College Of Natural And Computational Science
Department Of Mathematics

Title:System Of Linear Equation By LU Decomposition And Jacobi's Iteration Method

Prepared By: Lomitu Tadesse
Advisor: Degsaw Demmisse

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Contents

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The undersigned here by certify that they have read and recommend to the Department of Mathematics for acceptance of a project entitled **System Of Linear Equation by *LU* Decomposition and Jacobi,s Iteration** by Lomitu Tadesse in partial fulfillment of the requirements for the degree of Bachelor of Science.

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Advisor: ______________________

Advisor name

Examining committee: ______________________

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Abstract

This Project deals about System of linear equation. It contains three Chapter's, the first chapter is introduction. The second chapter is about system of linear equations. It has two section; Direct and Indirect method.Direct method has one subsection.that is LU Decomposition. Indirect method has also one subsection.that is Jacobi's Iteration method. The third and last chapter deals summary.

Notations

$$A = LU$$

A............................cofficient matrix
L............................lower triangular matrix
U............................upper triangular matrix

$$AX = B$$

A............................matrix
X..............................variable
B..............................constant

Chapter One

1 Introduction

In this chapter we present the solution of m linear simultaneous Algebraic Equations in n unknowns. Systems of linear equation are associated with many problems in engineering and science,as well as with applications of mathematics to the social sciences and quantitative study of business and economic problems.

A system of algebraic equations has the form

$a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n = b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \dots a_{mn}x_n = b_m$

where the coefficients a_{ij} and the constants b_j are known x_j Represents the unknowns. In the matrix notation the equations are written as

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{1}$$

or simply $Ax = b$

A system of linear equation in n unknowns has a unique solution,provided that the determinant of the coefficient matrix is non singular i.e,if $|A| \neq 0$. The rows and columns of a non singular matrix are linearly Independent in the sense that no row(column) is a linear combination of the other rows(columns). If the coefficient matrix is singular, the equations may have Infinite number of solutions, or no solution at all, depending on the constant vector.

Linear Algebraic equations occur in almost all branches of engineering.their most important application in engineering is in the analysis of linear system(any system whose response is proportional to the input is deemed to be linear).

Linear Systems include structures,elastic solids, heat flow,seepage of fluids,electromagnetic fields and electric circuits i,e.,most topic taught in an engineering curriculum.If the system is descrete, such as a truss or an electric circuit, then its analysis lead directly to linear algebraic equations.

Summarysing the modeling of linear systems Invariably gives rise to equations of the form $Ax = b$, where b is the input and x represents the response of the system.The coefficient matrix A, which reflects the characteristics of the system, is independent of the input. In other words, if the input is changed,the equations have to be solved again with a different b, but the same A.Hence,it is desirable to have an equation solvind algorithm that can handle any number of constant vectors with minimal computational effort.

1.1 Statement Of the Problem

A solution to a system of linear equation is a set of numbers that,when we substitute numbers for specified variables in the system.A system of equation of two algebraic expression which involves one or more literals(variables) is called an equation. The set of values of variable which makes the open sentence true is called the solution set.

1.2 Objective of the project

Objective of the project of system of linear equation is two.General objective and Specific objective.

1.2.1 General objective

The main objective of this project is to understand the linear equation by LU Decomposition and Jacobi's Iteration method.

1.2.2 specific objective

The specific objective of this project are:

- define LU decomposition and jacobi's iteration method
- explain LU decomposition and jacobi's iteration method

Chapter Two

2 System Of Linear Equation

Important field of application for numerical method is numerical linear algebra that deals with solving problems of linear algebra numerically(matrix vector product, finding eigen values, solving system of linear equation).

A system of m linear equations in n unknowns is a set of m equations,numbered from 1 to n going down, each in variables x_i which are multiplied by coefficients $a_{ij} \in F$,whose sum equals some $b_j \in R$:
$a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n = b_1$
$a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n = b_2$
$\vdots$
$a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n = b_m$
Where the coefficients a_i and the constants b_j are known and x_i represents the unknowns. In matrix notation,the equations are written as.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ \vdots & & \\ a_{m1} & a_{m2}\ldots & a_{mn} \end{bmatrix} X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{2}$$

If we condense this to matrix notation by writing $x = (x_1, ..., x_n), b = (b1, ..., bm)$ and $A \in R^{m \times n}$ the coefficient matrix of the system, the matrix whose elements are the coefficients a_{ij} of the variables in (S), then we can write (S) as

$m \times n$

2.1 method of solution

There are two classes of method for solving system of linear equations direct method and iterative or indirect methods. the common characteristics of direct method in this project we will present one direct method and one indirect(iterative) method.

2.2 Direct method

Direct method are that they transform the original equation into equivalent equations(equations that have the same solution) that can be solved more easily. the transformation is carried out by applying certain operations there are one direct method LU decomposition method.

2.2.1 LU Decomposition

It is possible to show that any square matrix A can be expressed as a product of a lower triangular matrix L and an upper triangulr matrix u.

$$A{=}LU$$

For instance

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{L32} & L33 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} \tag{3}$$

The process of computing L and U for a given A is known as LU Decomposition or LU Factorization.

LU Decomposition is not unique (the combination of L and U for a prescribed A are endless),unless certain constraints are placed on L or U. These constraints distinguish one type of decomposition from another.

Two commonly used decomposition are given below:
1. Cholesky's decomposition:Constraints are $L{=}U^T$
2.Crout's decomposition:Constrains are $U_{ii}{=}1,i{=}1,2,...,n$.

After decomposing the matrix A,it is easier to solve the equations A_x=b. We can rewrite the equation as

$$LU_x{=}b$$

or denoting U_x=y, the above equation becomes

$$L_y{=}b$$

This equation L_y=b can be solved for y by forward substitution. Then U_x=y will yield x by the backward substitution process. The advantage of LU decomposition method over the Gauss elimination method is that once A is decomposed,we can solve $AX = b$ for as many constant vectors b as we please.

Also, the forward and backward substitutions operations are much less time consuming than the decomposition process.

2.2.2 Cholesky's Triangularisation Method

Cholesky's decomposition method is faster than the LU decomposition. There is no need for pivoting. If the decomposition fails, the matrix is not positive definite.
consider the system of linear equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{aligned}$$

the above system can be written as

$$Ax = b$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{4}$$

Let A=LU...

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \textit{and} \;\; U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \tag{5}$$

The system can be written as

$$LUX = b$$

If we write

$$UX = V$$

Equation $LUX = b$ becomes

$$LV = b$$

Equation $LV = b$ is equivalent to the system

$$\begin{aligned} v_1 &= b_1 \\ l_{21}v_1 + v_2 &= b_2 \\ l_{31}v_1 + l_{32}v_2 + v_3 &= b_3 \end{aligned}$$

The above system can be solved to find the values of v_1, v_2 and v_3 which give us the matrix V.

$$UX = V$$

then becomes

$$u_{11}x_1 + u_{12}x_2 + u_{13}x_3 = v_1$$
$$u_{22}x_2 + u_{23}x_3 = v_2$$
$$u_{33}x_3 = v_3$$

which can be solved for x_3, x_2 and x_1 by the backward substitution process. In order to compute the matrices L and U, we write as

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (6)$$

Multiplying the matrices on the left equating the corresponding elements of both sides, we obtain

$$u_{11} = a_{11}, u_{12} = a_{12}, u_{13} = a_{13}$$

$$\left.\begin{array}{l} l_{21}u_{11} = a_{21} \Longrightarrow l_{21} = \frac{a_{21}}{a_{11}} \\ l_{31}u_{11} = a_{31} \Longrightarrow l_{31} = \frac{a31}{a_{11}} \end{array}\right\}$$

$$\left.\begin{array}{l} l_{21}u_{12} + a_{22} \Longrightarrow u_{22} = a_{22} - \frac{a_{21}}{a_{11}}a_{12} \\ l_{21}u_{13} + a_{23} \Longrightarrow u_{23} = a_{23} - \frac{a21}{a_{11}}a_{13} \end{array}\right\}$$

$$l_{31}u_{12} + l_{32}u_{22} = a_{32} \Longrightarrow l_{32} = \frac{1}{u_{22}}\left[a_{32} - \frac{a_{31}}{a_{11}}a_{12}\right]$$

and

$$l_{31}u_{13} + l_{32}u_{23} + u_{33} = a_{33}$$

The value of u_{33} can be computed from above equation.

To obtain the elements of L and U,we first find the first row of U and the first column of L.Then,we determine the second row of U and the second column of L. Finally,we compute the third row of U.

Cholesky's triangularisation method is also known as Crout's triangularisation method or method of factorisation.

Example.1

Solve the system of linear equations using Cholesky's factorisation method.

$$\begin{aligned} 2x - 6y + 8z &= 24 \\ 5x + 4y - 3z &= 2 \\ 3x + y + 2z &= 16 \end{aligned}$$

solution:

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix} = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix} \tag{8}$$

$u_{11} = 2, \qquad u_{12} = -6, \qquad u_{13} = 8$

$l_{21} = \frac{5}{u_{11}} = 2.5$
$l_{31} = \frac{3}{u_{11}} = 2.5$

$u_{22} = 4 - l_{21}u_{12} = 19$
$u_{23} = -3 - l_{21}u_{13} = -23$

$l_{32} = \frac{1 - l31u_{12}}{u_{22}} = \frac{10}{19}$
$l_{33} = 2 - l_{32}u_{13} - l_{32}u_{23} = \frac{40}{19}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2.5 & 1 & 0 \\ 1.5 & \frac{10}{19} & 1 \end{bmatrix}, \; U = \begin{bmatrix} 2 & -6 & 8 \\ 0 & 19 & -23 \\ 0 & 0 & \frac{40}{19} \end{bmatrix} \tag{9}$$

$$LV = B \Longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2.5 & 1 & 0 \\ 1.5 & \frac{10}{19} & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix} \tag{10}$$

$v_1 = 24$
$v_2 = 2 - 2.5 \times 24 = -58$
$v_3 = 16 - 1.5 \times 24 - \frac{10}{19}(-58) = \frac{200}{19}$

$$UX = V \Longrightarrow \begin{bmatrix} 2 & -6 & 8 \\ 0 & 19 & -23 \\ 0 & 0 & \frac{40}{19} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ -58 \\ \frac{200}{19} \end{bmatrix} \tag{11}$$

$2x - 6y + 8z = 24$
$19y - 23z = -58$
$\frac{40}{19}z = \frac{200}{19} \Longrightarrow z = 5$
$y = 3$
$x = 1$

2.2.3 Crout's Method

This method is based on the fact that every square matrix A can be expressed as the product of a lower triangular matrix and an upper triangular matrix,provided all the principle minors of A are non-singular. Also,such a factorisation,if exists,is unique.

This method is also called $triangularisation\ or\ factorisation\ method$.Here,we factorise the given matrix as $A = LU$, where L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. Then,

$A^{-1} = (LU)^{-1} = U^{-1}L^{-1}$

Consider the system

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

The above system can be written as

Let

$$\begin{aligned} A_x &= b \\ A &= LU \end{aligned}$$

where

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} and \;\; U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Here,L is a lower triangular matrix and U is an upper triangular matrix with diagonal elements equal to unity.

$A = LU \Longrightarrow A^{-1} = U^{-1}L^{-1}$

Now

The process of computing L and U for a given A is known as LU Decomposition or LU Factorization.

LU Decomposition is not unique (the combination of L and U for a prescribed A are endless),unless certain constraints are placed on L or U. These constraints distinguish one type of decomposition from another.

$$A = LU \Longrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{L32} & L33 \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

or

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{21}u_{12} + l_{22} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix} \tag{14}$$

Equation the corresponding elements, we obtain

$$\begin{aligned} l_{11} &= a_{11} \qquad l_{21} = a_{21} \qquad l_{31} = a_{31} \\ l_{11}u_{12} &= a_{12} \qquad l_{11}u_{13} = a_{13} \\ l_{21}u_{12} + l_{22} &= a_{22} \qquad l_{31}u_{12} + l_{32} = a_{32} \\ l_{31}u_{13} + l_{32}u_{23} + l_{33} &= a_{33} \end{aligned} \tag{15}$$

from equation (15) we find
$u_{12} = a_{12}/l_{11} = a_{12}/a_{11}$
$l_{22} = a_{22} - l_{21}u_{12}$
$l_{32} = a_{32} - l_{31}u_{12}$
$u_{13} = (a_{23} - l_{21}u_{23}/l_{22})$
$l_{33} = a_{33} - l_{31}u_{13} - l_{32}u_{23}$

Thus, we have determined all the elements of L and u.
From equation(12) we have

$$LU_x = b$$

Let

$$UX = V$$

where

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \tag{16}$$

From equation $LU_x = b$ we have $LV = b$,which on forward substitution yields V.
From $UX = V$,we find x (by backward substitution).

Example.2
Solve the following set of equations by Crout's method:

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

solution:

Where

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, = B \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix} \tag{17}$$

$AX = B$
Let

$A = LU$

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \qquad U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{L32} & L33 \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix} \tag{20}$$

$$l_{11}u_{12} = 1 \Longrightarrow u_{12} = \tfrac{1}{2}$$
$$l_{11}u_{12} = 4 \Longrightarrow u_{13} = \tfrac{4}{2} = 2$$
$$l_{22} + l_{21}u_{12} = -3 \Longrightarrow l_{22} = -3 - 8(\tfrac{1}{2}) = -7$$
$$l_{32} + l_{21}u_{12} = -3 \Longrightarrow l_{32} = 11 - 4(\tfrac{1}{2}) = 9$$
$$l_{21}u_{13} + l_{22}u_{23} = 2 \Longrightarrow u_{23} = \tfrac{2-8\times2}{-7} = 2$$
$$l_{31}u_{13} + l_{32}u_{23} = -1 \Longrightarrow L_{33} = -1 - 4\times2 - 9\times2 = -27$$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} and \;\; U = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \tag{21}$$

$LV = B$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix} \tag{22}$$

$2v_1 = 12 \Longrightarrow v_1 = 6$
$8v_1 - 7v_2 = 20 \Longrightarrow v_2 = \frac{-20+8\times6}{7} = 4$
$4v_1 + 9v_2 - 27v_3 = 33 \Longrightarrow v_3 = \frac{-33+4\times6+9\times4}{27} = 1$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} ; U_X = V \tag{23}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} \tag{24}$$

$$x + \frac{1}{2}y = z$$
$$y + 2z = 4$$
$$\Longrightarrow z = 1$$
$$y = 4 - 2 \times 1$$
$$\Longrightarrow y = 2$$
$$x = 6 - \frac{1}{2} \times 2 - 2 \times 1$$
$$\Longrightarrow x = 3$$

2.3 Indirect method or Iteration method

Indirect method means the indirect solution technique(iterative) is more useful it solve a set of ill-conditioned equation start with a guess of the solution x and then repeatedly refine the solution until a certain convergence criterion is reached iterative methods are generally less efficient the direct method due to the large number of operation or iteration required.

˜ There are two types of indirect method;

1.Jacobi's methods

2.Gauss-siedal iteration method

˜ In this section I will present only Jacobi's method.

2.3.1 jacobi's iteration method

Jacobi's method a matrix said to be diagonally dominant if the absolute value of each main diagonal element is greater than the sum of the absolute value of each element in the same row. these are methods which compute a sequence of progressively accurate iterats to approximate the solution of $AX = B$. We need such methods for solving many large linear systems sometime the matrix is to large to be stored in the computer memory making a direct method too difficult to use. this method is also known as the method of simultaneous displacemets consider the system of linear equation.

$$a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{31}x_1 + a_{32}x_2 + \dots a_{nn}x_n = b_n$$

we assume that the coefficients
a_{11}, a_{22} and a_{33} are the largest cofficients in the respective equations so that.

$$|a_{11}| > |a_{12}| + |a_{13}|$$
$$|a_{22}| > |a_{21}| + |a_{23}|$$
$$|a_{33}| > |a_{31}| + |a_{32}|$$

Jacobis iteration method is applicable only if the conditions given above are satisfied. Now, we can write

$$x_1 = \tfrac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3)$$
$$x_2 = \tfrac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}x_3)$$
$$x_3 = \tfrac{1}{a_{33}}(b_3 - a_{31}x_1 - a_{32}x_2)$$

Let the initial approximations be
x_1^0 x_2^0 and x_3^0 respectively The following iterations are then carried out
Iteration 1:
The first improvements are found as

$$x_1^1 = \tfrac{1}{a_{11}}(b_1 - a_{12}x_2^0 - a_{13}x_3^0)$$
$$x_2^1 = \tfrac{1}{a_{22}}(b_2 - a_{21}x_1^0 - a_{23}x_3^0)$$
$$x_3^1 = \tfrac{1}{a_{33}}(b_3 - a_{31}x_1^0 - a_{32}x_2^0)$$

Iteration 2:

$$x_1^2 = \tfrac{1}{a_{11}}(b_1 - a_{12}x_2^1 - a_{13}x_3^1)$$
$$x_2^2 = \tfrac{1}{a_{22}}(b_2 - a_{21}x_1^1 - a_{23}x_3^1)$$
$$x_3^2 = \tfrac{1}{a_{33}}(b_3 - a_{31}x_1^1 - a_{32}x_2^1)$$

$(n+1)^{th} iteration\ n:$

$$\begin{aligned} x_1^n + 1 &= \tfrac{1}{a_{11}}(b_1 - a_{12}x_2^n - a_{13}x_3^n) \\ x_2^n + 1 &= \tfrac{1}{a_{22}}(b_2 - a_{21}x_1^n - a_{23}x_3^n) \\ x_3^n + 1 &= \tfrac{1}{a_{33}}(b_3 - a_{31}x_1^n - a_{32}x_2^n) \end{aligned}$$

the above iteration process is continued until the values of x_1, x_2 and x_3 are found to a pre-assigned degree of accuracy. That is, the procedure is continued until the relative error between two consecutive vector norm is satisfactorily small. In Jacobis method, it is a general practice to assume $x_1^0 = x_2^0 = x_3^0$ The method can be extended to a system of n linear simultaneous equations in n unknowns.

Example.3

solve the system by using jacobi's iteration method.
$20x_1 + x_2 - 2x_3 = 17$
$3x_1 + 20x_2 - x_3 = -18$
$2x_1 - 3x_2 + 20x_3 = 25$

solution

$$\begin{aligned} |a_{11}| &> |a_{12}| + |a_{13}| \\ |20| &> |1| + |2| = 3 \\ |a_{22}| &> |a_{21}| + |a_{23}| \\ |20| &> |3| + |1| = 4 \\ |a_{33}| &> |a_{31}| + |a_{32}| \\ |20| &> |2| + |3| = 5 \end{aligned}$$

There fore diagonal dominant.
assume
$x_1^0 = 0$
$x_2^0 = 0$

$x_3^0 = 0$

1^{st} iteration:

$$x_1^1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^0 - a_{13}x_3^0)$$
$$x_1^1 = \frac{1}{20}(17 - 0 - 0)$$
$$x_1^1 = 0.85$$
$$x_2^1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^0 - a_{23}x_3^0)$$
$$x_2^1 = \frac{-18}{20}$$
$$x_2^1 = -0.9$$
$$x_3^1 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^0 - a_{32}x_2^0)$$
$$x_3^1 = \frac{25}{20}$$
$$x_3^1 = 1.25$$

2^{nd} iteration:

$$x_2^2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^1 - a_{23}x_3^1)$$
$$x_2^2 = \frac{1}{20}(-18 - 3)(0.85) + 1.25$$
$$x_2^2 = -0.965$$
$$x_3^1 = \frac{1}{a_{33}}(b_3 - a_{31}x_1^1 - a_{32}x_2^1)$$
$$x_3^1 = \frac{1}{20}(25 - (2)(0.85) - (3)(1.25))$$
$$x_3^2 = 1.03$$

3^{rd} iteration:

$$x_1^3 = \frac{1}{20}((17 + 0.965) + 2(1.03))$$
$$x_1^3 = 1.00125$$
$$x_2^3 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^1 - a_{23}x_3^1)$$
$$x_2^3 = \frac{1}{20}(-18 - 3(1.02) + 1.00125)$$
$$x_2^3 = -1.0015$$
$$x_3^3 = \frac{1}{20}(25 - 2(1.02) - 3(0.965))$$
$$x_3^3 = 1.00325$$

4^{th} iteration:

$$x_1^4 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^3 - a_{13}x_3^3)$$
$$x_1^4 = \frac{1}{20}(17 + 1.0015 - 2(1.00325))$$
$$x_1^4 = 1.004$$
$$x_2^4 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^3 - a_{23}x_3^3)$$
$$x_2^4 = \frac{1}{20}(-18 - 3(1.00125) + 1.00325)$$
$$x_3^4 = 0.99965$$

5^{th} iteration:

$$x_1^5 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^4 - a_{13}x_3^4)$$
$$x_1^5 = \frac{1}{20}(17 + 1.000025 + 2(0.99965))$$
$$x_1^5 = 0.99996$$
$$Therefore$$
$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = 1$$

chapter Three

3 summary

A system of linear equations consists of two or more equations up of two variables such that all equation in the system are considered simultaneously.
The solution to a system of linear equation in two variables is any ordered pair that satisfies each equation independently.
System of equation are classified as independent with one solution dependent with an infinite number of solution, or inconsistent with no solution.
One method of solving a system of linear equation in two variables is by graphing,in this method, we graph the equation on the sames set of axes.
Another method of solving a system of linear equation is by substitute on. In this method we solve for one variable in one equation and substitute the result in to the second equation.
A third method of solving asystem of linear equation is by addition in which we can eliminate a variable by adding opposite cofficients of corresponding variables.
This project presented an overview of the method of system equation specifically the LU decomposition and Jacob's iteration method these projects provided the relevant and use full elements of matrix analysis for the solution of linear simultaneous algebraic equation.
The solution of n linear simultaneous algebraic equation in n unknowns is presented.
There are two classes of method of solving system of linear equation direct and iterative or indirect method.
Direct method; transform the original equation into equivalent equations that can be solved more easily. one direct method(LU decomposition) are presented.
Iterative or indirect method; start with a guess of the solution X and then repeatedly refine the solution on until a certain convergence criterion is reached. one indirect method (Jacobi's iterative method) are presented.
this method given comparable results and is easy to compute further more it-

erative method is more applicable and better convergent if the size of matrix is very large or more variable.

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