

WOLKITE UNIVERSITY

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Binomial Theorem And Its Application

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The undersigned hereby certify that they have read and recommend to the Department of Mathematics for acceptance of a project entitled **Binomial theorem and it's application** By Student **Cherinet Birhanu** in partial fulfillment of the requirements for the degree of Bachelor of Science.

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Abstract

This project deals all about binomial theorem and its applications. It contains three chapters. The first chapter is deals about Introduction of binomial theorem. The second chapter is deals about theorems on binomial theorem, in general the two chapters contains subtopics ,and they refers about binomial theorem. The third chapter is about applications of binomial theorem.

Notations

$\binom{n}{r}$ *or* $C(n,r)$ The combinaton of n choose r (n combination r)

$\emptyset$ Empty set

$\Rightarrow$ Implication

$\sum$ The Summation

$\begin{bmatrix} n \\ r \end{bmatrix}$ The binomial coefficient of n choose r.

Objectives

General Objective

The main objective of this project is to understand the binomial theorem.

specific Objective

- To proof the binomial theorem.
- To solve some problem by using binomial expansion.
- To solve factorial of a positive integer.
- To solve the binomial theorem by using combination.
- To solve some applications of binomial theorem.

Chapter 1

Introduction

1.1 Historical Back Ground Of The Binomial Theorem

Special case of the binomial theorem were known since at least the 4th century BC when Greek mathematician Euclid mentioned the special case of the binomial theorem for exponent 2.The binomial theorem for cubes was known by the 6th century in India. The binomial expansion of small degree's were known in 13th century mathematical works of Yang Hui and also $Chushih_C hieh$. The first formulation of the binomial theorem and the table of binomial coefficients, to our knowledge, can be found in a work by $Al_K araji$ in (953 A.D$\cdots$1029A.D); $Al_K araji$ described the triangular pattern of the binomial coefficients and also provided a mathematical proof of both the binomial theorem and $Paskal's$ triangle, using an early form of mathematical induction. The Persian Poet and mathematician Omar Khayyam was probably familiar with the formula to higher orders in ($1048_1 113$). Isaac Newton is generally credited with the generalized binomial theorem, valid for any rational exponent in ($1642_1 727$).

1.2 Basic Definitions On Binomial Theorem

The binomial theorem is the method of expanding an expression which has been raised to any finite power. A binomial theorem is a powerful tool of expansion, which has application in Algebra, Probability, etc. An algebraic containing two terms is called a **binomial expression**, **Bi** means two and **Nom** means term. Thus the general type of a binomial is a+b, x-2, 3x+4 etc.The expression of a binomial raised to a small positive power can be solved by ordinary multiplication, but for large power the actual multiplication is laborious and for fractional power actual multiplication is not possible. Binomial theorem:statement that for any positive integer n, the nth power of the sum of two numbers a and b may be expressed as the sum of $n+1$ terms of the form $\begin{pmatrix} n \\ r \end{pmatrix} a^{n-r} b^r$; in the sequence of terms the index r takes on the successive values 0,1,2,...,n. The coefficients,Called **the binomial coefficients**,are defined by the formula $\begin{pmatrix} n \\ r \end{pmatrix}$ = n!/(n-r)!r!, in which n!(called n factorial) is the product of the first n natural numbers 1,2,3,...,n (and where 0! is defined as equal to 1). The coefficients may also be found in the array often called **pascal's triangle**. If we wanted to expand a binomial expression with large power, **eg**, $(1+x)^{32}$,use of pascal's triangle would not be recommended because of the need to generate a large number of rows of the triangle. An alternative method is to use the binomial theorem. The theorem enables us to expand $(a+b)^n$ in increasing powers of b and decreasing powers of a. We will look at expanding expressions of the form $(a+b)^2$,$(a+b)^3$, ...,$(a+b)^{32}$,..., that is when the power is a positive integer. Under certain conditions the theorem can be used when n

is negative or fractional and this is useful in more advanced applications, these conditions will not be studied here.

Definition The binomial theorem gives the coefficients of the expansion of powers of binomial expressions. A binomial expression is simply the sum of two terms,such as x +y.(The terms can be products of constants and variables, but that does not concern us here.) Example 1.1 illustrates how the coefficients in a typical expansion can be found and prepares us for the statement of the binomial theorem.

Example 1.1 The expansion of $(x+y)^3$ can be found using *Combinatorial* reasoning instead of multiplying the three terms out. When $(x+y)^3 =$ (x +y)(x +y)(x +y) is expanded, all products of a term in the first sum, a term in the second sum, and a term in the third sum are added. Terms of the form x^3, x^2y,xy^2, andy^3 arise. To obtain a term of the form x^3, and x must be chosen in each of the sums, and this can be done in only one way. Thus, the x^3 term in the product has a coefficient of 1. To obtain a term of the form x^2y, and x must be chosen in two of the three sums (and consequently a y in the other sum). Hence, the number of such terms is the number of 2-combinations of three objects, namely, $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Similarly, the number of terms of the form xy^2 is the number of ways to pick one of the three sums to obtain an x (and consequently take a y from each of the other two sums). This can be done in $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ ways. Finally, the only way to obtain a y^3 term is to choose the y for each of the three sums in the product, and this can be done in exactly one way. Consequently, it follows that

$(x + y)^3$ = (x +y)(x +y)(x +y)= (xx +xy+yx+yy)(x +y) = $xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy = x^3 + 3x^2y + 3xy^2 + y^3$.

Factorial of a positive integer

If n is a positive integer, then the factorial of 'n' denoted by n! or n and is defined as the product of n +ve integers from n to 1 (or 1 to n) i.e., n! = n(n – 1)(n – 2) ….. 3.2.1 For example, 4! = 4.3.2.1 = 24 and 6! = 6.5.4.3.2.1 = 720 one important relationship concerning factorials is that (n + 1)! = (n + 1) n! (1) for instance, 5! = 5.4.3.2.1 = 5(4.3.2.1) 5! = 5.4! Obviously, 1! = 1 and this permits to define from equation (1) (n+1)! n!= n+1 Substitute 0 for n, we obtain (0+1) ! 1 ! 1 0!= 0+1 1 1 0! = 1

Combination

Definition. Let n, r $\in N$ such that n $\geq$ r $\geq$ 0. An unordered selection of r elements from an n element set is called a **combination.**

Example1.2 List all the combinations of the set {a, b, c}.

Solution. The combinations will be of sizes 0, 1, 2, and 3. All combinations are $\emptyset$, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, and {a, b, c}.

THEOREM 1.1: The number of r-combinations of a set with n elements, where n is a non negative integer and r is an integer with 0 $\leq$ r $\leq$ n, equals

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

Proof:

The P(n,r) r-permutations of the set can be obtained by forming the C(n,r) r-combinations of the set, and then ordering the elements in each r-combination, which can be done in P(r, r)ways. Consequently, by the product rule, P(n,r) = C(n,r)·P(r,r).

This implies that

$$\text{C(n,r)} = \frac{p(n,r)}{p(r,r)} = \frac{n!/(n-r)!}{r!/(r-r)!} = \frac{n!}{r!(n-r)!}$$

We can also use the division rule for counting to construct a proof of this theorem. Because the order of elements in a combination does not matter and there are P(r,r) ways to order r elements in an r-combination of n elements, each of the C(n,r) r-combinations of a set with n elements corresponds to exactly P(r,r) r-permutations. Hence, by the division rule, C(n,r) $= \frac{p(n,r)}{p(r,r)}$, which implies as before that C(n,r) $= \frac{n!}{r!(n-r)!}$.
The formula in Theorem 1, although explicit, is not helpful when C(n,r) is computed for large values of n and r. The reasons are that it is practical to compute exact values of factorials exactly only for small integer values, and when floating point arithmetic is used, the formula in Theorem 2 may produce a value that is not an integer. When computing C(n,r), first note that when we cancel out (n-r)!from the numerator and denominator of the expression for C(n,r) in Theorem 1, we obtain C(n,r) $= \frac{n!}{r!(n-r)!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$
Consequently, to compute C(n,r) you can cancel out all the terms in the larger factorial in the denominator from the numerator and denominator, then multiply all the terms that do not cancel in the numerator and finally divide by the smaller factorial in the denominator. [When doing this calculation by hand, instead of by machine, it is also worthwhile to factor out common factors in the numerator n(n-1)$\cdots$(n-r+1) and in the denominator r!.] Note that many calculators have a built in function for C(n,r) that can be used for relatively small values of n and r and many computational programs can be used to find C(n,r). [Such functions may

be called choose (n,k) or *binom*(n,k)].

COROLLARY 2 Let n and r be non negative integers with r $\leq$ n. Then C(n,r) = C(n,n-r).

Proof:

From Theorem 1.1 it follows that

$$\text{C(n,r)} = \frac{n!}{r!(n-r)!}$$

and

$$\text{C(n,n-r)} = \frac{n!}{(n-r)![n-(n-r)]!} = \frac{n!}{(n-r)!r!}$$

Hence, C(n,r) = C(n,n-r).

General Term

The term $\begin{bmatrix} n \\ r \end{bmatrix} a^{n-r}b^r$ in expansion of binomial theorem is called the general term or $(r+1)^{th}$ term. It is denoted by T_{r+1} . Hence $T_{r+1} = \begin{bmatrix} n \\ r \end{bmatrix} a^{n-r}b^r$

- $(x+y)^n = nC_0x^ny^0 + nC_1x^{n-1}y^1 + \cdots + nC_rx^{n-r}y^r + \cdots + nC_nx^0y^n$
- $T_{r+1} = nC_rx^{n-r}y^r$

Note: The general term is used to find out the specified term or the required coefficient of the term in the binomial expansion.

Example1.3: Expand $(x+y)^4$ by binomial theorem

Solution:

$$(x+y)^4 = x^4 + \begin{pmatrix} 4 \\ 1 \end{pmatrix} x^{4-1}y + + \begin{pmatrix} 4 \\ 2 \end{pmatrix} x^{4-2}y^2 + \begin{pmatrix} 4 \\ 3 \end{pmatrix} x^{4-3}y^3 + y^4$$

$$= x^4 + 4x^3y + \frac{4*3}{2*1}x^2y^2 + \frac{4*3*2}{3*2*1}xy^3 + y^4$$

$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Middle Term In The Expansion $(a+b)^n$

In the expansion of $(a+b)^n$ there are (n+1) terms.

Case1: If n is even then (n+1) will be odd so $[\frac{n}{2}+1]^{th}$ term will be the middle term in the expansion.

Example1.4: If n=8(even), number of the terms will be 9(odd), therefore,$[\frac{8}{2}+1] = 5^{th}$ will be middle term.

Case2: If n is odd then (n+1) will be even, in this case there will not be a single middle term, but $[\frac{n+1}{2}]^{th}$ and $[\frac{n+1}{2}+1]^{th}$ terms is 10

i.e, $[\frac{9+1}{2}]^{th}$ and $[\frac{9+1}{2}+1]^{th}$

i.e, 5^{th} and 6^{th} terms are taken as middle terms and these middle terms are found by using the formula for the general term

Example1.5: Find the middle term of $[1-\frac{x^2}{2}]^{14}$.

Solution: We have n = 14, then number of terms is 15

$\therefore [\frac{14}{2}+1]$ **i.e** 8^{th} will be middle term.

a=1, b= $[\frac{-x^2}{2}]$, n=14, r=7, T_8 = ?

$$T_{r+1} = \begin{bmatrix} n \\ r \end{bmatrix} a^{n-r}b^r$$

$$T_{7+1} = \begin{bmatrix} 14 \\ 7 \end{bmatrix} [1]^{14-7} \, [\frac{-x^2}{2}]^7 = \frac{14!}{7!7!}(-1)^7 \, \frac{x^{14}}{2^7}$$

$$T_8 = \frac{14 * 13 * 12 * 11 * 10 * 9 * 8 * 7!}{7 * 6 * 5 * 4 * 3 * 2 * 1 * 7!} \frac{(-1)}{128} x^{14}$$
$$T_8 = -(2)(13)(11)(2)(3) \frac{(1)}{128} x^{14}$$
$$T_8 = \frac{-429}{16} x^{14}$$

Chapter 2

Theorems

We now prove a theorem that shows how to find all coefficients for the expansion of a binomial.

Theorem 2.1. (Binomial Theorem) Let n$\in$ N. For all x and y,

$(x + y)^n = c(n, 0)x^n + c(n, 1)x^{n-1}$y $+ c(n, 2)x^{n-2}y^2 + \ldots + c(n, n)y^n = \sum_{i=0}^{n} c(n, i)x^{n-i}y^i$

Proof. The proof is by induction on n. Let no = 0 and T = {n$\in$ N the identity is true).

(Base step) For n = 0, the formula is

$(x + y)^0 = \sum_{i=0}^{0} c(0, i)x^{o-i}y^i$

$= c(0, 0)x^0y^0$

$= 1$

which is true.

(Inductive step) Choose n$\geq$ no, and assume n$\in$ T. Now, prove that n + 1$\in$ T. Begin by setting

$(x + y)^{n+1} = (x + y)(x + y)^n$.

By the inductive assumption, this is

$(x + y)^{n+1} =$ (x+y).$\sum_{i=0}^{n} c(n, i)x^{n-i}y^i$

$=$ x$(\sum_{i=0}^{n} c(n, i)x^{n-i}y^i)$ + y$(\sum_{i=0}^{n} c(n, i)x^{n-i}y^i)$

$= c(n, 0)x^{n+1} + \sum_{i=1}^{n} c(n, i)x^{n-i+1}y^i + \sum_{i=0}^{n-i} x^{n-i}y^{i+1} + c(n, n)y^{n+1}$

Next, replace i by i-1 in the third term, giving

$\sum_{i=0}^{n-1} c(n,i)x^{n-i}y^{i+1} = \sum_{i=1}^{n} c(n,i-1)x^{n-i+1}y^{i}$

This makes the two sums start at the same value so that they can be added term by term to get

$\sum_{i=1}^{n}$(c(n,i)+c(n,i-1))$x^{n-i+1}y^{i}$

By Pascal's Identity, the sum becomes

$\sum_{i=1}^{n}$c(n+1,i) $x^{n-i+1}y^{i}$

Combining these results gives

$(x+y)^{n+1}$=y^{n+1} + $\sum_{i=1}^{n}$c(n+1,i)$x^{n-i+1}y^{i}$+x^{n+1}

=$\sum_{i=0}^{n+1}$c(n+1,i)$x^{n-i+1}y^{i}$

Therefore, n + 1 ∈ T.

By the Principle of Mathematical Induction, T = N. U Knowing the form of the coefficients of $(x+y)^n$ allows some interesting results to be proven.

Theorem 2.2: (Binomial Theorem)

For n ≥ 1,

$\sum_{i=0}^{n} c(n,i)(-1)^{i}$ =0

Proof:

The idea of the proof is to use the binomial theorem and expand $(x+y)^n$ with x = 1 and y=-1. By the binomial theorem with n ≥ 1, we have $(x+y)^{n+1} = \sum_{i=0}^{n+1} C(n,i)x^{n-i}y^{i}$. Let x=1 and y=-1. The identity then becomes

$(1+(-1))^{n+1} = \sum_{i=0}^{n+1} C(n,i)1^{n-i}(-1)^{i}$ $0 = \sum_{i=o}^{n+1} C(n,i)(-1)^{i}$

Corollary 1: For n even,

$\sum_{i=0}^{n/2}$C(n,2i) = $\sum_{i=0}^{\frac{n-1}{2}} C(n,2i+1)$

Corollary 2: For n odd

$\sum_{i=0}^{[n/2]}$C(n,2i) = $\sum_{i=0}^{[n/2]+1} C(n,2i+1)$

The corollaries to theorem 2 can be interpreted as saying that for any n-

element set there are as many sub sets with an even number of elements as there are sub sets with an odd number of elements. An obvious of the corollaries is that.

$\sum_{i=0}^{[n/2]}$C(n,2i) = $\sum_{i=0}^{[n/2]} C(n, 2i)$

Since it is also true that

$\sum_{i=0}^{n/2}$C(n,2i) + $\sum_{i=0}^{[n/2]} C(n, 2i) = 2^n$

it follows that $\sum_{i=0}^{n/2}$C(n,2i) = $\sum_{i=0}^{[n/2]} C(n, 2i) = 2^{n-1}$

A second technique for proving identities using binomial coefficients involves recognizing that the expression.

$(1 + x)^n = \sum_{i=0}^{n} C(n, i)xi$ can be viewed as a polynomial identity since this expression is an identity , its derivative will also be an identity, So

$n(x + 1)^{n-1} = \sum_{i=1}^{n} iC(n, i)x^{i-1}$

This fact is used in evaluating the next sum.

Example 2.1: Show that $\sum_{i=1}^{n} iC(n, i) = n2^{n-1}$

solution: Compute the derivative of $(x + 1)^n = \sum_{i=0}^{n} C(n, i)x^{n-i}$

getting

$n(x + 1)^{n-1} = \sum_{i=1}^{n} iC(n, i)x^{i-1}$

Now substituting x=1 gives

$n2^{n-1} = \sum_{i=1}^{n} iC(n, i)$

Theorem 2.3: (Binomial Theorem)

Let x and y be variables and let n be a non negative integer. Then,

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j}y^j = \binom{n}{0} x^n + \binom{n}{1} x^{n-1}y + \cdots + \binom{n}{n-1} xy^{n-1} + \binom{n}{n} y^n$$

Proof:

We use a *combinatorial* proof. The terms in the product when it is ex-

panded are of the form $x^{n-j}y^j$ for j=0,1,2,$\cdots$,n. To count the number of terms of the form $x^{n-j}y^j$, note that to obtain such a term it is necessary to choose $n-jxs$ from the n sums (so that the other j terms in the products are ys). There fore, the coefficient of $x^{n-j}y^j$ is $\begin{pmatrix} n \\ n-j \end{pmatrix}$, which is equal to $\begin{pmatrix} n \\ j \end{pmatrix}$.
This proves the theorem.

corollary3: Let n be a non negative integer. Then $\sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} = 2^n$.

proof:

using the binomial theorem with x=1 and y=1, we see that,

$2^n = (1+1)^n = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} 1^k 1^{n-k} = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix}$

This is the desired result.

corollary4: Let n be a positive integer.Then

$\sum_{k=0}^{n} (-1)^k \begin{pmatrix} n \\ k \end{pmatrix} = 0$

proof:

When we use the binomial theorem with x=-1 and y=1, we see that

$0 = 0^n = ((-1)+1)^n = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} (-1)^k 1^{n-k} = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} (-1)^k$

This proves the corollary.

Remark: Corollary 4 implies that

$\begin{pmatrix} n \\ 0 \end{pmatrix} + \begin{pmatrix} n \\ 2 \end{pmatrix} + \begin{pmatrix} n \\ 4 \end{pmatrix} + \cdots = \begin{pmatrix} n \\ 1 \end{pmatrix} + \begin{pmatrix} n \\ 3 \end{pmatrix} + \begin{pmatrix} n \\ 5 \end{pmatrix} + \cdots$

Corollary 5: Let n be a non negative integer. Then

$\sum_{k=0}^{n} 2^k \begin{pmatrix} n \\ k \end{pmatrix} = 3^n$

Proof:

We recognize that the left hand side of this formula is the expansion of $(1+2)^n$ provided by the binomial theorem. There fore, by the binomial theorem, we see that

$(1+2)^n = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} 1^{n-k}2^k = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} 2^k$

Hence $\sum_{k=0}^{n} 2^k \begin{pmatrix} n \\ k \end{pmatrix} = 3^n$

Chapter 3

Applications of Binomial Theorem

Binomial theorem has a wide range of application in mathematics like finding the remainder, finding digits of a number, etc. The most common binomial theorem applications are:

Finding Remainder Using Binomial Theorem

• **The general formula for the binomial theorem:** $(ax + by)^n = nC_0ax^nby^0 + nC_1ax^{n-1}by^1 + \cdots + nC_rax^{n-r}by^r + \cdots + nC_nax^0by^n$

Illustration: Finding the remainder when 7^{103} is divided by 25

Solution:

$$\frac{7^{103}}{25} = [\frac{7(49)^{51}}{25}] = [\frac{7(50-1)^{51}}{25}]$$

$$= [\frac{7(25k-1)}{25}] = [\frac{(175k-25-7)}{25}]$$

$$= [\frac{(25(7k-1)+18)}{25}]$$

Therefore The remainder is 18

Finding Digits Of a Number

• **The general formula for the binomial theorem:** $(ax + by)^n = nC_0ax^nby^0 + nC_1ax^{n-1}by^1 + \cdots + nC_rax^{n-r}by^r + \cdots + nC_nax^0by^n$

Illustration: Find the last two digits of the number $(13)^{10}$

Solution:

$(13)^{10} = (169)^5 = (170 - 1)^5$

$= 5C_0(170)^5 - 5C_1(170)^4 + 5C_2(170)^3 - 5C_3(170)^2 + 5C_4(170) - 5C_5$

$5C_0(170)^5 - 5C_4(170)^4 + 5C_2(170)^3 - 5C_3(170)^2 + 5(170) - 1$

A multiple of 100 + 5(170)-1 = 100K + 849

There fore The last two digits are 49

Relation Between Two Numbers

• **The general formula for the binomial theorem:** $(ax + by)^n = nC_0ax^nby^0 + nC_1ax^{n-1}by^1 + \cdots + nC_rax^{n-r}by^r + \cdots + nC_nax^0by^n$

Illustration: Find the larger of $99^{50} + 100^{50}$ and 101^{50}

Solution:

$101^{50} = (100 + 1)^{50} = 100^{50} + 50 * 100^{49} + 25 * 49 * 100^{48} + \cdots$

$\Rightarrow 99^{50} = (100 - 1)^{50} = 100^{50} = 50 * 100^{49} + 25 * 49 * 100^{48} - \cdots$

$\Rightarrow 101^{50} - 99^{50} = 2[50 * 100^{49} + 25(49)(16)100^{47} + \cdots]$

$= 100^{50} + 50 * 49 * 16 * 100^{47} + \cdots > 100^{50}$

$\therefore 101^{50}$ - $99^{50} > 100^{50}$

$\Rightarrow 101^{50} > 100^{50} + 99^{50}$

Divisibility Test

• **The general formula for the binomial theorem:** $(ax + by)^n = nC_0ax^nby^0 + nC_1ax^{n-1}by^1 + \cdots + nC_rax^{n-r}by^r + \cdots + nC_nax^0by^n$

Illustration: Show that $11^9 + 9^{11}$ is divisible by 10.

Solution:

$11^9 + 9^{11} = (10 + 1)^9 + (10 - 1)^{11}$

$= (9C_010^9 + 9C_110^8 + \cdots + 9C_9) + (11C_010^{11} - 11C_110^{10} + \cdots$ -$11C_{11})$

$9C_010^9 + 9C_110^8 + \cdots + 9C_810 + 1 + 10^{11}$ - $11C_110^{10} + \cdots + 11C_{10}10 - 1$

$10[9C_010^8 + 9C_110^7 + \cdots + 9C_8 + 11C_010^{10} - 11C_110^9 + \cdots + 11C_{10}]$

10K which is divisible by 10.

Where K= $9C_010^8 + 9C_110^7 + \cdots + 9C_8 + 11C_010^{10} - 11C_110^9 + \cdots + 11C_{10}$

Conclusion

Generally, from this project we understand all about binomial theorem,and also how we can proof the binomial theorem and how we can solve some applications of binomial theorem.

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