



COLLEGE OF NATURAL AND COMPUTATIONAL  
SCIENCE DEPARTMENT OF MATHEMATICS

Project Work On Game theory and its application  
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Bachelor of Science in Mathematics

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# Contents

Acknowledgement

Abstract

Introduction	1
<b>1 PRELIMINARIES</b>	<b>2</b>
1.1 Game Theory . . . . .	2
1.1.1 Classification and factors of game theory . . . . .	3
1.1.2 TWO PERSON ZERO SUM GAME . . . . .	4
<b>2 SOLUTION METHOD</b>	<b>6</b>
2.1 solving game theory . . . . .	6
2.1.1 pure strategies with saddle point . . . . .	7
<b>3 Game Theory and its application</b>	<b>30</b>
3.1 Application . . . . .	30
Summary	32
Reference	33

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The undersigned hereby certify that they have read and recommend to the department of Mathematics for acceptance of a project entitled —— by in partial fulfillment of the requirements for the Bachelor of Science in Mathematics.

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# Abstract

This project contains three chapters, the first chapter about Introduction of Game theory and its application, the second chapter about the Solution method, the third chapter about summary, Game theory is a theoretical framework for conceiving social situation among competing players. It is one of the creative methods of successful managing of communication risks as educational projects of different types and sizes. Who are assumed to act rationally. We will explore the basic ideas of game theory, including the ideas of pay offs and outcomes classification of games as zero sum or non zero sum, types of strategies. Game theory has many applications in subjects such as economics international relations and politics, and politics, and psychology as it can be used to analyse and predict the behavior and decisions of the player we will apply game theory to the decisions made by the players during the Triwizard tournament in Harry Potter and the Goblet of Fire.

# Introduction

Game theory is a theoretical framework for conceiving social situation among competing players.it is the branch of mathematics concerned with competitive situations where the outcome of a participant's choice of action depends critically on the actions of other participants. Game theory is used to denoting on study of mathematical decision makers .Game theory enables universal mathematical techniques for analysis situation which individual or competitors in same segment make decision that will influence one another interests.it is implementing that conflict analysis or interactive decision theory.the foundation of game theory enables through its position in the mathematical narration of social science with the advantage of this theory economist can able to understand conflict and cooperation by understanding quantitative models and real life situations.finally it provides a way of elucidating the logical difficulty of philosophers such as Hobbes,Rousseau,other social and political theorists.so it develops a framework for analysing decision making in such situations where inter dependence of firms is considered.

.Generally game theory is the mathematical study of strategic decisions making it can be used to analyse the options motivations and reward involved in a decision.

# Chapter 1

## PRELIMINARIES

### 1.1 Game Theory

Life is full of conflict and competition. Numerous examples involving adversaries in conflict include parlor games, military battles, political campaigns, advertising and marketing campaigns by competing business firms, and so forth. A basic feature in many of these situations is that the final outcome depends primarily upon the combination of strategies selected by the adversaries. Game theory is a mathematical theory that deals with the general features of competitive situations like these in a formal, abstract way. It places particular emphasis on the decision-making processes of the adversaries. research on game theory continues to delve into rather complicated types of competitive situations. However, the focus in this chapter is on the simplest case, called two-person, zero-sum games. As the name implies, these games involve only two adversaries or players (who may be armies, teams, firms, and so on). They are called zero-sum games because one player wins whatever the other one loses, so that the sum of their net winnings is zero. introduces the basic model for two-person, zero-sum games, In a game in strategic form, a strategy is one of the given possible actions of a player. In an extensive game, a strategy is a complete plan of choices, one for each decision point of the player. A game is said to be zero-sum if for any outcome, the sum of the payoffs to all players is zero. In a two-player zero-sum game, one players gain is the other players loss, so their interests are diametrically opposed.

### 1.1.1 Classification and factors of game theory

The models in the theory of games can be classified depending up on the following factors.

1. number of players- It is number of competitive decision makers, involved in the game. A game involving two players is referred to as a two-person game, however if the number of players is more than 2, then the game is called an N person game.
  
2. Total payoff
  - It is the sum of gains and losses from the game that are available to the players, if in a game the sum of the gains to one player is exactly equals to the sum of the losses to on other players. So that the sum of the gains and losses equals zero, then the game is said to be a zero-sum game. There are also games in which the sum of the players gains and losses does not equal zero, and those games are expressed as non-zero sum game.
  
3. Strategy
  - In a game situation, each the player has a set of strategies available. The strategy for the player is the set of alternative course of action that he will take for every payoff (outcomes) that might a rises. It is assumed that the players know the rules governing the choice in advance, the different outcomes resulting from the choices are also know to the players in advance and are expressed in terms of the numerical values. If the players select the same strategy in each time, then it is referred to as pure strategy. In this case each player knows exactly what the opponent is going to do and the objectives of the players are to maximize gains or to minimize losses.
  
4. Optimal strategy
  - A strategy which when adopted puts the players in the most preferred position, irrespective of the strategy of his competitors are called an optimal strategy. The optimal strategy involves maximal payoff to the players.

5. Number of strategies

- A game is said to be finite, if each player has the option of choosing from only a finite number of strategies otherwise infinite. If the strategy of the player can be discovered by his competition, then it is known as perfect information game. In a case of imperfect information games, no player has complete information and tries to guess the real situation.

### 1.1.2 TWO PERSON ZERO SUM GAME

If a game involves only two players (who may be armies, teams, firms, etc.), they are called zero-sum games because one player wins whatever the other one loses, so that their net winning is zero. That means that the winnings of the winning players are paid by the losses of the losing players. In general, a two-person game is characterized by the strategies of player 1, the strategies of player 2 and the payoff table. There are two players (called the row player and the column player). The row player must choose 1 of  $m$  strategies. Simultaneously, the column player must choose 1 of  $n$  strategies. If the row player chooses his  $i$ th strategy and the column player chooses his  $j$ th strategy. Then the row player receives a reward of  $a_{ij}$  and the column player loses an amount  $a_{ij}$ . Thus, we may think of the row player's reward of  $a_{ij}$  as coming from the column player. Such a game is called a two-person zero-sum game, which is represented by the matrix in table 1 (the game's reward matrix).

Such a game is called a two-person zero-sum game, which is represented by the matrix in table 1 (the game's reward matrix).

#### PAY OFF GAME TABLE

		Column player's strategy			
		Column 1	Column 2	...	Column n
Row player's Strategy	Row 1	$a_{11}$	$a_{12}$	...	$a_{1n}$
	Row 2	$a_{21}$	$a_{22}$	....	$a_{2n}$
	.	.	...	...	.
	.	.			.
	Row m	$a_{m1}$	$a_{m2}$	...	$a_{mn}$

as previously stated,  $a_{ij}$  is the row player's reward (and the column player's loss) if the row player chooses his  $i$ th strategy and the column player chooses his  $j$ th strategy.

# Chapter 2

## SOLUTION METHOD

### 2.1 solving game theory

solving by min max strategies

**Example 1.** *Example 1 consider the to solve the game whose pay off matrix is given in the following table as follows*

		<i>player B</i>
<i>player A</i>	$B_1$	$B_2$
$A_1$	1	3
$A_2$	-1	6

solution of 2 by 2 matrix the game is worked out using mini procedure.find the smallest value in each row and select the largest value of these value next,find the largest value in each column and select the smallest of these numbers the procedure shown

		player B	
player A	$B_1$	$B_2$	<i>rowmin</i>
$A_1$	1	3	1
$A_2$	-1	6	-1
column max	1	6	

if maximum value in row is equal to the minimum value of column,then saddle point exists (1,2) and the value is 1

$$\min(\text{columnmaximum}) = \min(1, 6) = 1 \quad (2.1)$$

by choosing column 1. We have shown that the row player can ensure that he will win at least one unit and the column player can hold the row player's winnings to at most one unit. Thus, the only rational outcome of this game is for the row player to win exactly one unit; the row player cannot expect to win more than one unit, because the column player (by choosing column 1) can hold the row player's winnings to one unit.

The game matrix we have just analyzed has the property of satisfying the Saddle Point Condition.

$$\max(\text{row minimum}) = \min(\text{column maximum}) \quad (2.2)$$

Any two-person zero-sum game satisfying (1) is said to have a saddle point, if a two-person zero-sum game has a saddle point, then the row player should choose any strategy (row) attaining the maximum on the left side of (1). The column player should choose any strategy (column) attaining the minimum on the right side of (1).

### 2.1.1 pure strategies with saddle point

A saddle point is the smallest entry in its row and the largest entry in its column in a game with a saddle point the optimum pure strategies for player A is to choose the row containing the saddle point, while the optimum pure strategies for B is to choose the column saddle point.

Given a matrix of  $n \times n$  size, the task is to find the saddle point of the matrix. A saddle point is an element of the matrix such that it is the minimum element in its row and maximum in its column.

**Example** solve pay off matrix

		B'S strategies	
A's strategies	$Y_1$	$Y_2$	$Y_3$
$X_1$	5	7	4
$X_2$	2	3	5
$X_3$	10	6	6

solution of pay off matrix at saddle point

		B'S strategies		
A's strategies	$Y_1$	$Y_2$	$Y_3$	row min
$X_1$	5	7	4	4
$X_2$	2	3	5	2
$X_3$	10	6	6	6
column max	10	7	6	

the saddle point solution (2,3) and the value of the game is 6.

**Example 3** solve the game with pay off matrix for player A as given in table

	player B		
player A	B1	B2	B3
A1	-4	0	4
A2	1	4	2
A3	-1	5	-3

solution find the smallest element in rows and largest element in columns as shown in table min max procedure.

	player B			
player A	$B_1$	$B_2$	$B_3$	row min
$A_1$	-4	0	4	-4
$A_2$	1	4	2	1
$A_3$	-1	5	-3	-3
column max	1	5	4	

select the largest element in row and smallest element in column check for the mini max criteria, (2,1) the value of the game is 1

**Example 4** solve game with saddle point

	player B		
player A	$B_1$	$B_2$	$B_3$
$A_1$	-3	-2	6
$A_2$	2	0	2
$A_3$	5	-2	-4

solution of game with saddle point

	player B			
player A	$B_1$	$B_2$	$B_3$	row min
$A_1$	-3	-2	6	-3
$A_2$	2	0	2	0
$A_3$	5	-2	-4	-4
column max	5	0	6	

the game is 0 if the value of the game is 0 then that

$$0 = 0 \quad (2.3)$$

$$\text{saddlepoint}, (1, 2) \quad (2.4)$$

**Example 5** solve the game with the pay off matrix given in table and determine the best strategies for the companies a and b and find the value of the game for the them.

		company B	
company A	$B_1$	$B_2$	$B_3$
$A_1$	2	4	2
$A_2$	1	-5	-4
$A_3$	2	6	-2

the matrix is solved using saddle point criteria,

		company B		
company A	$B_1$	$B_2$	$B_3$	row min
$A_1$	2	4	2	2
$A_2$	1	-5	-4	-5
$A_3$	2	6	-2	-2
column max	2	6	2	

therefore,there is a saddle point optimum strategy for company A is  $A_1$  and the optimum strategy for company B is  $B_1$  or B  
 $2=2$

### pure strategies with out saddle point

No saddle point (not strictly determined) no pure strategy More than 2 choices for a player and matrix cannot be reduced by dominance.

solve a pay off matrix

		strategy B	
strategy A	$B_1$	$B_2$	$B_3$
$A_1$	1	2	3
$A_2$	2	5	4
$A_3$	3	2	6

solution of the game with out saddle point

		strategy B		
strategy A	$B_1$	$B_2$	$B_3$	row min
$A_1$	1	2	3	1
$A_2$	2	5	4	2
$A_3$	3	2	6	2
column max	3	5	6	

the strategies is null

## TWO PERSON ZERO SUM GAME ( WITHOUT A SADDLE POINT)

You have observed that in game situations which have a saddle point(s) are provided with an adequate theory of how best to play the game. It is possible that there is no saddle point of a game and then it is not possible to find its solution in terms of the pure strategies the max min and the min max. Games without saddle point are not strictly determined. The solution to such problems calls for employing mixed strategies i.e. to solve such games both the players must determine an optimal mixture of strategies to find a saddle point. A mixed strategy therefore represents a combination of two or more strategies that are selected one at a time, according to pre-determined probabilities. Thus in employing a mixed strategy, a player decides to mix his choices among several alternative with certain ratio.

**Example 2.** *pay off game with out saddle point*

		A's Payoff			Row	A's
		B's Strategies			minimum	Maximini
		1	2	3		
A's Strategies	1	20	8	-6	-6	
	2	12	10	2	2	3
	3	3	5	6	3	
Col.Max.		20	10	6		
B's Minimax			6			
Saddle point solution Does Not Exist						

*Solution by Mixed Strategy*

From the example above there is no saddle point solution. For such models we can always find a mixed strategy solution by using theory of probabilities. Let player A play strategies 1, 2, 3 with probabilities  $x_1, x_2, x_3$  respectively subject to  $x_1 + x_2 + x_3 = 1$ . Similarly, let player B play strategies 1, 2, 3 with probabilities  $y_1, y_2, y_3$  respectively subject to  $y_1 + y_2 + y_3 = 1$ . Once the probabilities are assigned, we can always calculate the expected payoffs called value of the game by mere substitution.

		A's Payoff			Row	A's
		B's Strategies			minimum	Maximini
		1	2	3		
		y1	y2	y3		
A's Strategies	1	x1	20	8	-6	
	2	x2	12	10	2	3
	3	x3	3	5	6	
Col.Max			20	10	6	
B's Minimax				6		

Saddle point solution Does Not Exist  
Value of Game=Nil

Method of getting optimum strategic Probabilities Since A is a maximini player he will

		A's Payoff	
		B's Strategies	
		1	2
A's Strategies	1	4	1
	2	3	5

according to pre-determined probabilities. Thus in employing a mixed strategy, a player decides to mix his choices among several alternative with certain ratio.

According to minimax theorem for any given problem there will always be at least one mixed strategy solution. Using any one of the following methods we can solve such games.

1. Algebraic Method
2. Sub game method
3. Dominance method
4. Graphical method
5. Linear programming method

only seen three of them

1) Solution by Algebraic Method

This method is often used for a very simple 2 x2 type of game

**step 1** First let us Check whether saddle point solution exist or not.

		A's Payoff			Row Minimum	A's Maximin
		B's Strategies				
		1	2			
A's Strategies	1	4	1	1		
	2	3	5	3		3
Col. Max		4	5			
B's Minimax			5			
Saddle point solution Does Not Exist						

**Step2:** Since the problem does not have the saddle point, we proceed further for a strategic mixed solution. Let  $x_1$  and  $x_2$  are the two strategic probabilities for player A and  $y_1$ ;  $y_2$  be strategic probabilities for player B. Now, since  $x_1 + x_2 = 1$  and  $y_1 + y_2 = 1$  we have  $x_2 = 1 - x_1$  and  $y_2 = 1 - y_1$

		<b>A's Payoff</b>	
		<b>B's Strategy</b>	
		<b>1</b>	<b>2</b>
		$y_1$	$y_2$
<b>1</b>	$x_1$	<b>4</b>	<b>1</b>
<b>2</b>	$x_2$	<b>3</b>	<b>5</b>

### Calculation of A's Strategic probabilities

Since A is a maxi mini row player he will play strategy 1 and 2 in such a way expected earnings from both types are equal to one another irrespective of his opponent B's Play.

If B plays his pure strategy 1, then the expected earnings by playing strategy 1 and 2 by A is obtained as:

$$4x_1 + 3x_2 = v \quad (2.5)$$

$$4x_1 + 3(1 - x_1) = v \quad (2.6)$$

$$x_1 + 3 = v \quad (2.7)$$

If B plays his pure strategy 2, then the expected earnings by playing strategy 1 and 2 by A is obtained as:

$$x_1 + 5x_2 = v \quad (2.8)$$

$$x_1 + 5(1 - x_1) = v \quad (2.9)$$

$$-4x_1 + 5 = v \quad (2.10)$$

Now from using equation (2.7) and (2.10)

$$x_1 + 3 = -4x_1 + 5 \quad (2.11)$$

$$5x_1 = 2 \quad (2.12)$$

$$x_1 = \frac{2}{5} \quad (2.13)$$

$$x_2 = 1 - \frac{2}{5} = \frac{3}{5} \quad (2.14)$$

Hence player  $A$  will play strategy 1 by  $\frac{2}{5}$  times of his total play and strategy 2 by  $\frac{3}{5}$  times of his total play.

### Calculation of B's Strategic probabilities

Since  $B$  is a column player he will play strategy 1 and 2 in such a way that his expected earnings from both types are equal to one another irrespective of his opponent  $A$ 's play.

If  $A$  plays his pure strategy 1, then the expected earnings by playing strategy 1 and 2 by  $B$  is obtained as:

$$4y_1 + y_2 = v \quad (2.15)$$

$$4y_1 + (1 - y_1) = v \quad (2.16)$$

$$3y_1 + 1 = v \quad (2.17)$$

If  $A$  plays his pure strategy 2, then the expected earnings by playing strategy 1 and 2 by  $B$  is obtained as:

$$3y_1 + 5y_2 = v \quad (2.18)$$

$$3y_1 + 5(1 - y_1) = v \quad (2.19)$$

$$-2y_1 + 5 = v \quad (2.20)$$

Now from the last two equations

$$3y_1 + 1 = -2y_1 + 5 \quad (2.21)$$

$$5y_1 = 4 \quad (2.22)$$

$$y_1 = \frac{4}{5} \text{ and } y_2 = 1 - \frac{4}{5} = \frac{1}{5} \quad (2.23)$$

Hence player  $B$  will play strategy 1 by  $\frac{4}{5}$  times of his total play and strategy 2 by  $\frac{1}{5}$  times of his total play.

Therefore,

$$v = 4y_1 \quad (2.24)$$

$$y_2 = 4\left(\frac{4}{5}\right) + 1\left(\frac{1}{5}\right) = 3.4 \quad (2.25)$$

**Sub game method**

<b>A's Payoff</b>			
<b>B's Strategy</b>			
	1	2	3
1	275	-50	-75
2	125	130	150

**A's Strategy**

In this method first we divide the given big game into simple 2x2 games. So the obtained simple games are solved independently to arrive at the final solution. Now since B has three strategies in total he has better capacity to manipulate. So finally B will play that simple game that gives him the payoff.

**Example 3.** *for the given A's pay off, find the mixed strategy solution by algebraic method using sub games*

<b>A's Payoff</b>					
<b>B's Strategy</b>				<b>Row minimum</b>	<b>A's maximini</b>
	1	2	3		
1	275	-50	-75	-75	
2	125	130	150	125	125

**A's Strategy**

<b>Col. maximum</b>	275	130	150
<b>B's minimax</b>	130		

**Saddle point doesnot exists**  
**Value of the game = Null**

*Since there is no saddle point solution to this problem let us split this 2x3 game into three 2x 2 games first and try to solve them by algebraic method*

discussed.

Sub-Game I				Sub-Game II			
		A's Payoff				A's Payoff	
		B's Strategy				B's Strategy	
			1	2			
			$y_1$	$y_2$			$y_1$
1	$x_1$	275	-50	1	$x_1$	275	-75
2	$x_2$	125	130	2	$x_2$	125	150
A's Strategy				A's Strategy			

Sub-Game III			
		A's Payoff	
		B's Strategy	
		2	3
		$y_2$	$y_3$
1	$x_1$	-50	-75
2	$x_2$	130	150
A's Strategy			

Solution: For Sub-Game I

**Step 1:** first let us Check whether the sub-problem 1 has a saddle point solution or not.

**Sub-Game I**

		A's Payoff		Row minimum	A's maximin
		B's Strategy			
		1	2		
A's Strategy	1	275	-50	-50	
	2	125	130	125	125
Col. maximum		275	130		
B's minimax		130			

Saddle point solution does not exist  
Value of the game = Nil

**Step 2:** Since the problem does not have the saddle point, we proceed further for a mixed strategy solution.

**Sub-Game I**

		A's Payoff		
		B's Strategy		
		1	2	
		$y_1$	$y_2$	
A's Strategy	1	$x_1$	275	-50
	2	$x_2$	125	130

Let  $x_1$  and  $x_2$  are two strategic probabilities for player A. Similarly, let  $y_1$  and  $y_2$  are the two strategic probabilities for the player B. Now since  $x_1 + x_2 = 1$  we can always replace

$x_2 = 1 - x_1$ . Similarly since  $y_1 + y_2 = 1$  we can always replace  $y_2 = 1 - y_1$ .

### Calculation of A's Strategic Probabilities

Since A is a minimax row player he will play strategy 1 and 2 in such a way that his expected earnings from both types are equal to one another irrespective of his opponent Bs play.

If B plays his pure strategy 1 then the expected earnings by playing strategy 1 and 2 by A is obtained as

$$275x_1 + 125x_2 = v \quad 275x_1 + 125(1 - x_1) = v \quad 150x_1 + 125 = v \quad (2.26)$$

If B plays his pure strategy 2 then the expected earnings by playing strategy 1 and 2 by A is obtained as

$$-50x_1 + 130x_2 = v \quad (2.27)$$

$$-50x_1 + 130(1 - x_1) = v \quad (2.28)$$

$$-180x_1 + 130 = v \quad (2.29)$$

Now from equation using we have

$$150x_1 + 125 = -180x_1 + 130 \quad (2.30)$$

$$330x_1 = 5 \quad (2.31)$$

$$x_1 = \frac{1}{66} \text{ and } x_2 = \frac{65}{66} \quad (2.32)$$

Hence, player A will play strategy 1 by  $\frac{1}{66}$  times of his total play and strategy 2 by  $\frac{65}{66}$  times of his total play. Value of the game  $v = (150 \times \frac{1}{66}) + 125 = 127.27$

### Calculation of B's Strategic Probabilities

Since B is a maximini column player he will play strategy 1 and 2 in such a way that his expected earnings from both types are equal to one another irrespective of his opponent As play.

If A plays his pure strategy 1 then the expected earnings by playing strategy 1 and 2 by B is obtained as

$$275y_1 - 50y_2 = v \quad (2.33)$$

$$275y_1 - 50(1 - y_1) = v \quad (2.34)$$

$$325y_1 - 50 = v \quad (2.35)$$

Similarly, if A plays his pure strategy 2 then the expected earnings by playing strategy 1 and 2 by B is obtained as

$$125y_1 + 130y_2 = v \quad (2.36)$$

$$125y_1 + 130(1 - y_1) = v \quad (2.37)$$

$$-5y_1 + 130 = v \quad (2.38)$$

Now from equation we get

$$325y_1 - 50 = -5y_1 + 130 \quad (2.39)$$

$$330y_1 = 180 \quad (2.40)$$

$$y_1 = \frac{36}{66} \text{ and } y_2 = \frac{30}{66} \quad (2.41)$$

Hence, player B will play strategy 1 by  $\frac{36}{66}$  times of his total play and strategy 2 by  $\frac{30}{66}$  times of his total play.

Value of the game  $v = (325 \times \frac{36}{66}) - 50 = 127.30$

Solution: For Sub-Game II

**step 1** check whether the sub problem it has a saddle point solution or not

		A's Payoff		Row minimum	A's maximini
		B's Strategy			
		1	3		
A's Strategy	1	275	-75	-75	
	2	125	150	125	125
Col. maximum		275	150		
B's minimax		150			
Saddle point solution does not exists					
Value of the game = Nil					

**step 2** since the problem does not have the saddle point we processed further for a mixed strategy solution

Let  $x_1$  and  $x_2$  are two strategic probabilities for player A. Similarly, let  $y_1$  and  $y_3$  are the two strategic probabilities for the player B. Now since  $x_1 + x_2 = 1$  we can always replace  $x_2 = 1 - x_1$ . Similarly since  $y_1 + y_3 = 1$  we can always replace  $y_3 = 1 - y_1$ .

**Calculation of A's Strategic Probabilities** Since A is a maximin row player he will play strategy 1 and 2 in such a way that his expected earnings from both types are equal to one another irrespective of his opponent B's play.

If B plays his pure strategy 1 then the expected earnings by playing strategy 1 and 2 by A is obtained as:

$$275x_1 + 125x_2 = v \quad (2.42)$$

$$275x_1 + 125(1 - x_1) = v \quad (2.43)$$

$$150x_1 + 125 = v \quad (2.44)$$

If B plays his pure strategy 3 then the expected earnings by playing strategy 1 and 2 by A is obtained as

:

$$-75x_1 + 150x_2 = v \quad (2.45)$$

$$-75x_1 + 150(1 - x_1) = v \quad (2.46)$$

$$-225x_1 + 150 = v \quad (2.47)$$

Now from using by equation (2.44) and (2.47) we have

$$150x_1 + 125 = -225x_1 + 150 \quad (2.48)$$

$$375x_1 = 25 \frac{1}{15} \quad (2.49)$$

$$x_1 = \frac{1}{15} \text{ and } x_2 = \frac{14}{15} \quad (2.50)$$

Hence, player A will play strategy 1 by  $\frac{1}{15}$  times of his total play and strategy 2 by  $\frac{14}{15}$  times of his total play. Value of the game  $v = (150 \times \frac{1}{15}) + 125 = 135$ .

### Calculation of B's Strategic Probabilities

Since B is a minimax column player he will play strategy 1 and 3 in such a way that his expected earnings from both types are equal to one another irrespective of his opponent

As play.

If A plays his pure strategy 1 then the expected earnings by playing strategy 1 and 3 by B is obtained as:

$$275y_1 - 75y_2 = v \quad (2.51)$$

$$275y_1 - 75(1 - y_1) = v \quad (2.52)$$

$$350y_1 - 75 = v \quad (2.53)$$

Similarly, if A plays his pure strategy 2 then the expected earnings by playing strategy 1 and 3 by B is obtained as:

$$125y_1 + 150y_2 = v \quad (2.54)$$

$$125y_1 + 150(1 - y_1) = v \quad (2.55)$$

$$-25y_1 + 150 = v \quad (2.56)$$

Now from equation we get

$$350y_1 - 75 = -5y_1 + 150 \quad (2.57)$$

$$375y_1 = 225 \quad (2.58)$$

$$y_1 = \frac{9}{15} \text{ and } y_2 = \frac{6}{15} \quad (2.59)$$

Hence, player B will play strategy 1 by  $\frac{9}{15}$  times of his total play and strategy 3 by  $\frac{6}{15}$  times of his total play. Value of the game  $v = (350 \times \frac{9}{15}) - 75 = 135$ .

Solution: For Sub-Game III

		A's Payoff		Row minimum	A's maximin
		B's Strategy			
		2	3		
A's Strategy	1	-50	-75	-75	
	2	130	150	130	130
Col. maximum		130	150		
B's minimax		130			

Saddle point solution does not exist

Value of the game = 130  
A' optimal strategy = 2  
B's optimal strategy = 2

Since this sub-game has a saddle point our search stops at this stage.  
 Final Solution: Now since B is having 3 strategies at his command he has better flexibility in handling the situation. Since the given payoff matrix is for A, as usual B will choose the sub-game that gives him the least payoff. In our example since sub-game I gives the least payoff (127.30) B will play always sub-game I after ignoring the 3rd strategy completely.

		<b>A's Payoff</b>	
		<b>B's Strategy</b>	
<b>A's Strategy</b>		1	2
		1	1
		2	-2
		3	5

Remark 1. Since in all the 3 games the expected payoff are positive, A is the ultimate winner of the game. 2. If it is a 3x2 game then invariably A has the better chance of handling the situation. So he will choose the sub-game that gives him the highest payoff.

Dominance Method:  
 Dominance Reduction Rule

1. If all the elements in a row of a given As payoff matrix are greater than the corresponding elements of any other rows, then player A would prefer to play the larger elements row and discard the smaller elements rows. Here the row that contains the larger elements is often called the dominant row and the associated strategy is called the dominant strategy for player A.

2. If all the elements in a column of a given As payoff matrix are less than the corresponding elements of any other columns then player B would prefer to play the smaller elements column and discard the larger elements column.

Here the column that contains the smaller elements is often called the dominant column and the associated strategy is called the dominant strategy for player

**Example 4.** Reduce the following  $3 \times 2$  game by using dominance rule. Also obtain the solution.

<b>A's Payoff</b>		
<b>B's Strategy</b>		
	<b>1</b>	<b>2</b>
<b>1</b>	4	1
<b>2</b>	-1	-2
<b>3</b>	3	5

**A's Strategy**

*Solution: **Step 1:** Reduction by dominance In the  $3 \times 2$  game stated above, A has 3 strategies so he has better capability of handling the situation. Further, since both the entries are negative, playing strategy 2 by A clearly involves payment to B irrespective of his opponents play. Since in strategy 1 the corresponding entries are greater than strategy 2 entries, we say that strategy 1 dominates strategy 2 for A. By the reasoning we notice that strategy 3 also dominates strategy 2 for A so ultimately, player A will discard strategy 2 and play only strategy 1 and 3 these reduced  $2 \times 2$  game as*

A's Payoff		
B's Strategy		
	1	2
A's Strategy	1	1
	3	5

**step 2** search for saddle point solution

A's Payoff			Row minimum	A's maximini
B's Strategy				
	1	2		
A's Strategy	1	1	1	
	3	5	3	3
Col. maximum	4	5		
B's minimax	4			
Saddle point solution does not exists				
Value of the game = Nill				

**Step 3:** Algebraic solution Let  $x_1$  and  $x_3$  are two strategic probabilities for player  $A$ . Similarly, let  $y_1$  and  $y_2$  are the two strategic probabilities for the player  $B$ . Now since

$x_1 + x_3 = 1$  we can always replace

$x_3 = 1 - x_1$ . Similarly since  $y_1 + y_2 = 1$  we can always replace  $y_2 = 1 - y_1$ .

		<b>A's Payoff</b>	
		<b>B's Strategy</b>	
		<b>1</b>	<b>2</b>
		$y_1$	$y_2$
<b>A's Strategy</b>	<b>1</b>	$x_1$	4    1
	<b>3</b>	$x_3$	3    5

### Calculation of A's Strategic Probabilities

Since A is a maximini row player he will play strategy 1 and 3 in such a way that his expected earnings from both types are equal to one another irrespective of his opponent Bs play.

If B plays his pure strategy 1 then the expected earnings by playing strategy 1 and 3 by A is obtained as:

$$4x_1 + 3x_3 = v \tag{2.60}$$

$$4x_1 + 3(1 - x_1) = v \tag{2.61}$$

$$x_1 + 3 = v \tag{2.62}$$

If B plays his pure strategy 2 then the expected earnings by playing strategy 1 and 3 by A is obtained as:

$$1x_1 + 5x_3 = v \tag{2.63}$$

$$1x_1 + 5(1 - x_1) = v \tag{2.64}$$

$$-4x_1 + 5 = v \tag{2.65}$$

Now from equation we have

$$x_1 + 3 = -4x_1 + 5 \tag{2.66}$$

$$5x_1 = 2 \quad (2.67)$$

$$x_1 = \frac{2}{5} \text{ and } x_3 = \frac{3}{5} \quad (2.68)$$

Hence, player A will play strategy 1 by  $\frac{2}{5}$  times of his total play and strategy 3 by  $\frac{3}{5}$  times of his total play. Value of the game  $v = \frac{2}{5} + 3$ .

**Calculation of B's Strategic Probabilities** Since B is a minimax column player he will play strategy 1 and 2 in such a way that his expected earnings from both types are equal to one another irrespective of his opponent A's play.

If A plays his pure strategy 1 then the expected earnings by playing strategy 1 and 2 by B is obtained as:

$$4y_1 + 1y_2 = v \quad (2.69)$$

$$4y_1 + 1(1 - y_1) = v \quad (2.70)$$

$$3y_1 + 1 = v \quad (2.71)$$

Similarly, if A plays his pure strategy 3 then the expected earnings by playing strategy 1 and 2 by B is obtained as:

$$3y_1 + 5y_2 = v \quad (2.72)$$

$$3y_1 + 5(1 - y_1) = v \quad (2.73)$$

$$-2y_1 + 5 = v \quad (2.74)$$

Now from equation we get

$$3y_1 + 1 = -2y_1 + 5 \quad (2.75)$$

$$5y_1 = 4 \quad (2.76)$$

$$y_1 = \frac{4}{5}, \text{ and } y_2 = \frac{1}{5} \quad (2.77)$$

Hence, player B will play strategy 1 by  $\frac{4}{5}$ , times of his total play and strategy 2 by  $\frac{1}{5}$  times of his total play. Value of the game  $v = (3 \times \frac{4}{5}) + 1 = 3.4$

**Example 5.** reduce the following  $2 \times 4$  game by using dominance rule also obtain the solution

		A's Payoff			
		B's Strategy			
A's Strategy		1	2	3	4
	1	0	2	-4	-7
2	1	3	-4	-1	

Solution

**Step 1:** Reduction by dominance

In the 2x4 game stated above, since B has 4 strategies he has better capability of handling the situation. Further, since we are given with A's payoff negative entries refers gains to B. Clearly third and fourth strategies are dominated by B. So B will not play strategies 1 and 2 and make payments to A. So the reduced 2x2 game is obtained as

		A's Payoff	
		B's Strategy	
A's Strategy		3	4
	1	-4	-7
2	-4	-1	

**step 2** search for saddle point solution

		A's Payoff			
		B's Strategy		Row minimum	A's maximini
		3	4		
A's Strategy	1	-4	-7	-7	
	2	-4	-1	-4	-4
Col. maximum		-4	-1		
B's minimax		-4			
Saddle point solution exists					
Value of the game = -4					
A's optimal strategy = 2					
B's optimal strategy = 3					

# Chapter 3

## Game Theory and its application

### 3.1 Application

Game theory is a mathematical tool that helps to study strategic situations in which players optimize a variable not only the basis of their own preference, but also on other players decisions and reactions.

If solution in one way is maximum and another way is minimum, it is often called the saddle point solution.

primary objective of game theory is the development of rational criteria for selecting a strategies two key assumption are made.both player choose their strategies solely to promote.

Game theory is used to denoting on study of mathematical decision makers .it is enables universal mathematical techniques for analysis situation which individual or competitors in same segment make decision that will influence one another interests.it is implementing that conflict analysis or interactive decision theory.the foundation of game theory enables through its position in the mathematical narration of social science with the advantage of this theory economist can able to understand conflict and cooperation by understanding quantitative models and real life situations.finally it provides a way of elucidating the logical difficulty of philosophers such as Hobbes,Rousseau,other social and political theorists.so it develops a framework for analysing decision making in such situations where inter dependence of firms is considered.

game theory was invented to analyse situations where parties with potentially conflicting interests are interacting.

game theory has been successfully used to analysis incentives and explain some phenomena in the field of security.

A practical application of game-theoretic analysis may be to reveal the potential effects of changing the rules of the game. This has been illustrated with three versions of the quality choice game, with the analysis resulting in three different predictions for how the game might be played by rational players. Changing the original quality choice game the automation of strategic choices enhances the need for these choices to be made efficiently, and to be robust against abuse. Game theory addresses these requirements. As a mathematical tool for the decision-maker the strength of game theory is the methodology it provides for structuring and analyzing problems of strategic choice. The process of formally modeling a situation as a game requires the decision-maker to enumerate explicitly the players and their strategic options, and to consider their preferences and reactions. The discipline involved in constructing such a model already has the potential of providing the decision-maker with a clearer and broader view of the situation. This is a prescriptive application of game theory, with the goal of improved strategic decision making. With this perspective in mind, this article explains basic principles of game theory, as an introduction to an interested reader without a background in economics. The object of study in game theory is the game, which is a formal model of an interactive situation. It typically involves several players; a game with only one player is usually called a decision problem. The formal definition lays out the players, their preferences, their information, the strategic actions available to them, and how these influence the outcome. Generally game theory is a tool used to analyze strategic behavior by taking into account how participants expect others to behave. It is used to find the optimal outcome from a set of choices by analyzing the costs and benefits to each independent party.

In general, all the competitive situation with the following characteristic game theory

1. There are finite number of competitors in the players list
2. All the players know the rules of the game
3. All the player plays the strategy simultaneously so that no one knew the opponents strategy until it is being played
4. The final outcome of the actions and reactions of all of the participants ends with a no loss (zero sum game). definite outcome may be positive, negative

# Summary

you can see so many things for example, pure strategies with saddle point, application of game theory, definition of game theory and summarize those things.

Game Theory: A branch of mathematics that provides a systematic way to attack problems of decision making when some alternatives are unclear or ambiguous. The general problem of how to make decisions in a competitive environment is a very common and important one. The fundamental contribution of game theory is that it provides a basic conceptual framework for formulating and analyzing such problems in simple situations. However, there is a considerable gap between what the theory can handle and the complexity of most competitive situations arising in practice. Therefore, the conceptual tools of game theory usually play just a supplementary role in dealing with these situations. Because of the importance of the general problem, research is continuing with some success to extend the theory to more complex situations. Game theory is a theoretical framework for conceiving social situations among competing players. It is the branch of mathematics concerned with competitive situations where the outcome of a participant's choice of action depends critically on the actions of other participants. Game theory is the study of situations of conflict and cooperation. We define a game as a situation where

(i) there are at least two players; (ii) each player has a number of possible strategies that determine the outcome;

(iii) associated with each outcome is a set of numerical payoffs to each player.

Of course, games can encompass a huge variety of situations, ranging from traditional games like chess or poker, to social, psychological, biological, or economic games. With only two lectures we are forced to give only a taste of some of the fascinating topics in the mathematical theory of games. Our topics are roughly divided into three topics:

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