and junctions are used as models in such fields as economics, traffic analysis, and electrical engineering. In a network model, we assume that the total flow into a junction is equal to the total flow out of the junction. For instance, the junction shown in Figure 3.1.3 has 25 units flowing into it, so there must be 25 units flowing out of it. You can represent this with the linear equation.
$x_1 + x_2 = 25$

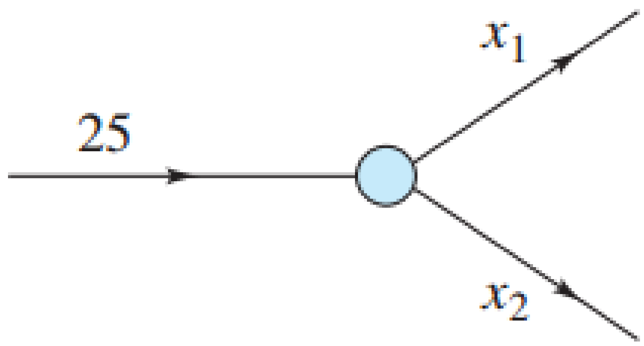

figure 3.1.3

Because each junction in a network give rise to a linear equation can analyze the flow through network composed of several junction by solving a system of linear equation. the following example illustrates this procedure

Example: Analysis of a network

Set up a system of linear equations to represent the network shown in Figure 1.10. Then solve the system

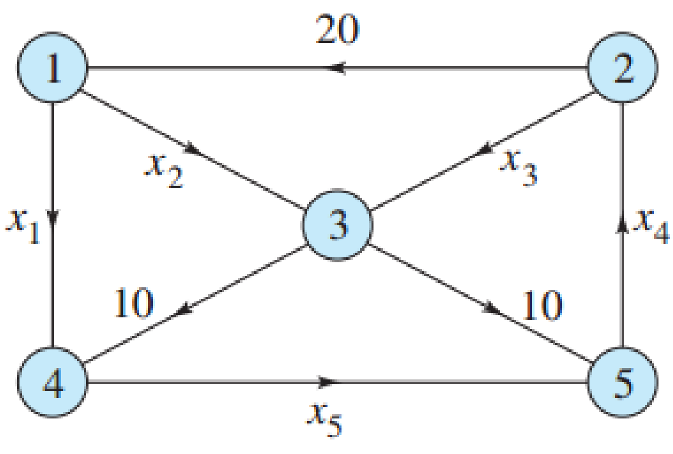

figure 3.1.4

solution

Each of the network's five junctions gives rise to a linear equation, as follows.

$x_1 + x_2 = 20 \qquad junction1$

$x_3 - x_4 = -20 \qquad junction2$

$x_2 + x_3 = 20 \qquad junction3$

$x_1 - x_5 = -10 \qquad junction4$

$-x_4 + x_5 = -10 \qquad junction5$

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & -1 & 1 & -10 \end{bmatrix} \tag{3.1}$$

Gauss Jordan elimination produces the matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.2}$$

from the matrix above, we can see that

$x_1 - x_5 = -10$

$x_2 + x_5 = 30$

$x_3 - x_5 = -10$

$x_4 - x_5 = 10$

letting $t = x_5$

$x_1 = t - 10$

$x_2 = -t + 30$

$x_3 = t - 10$

$x_4 = t + 10$

$x_5 = t$

Where t is any real number so this system has infinitely many solutions.

In the above example suppose we could control the amount of flow along the branch labeled using the solution of the example, we could then control the flow represented by each of the other variables. For instance, letting would reduce the flow of and to zero, as shown in the below figure

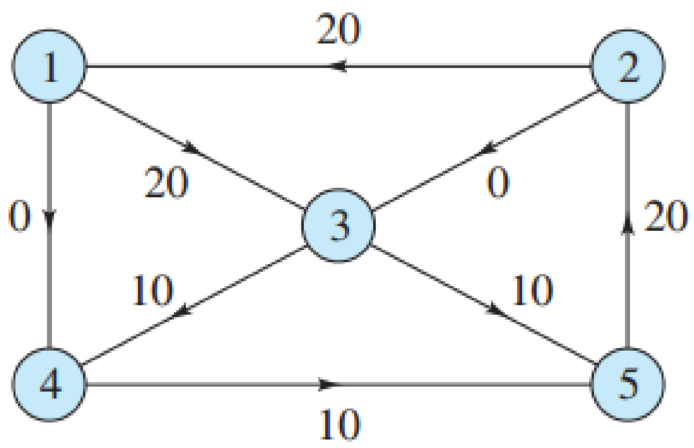

figure 3.1.5

An electrical network is another type of network where analysis is commonly applied. An analysis of such a system uses two properties of electrical networks known as **Kirchhoff's** Laws.

1. All the current flowing into a junction must flow out of it.
2. The sum of the products (is current and is resistance) around a closed path is equal to the total voltage in the path.

Example: Analysis of an Electrical Network

Determine the currents and for the electrical network shown in the Figure below

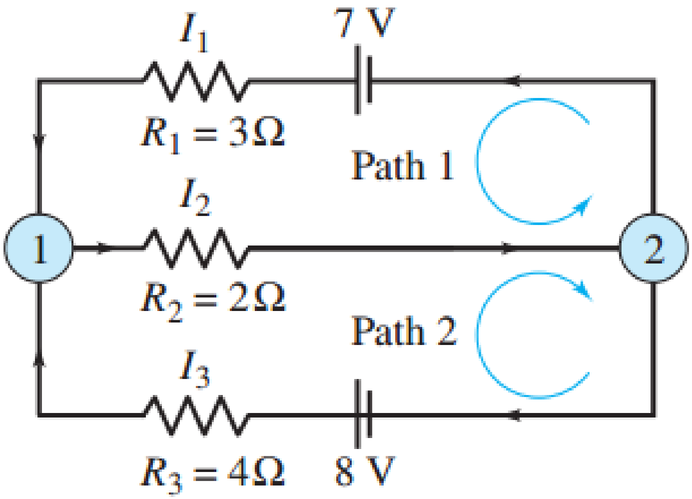

figure 3.1.6

solution

Applying Kirchhoff's first law to either junction produces

$I_1 + I_3 = I_2$ Junction 1 or Junction 2

and applying Kirchhoff's second law to the two paths produces

$R_1 I1 + R_2 I_2 = 3I_1 + 2I_2 = 7$ path 1

$R_2 I2 + R_3 I_3 = 2I_1 + 4I_3 = 7$ path 2

So, we have the following system of three linear equations in the variables I_1, I_2 and I_3.

$I_1 - I_2 + I_3 = 0$

$3I_1 + 2I_2 = 7$

$2I_2 + 4I_3 = 8$

Applying Gauss-Jordan elimination to the augmented matrix

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 2 & 0 & 7 \\ 0 & 2 & 4 & 8 \end{bmatrix} \tag{3.3}$$

produces the reduced row-echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{3.4}$$

which means $I_1 = 1$ amp, $I_2 = 2$ amps and$I_3 = 1$ amp.

Conclusion

In this project, I have presented an overview of the methods of system of linear equation and its application. specifically the Numerical method which is widely used in numerous real-world application. these project provided the relevant and useful elements of matrix analysis for the solution of linear simultaneous algebraic equation and more about application of algebraic equation and solutions. the solution of n linear simultaneous algebraic equation in n unknowns is presented. There are two classes of method of solving system of linear equation direct and iterative method. Direct methods transform the original equation in to equivalent equation that can be solved more easily. Iterative or indirect method start with a guess of the solution x and then repeatedly refine the solution until a certain convergence criterion is reached. Three direct methods (matrix inversion method, Gauss elimination method and Gauss Jordan method) are presented. two indirect or iterative method (Jacobi's iterative method and Gauss-seidal iterative method) are presented. We conclude that the Numerical method(iterative) method is an applicable technique and approximate the solution very well. This method give comparable results and is easy to compute. Further more, Iterative method is more applicable and better convergent if the size of matrix is very large or more variables.

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