

WOLKITE UNIVERSITY

College of Engineering and Technology

Department of Electrical and Computer Engineering

Industrial Control Stream

Design and simulation of a PID Controller for Inverted Pendulum

(Human Transportation Application)

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Declaration

We here by certify that the work titled by Design and simulating Inverted Pendulum (Human Transportation Application) Based On PID Controller which submitted to Wolkite University Department of Electrical and Computer Engineering is an authentic record of the work done by Temesgen Haile**,** Bereket Bekele and Nuri Ali under the supervision of Mr. Misker Shumeye. All the sources of the materials used by our work are properly referred.

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Abstract

The inverted pendulum is a classical control problem in dynamics and control theory which is the system encounter highly nonlinear and unstable system. Basically, the design of an inverted pendulum system consist of a cart or carriage which it can move horizontally along its track, where there is DC motor embedded in the cart to drive it, while on the cart, there is pendulum that mounted on it. For visualization purpose, the inverted pendulum system had similar concept when trying to balance a broomstick with hand. Since inverted pendulum is unstable system, the controller was introduced. So, a systematic iterative method for the state feedback design by choosing weighting matrices key to proportional integrated derivative (PID) design is presented assuming all the states to be available at the output. Designing the PID controller and simulating in MATLAB for proper tuning and verification. Proportional integrated derivative (PID) will be developed as an optimal and effective controller that will be used to place cart at desired position and at the same time to make sure the pendulum is always erected in its inverted position during movement of cart.

***Key words**:* Inverted pendulum, PID, MATLAB/ SIMULINK.

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Acronomy

PID	proportional integral derivative
MATLAB	matrix library
SIMO	single input multiple output
DC	direct current
KP	proportional controller
KI	integral controller
KD	derivative controller
LQR	linear quadratic regulator
IP	inverted pendulum
TWIPHT	two wheel inverted pendulum human transportation
CIPS	cart inverted pendulum system

Chapter 1. Introduction

1.1 Background

Inverted Pendulum is a very good platform for control engineers to verify and apply different logics in the field of control theory. Most of the modern technologies use the basic concept of inverted Pendulum, such as attitude control of space satellites and rockets, landing of aircrafts, balancing of ship against tide, seismometer (which monitors motion of the ground due to earthquake and nuclear explosions) etc. [1]

An inverted Pendulum has its mass above the pivoted point, which is mounted on a cart which can be moved horizontally. The pendulum is stable while hanging downwards, but the inverted pendulum is inherently unstable and need to be balanced. In this case the system has one input - the force applied to the cart, and two outputs, which are speed of the cart and the angle of the pendulum, making it as a SIMO system. As shown in Figure1, the physical system consists of a cart, driven by a Dc motor, and a pendulum attached to the cart.

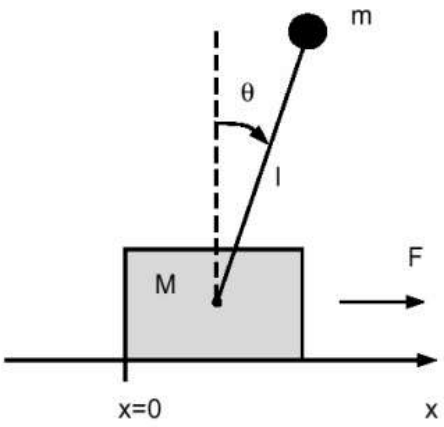


Figure 1. 1An Inverted Pendulum System Schematic

The Inverted Pendulum is a single input multi output (SIMO) system with control voltage as input, motor speed and pendulum angle as outputs. Even though the system is simple from construction point of view, but there lies a lot of control challenge owing to following characteristics .

Highly Unstable – The inverted position is the point of unstable equilibrium as can be seen from the non-linear dynamic equations.

Highly Non-linear – The dynamic equations of the CIPS consists of non-linear terms. Non-minimum phase system – The system transfer function of CIPS contains right hand plane zeros, which affect the stability margins including the robustness.

Under actuated – The system has two degrees of freedom of motion but only one actuator i.e. the D.C. Motor. Thus, this system is under-actuated. This makes the system cost effective but the control problem becomes challenging.

Additionally, there are constraints imposed by track length, control voltage etc. These make the problem still more complex. This control problem attracts attention explains the various control approaches that is in attempt to stabilize the unstable system.

1.2 Statement of the problem

The challenge of this project is to keep the inverted pendulum balanced and track the linear cart to a commanded position. In practice, it is interesting to point out that similar dynamics and control problem apply to rudder roll stabilization of ships. During this project, the controller that will be used is PID. So the proposed controllers with proper design will be able to overcome the above mentioned challenge.

The problem involves a cart, able to move backwards and forwards, and a pendulum, hinged to the cart at the bottom of its length such that the pendulum can move in the same plane as the cart. That is, the pendulum mounted on the cart is free to fall along the cart's axis of motion. The system is to be controlled so that the pendulum remains balanced and upright, and is resistant to a step disturbance.

This problem involves a simple coupled system. If the pendulum starts off-center, it will begin to fall. The pendulum is coupled to the cart, and the cart will start to move in the opposite direction, just as moving the cart would cause the pendulum to become off center.as change to one of parts of the system results in change to the other part, this is a more complicated control system than it appears at first glance. For this reason, this problem is often used as a demonstration of PID control.

The inverted pendulum cart runs along a track and is pulled by a belt connected to an electric motor. A potentiometer measures the cart speed from its rotation and another potentiometer measures the angle of the pendulum. If the output is the angle of the pendulum relative to the vertical axis (in upright position), we realize that the system is unstable, since the pendulum will fall down if we release it with a small angle. To stabilize the system, i.e., to keep the pendulum in upright position, a feedback control system must be used.

The inverted pendulum control problem is a solid starting point for testing different control algorithms on a physical system. Also, the inverted pendulum system can easily be complicated further to test control algorithms on more complicated systems. advantages of a PID controller for the control problems are unstable system to stable, non-linearize to linearize and adjust upright position.

1.3 Objectives

1.3.1General objective

The main objective of this project is to design and simulate a PID controller for an inverted pendulum considering a human transportation using MATLAB/SIMULINK software.

1.3.2 Specific objective

- To analyze the mathematical model and analyze controllability ranges of the Inverted Pendulum System.
- To design PID controller for speed and angle control of the inverted pendulum.
- To stabilize the inverted pendulum in upright position.

1.4 Significance of the project

The inverted pendulum makes its study even more interesting and important. In this project human transportation application balancing systems are developed using inverted pendulum. These find application in transport machines that need to balance objects, in systems that support walking for patients, that are used in domestic and It is the most easily available system (in most academia) for laboratory usage. even large scale constructions such as buildings are modeled as inverted pendulums [1]. The above applications along with many more render the inverted pendulum a system of great importance for engineers. This is the main reason why the inverted pendulum is chosen frequently for testing new control methods [2]. There exist a great number of methods used

in controlling the inverted pendulum. It is a nonlinear system, which can be treated to be linear, without much error. This method we used PID controller.

1.5 Scope and limitation of the project

The scope of this project is concentrates on design mathematical model and inverted pendulum system for human transportation application, obtain optimal linear quadratic controller gain for a system, design proportional integrated derivative controller, and the design of the system and controlling by MATLAB/SIMULINK

The aim of the study is to stabilize the pendulum such that the position of the carriage on the track is controlled quickly, accurately and that the pendulum is always maintained tightly in its inverted position during such movements.

The goal of controlling IP is to balance the pendulum in upright position when it initially starts with some nonzero angle off the vertical position.

There are certain limitations which are to be kept in mind while preparing a model for this system. In this case the limitations are:

- Human mass is not more than 85 kg.
- In this project high speed of inverted pendulum is not considered
- The acute angle of the pendulum w.r.t vertical position, therefore θ should be in the range of 0.4 radian.
- The applied voltage to the Dc motor should remain of 5V.

1.6 Methodology of the project

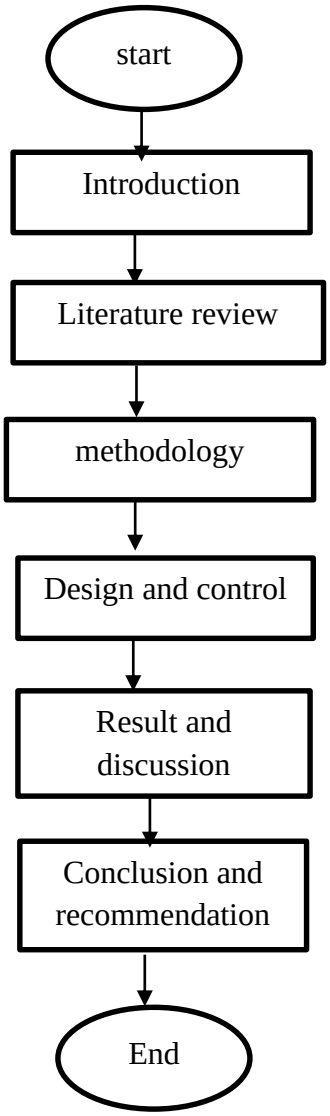


Figure 1. 2 methods of system

1.7 Organization of the project

The project contains six chapters as follows: -

Chapter 1 – Introduces the classical Inverted Pendulum Control problem, its dynamics, its introduce PID controller and its significance of project. It also describes the MATLAB, SIMULINK,

Chapter 2- describes literature review of some controlling methods. It describes the proportional integral derivative based state feedback control discussion.

Chapter 3 – This chapter deals modeling transfer function of the inverted pendulum, modeling motor equation and block diagram of inverted pendulum.

Chapter 4 - Design PID controller using pole placement, speed and angle control of inverted pendulum by using MATLAB/ SIMULINK.

Chapter 5 –This chapter deals with MATLAB/SIMULINK results and discussion of cart speed and pendulum angle of inverted pendulum human transportation application.

Chapter 6 – This chapter deals draws conclusions on the various works presented and aptly suggests the scope of future work (recommendation)

Chapter 2. Literature Review

There are several sources that take part in the project design. Many sources have contributed to be the references for this project for example, conference paper and project report. by taking the other project as a reference, it may be helpful in order to design this whole project.

The inverted pendulum since is an important control problem which the researchers have been trying to solve worldwide for last few decades. Historically, the Inverted Pendulum was used first by seismologists in design of a "seismometer" in the year 1844 in Great Britain. Since, the system is inherently in unstable equilibrium when mounted on a stiff wire it can sense even the slightest of vibrations. Linear Quadratic Regulator (LQR) for inverted pendulum is simplest of all linear control techniques. It is equivalent to a two loop PD control design. In [8], stabilization of the cart- pendulum system was carried out by linearization of the state model and designing a LQR after swing-up by an energy based controller. The velocity states were less penalized compared to the position states in [1] so that the resulting stabilized system will have almost zero position as a zero velocity is only a secondary priority. This logic will lead to an almost upright system.

LQG/LTR Controller Design for Rotary Inverted Pendulum Quanser. this design consists of a rigid link (pendulum) rotating in a vertical plane. The rigid link is attached to a pivot arm, which is mounted on the load shaft of a DC-motor [2]. The pivot arm can be rotated in the horizontal plane by the DC-motor. The DC-motor is instrumented with a potentiometer. In addition, a potentiometer is mounted on the pivot arm to measure the pendulum angle. The principal objective of this design is to balance the pendulum in the vertical upright position and to position the pivot arm. Since the plant has two degrees of freedom but only one actuator, the system is under-actuated and exhibits significant nonlinear behavior for large pendulum excursion. The purpose is to design a robust controller in order to realize a real-time control of the pendulum position using a Quanser PC board and power module and the appropriate WinCon real-time software. For the controller design is used a well-known robust method, called LQG/LTR (Linear Quadratic Gaussian/Loop Transfer Recovery) which implements an optimal state feedback. The real-time experiment is realized in the Automatic Control laboratory.

The Inverted Pendulum System is an under actuated, unstable and nonlinear system [3]. Therefore, control system design of such a system is a challenging task. To design a control system, this project report first obtains the nonlinear modeling of this system. Then, a linearized

model is obtained from the nonlinear model about vertical (unstable) equilibrium point. Next, for this linearized system, an LQR controller is designed.

A hybrid controller is designed in [10] that ensures global stabilization, this approach has a linear controller for stabilization, a linear cart controller and a combination of various bang-bang controllers for swing-up in minimum time. A non-linear controller is described in [14], in which the controller swings up the pendulum from the pendant position and stabilizes the pendulum in the unstable equilibrium and simultaneously restricts the cart excursion on the track. A simple controller for balancing the inverted pendulum to the upper equilibrium point and minimize the cart position to zero is discussed in [16]. A near optimal controller, non-linear control law has been designed based on linear quadratic optimal control yielding a near optimal gain schedule.

In this paper modeling of an inverted pendulum is done using Euler- Lagrange energy equation for stabilization of the pendulum [4]. The controller gain is evaluated through state feedback and proportional derivative integral controller techniques and also the results for both the controller are compared. The controller is designed by Pole-Placement technique. An advantage of Quadratic Control method over the pole-placement techniques is that the former provides a systematic way of computing the state feedback control gain matrix. PID controller is designed by the selection on choosing. The proposed system extends classical inverted pendulum by incorporating two moving masses.

Based on the pole placement design technique, a full state feedback controller using separation factor is proposed to stabilize a real single inverted pendulum [5]. The strategy is to start with selection two dominant poles that achieve a certain desired performance, using a separation factor between the selected dominant poles and the other poles to eliminate their effect on the system performance, and finally state space formula can be used to calculate the feedback gain matrix to place the system poles at the desired locations. Simulation and results demonstrate the effectiveness of the proposed controller, which offers an excellent stabilizing and also ability to overcome the external resistance acting on the pendulum system.

The objective of this paper is to determine which proportional integral derivative controllers bring better performance. The output of the inverted pendulum that had been analyzed is the speed of the cart and the angle of the pendulum. For the proportional integral derivative controller, the author used pole placement method in order to determine the best pole that the system should be. The performance was analyzing through specific requirement there are, the settling time, should

less than 5 seconds, the maximum overshoot is 18 degrees (0.4 radian) and the rise time must less than 0.5 seconds. In this project also, there are some assumption made by the authors. The assumptions are there is no friction either between the wheels and the horizontal plane or between the pendulum and the cart. Second assumption is small angle approximation is used and lastly, assume that the sensors for measuring all states are available. The analysis has done by using MATLAB Simulink.

In this paper modeling of an inverted pendulum has been done and then the controller PID has been used for stabilization of the pendulum [11]. The proposed system extends classical inverted pendulum by incorporating two moving masses. The motion of two masses that slide along the horizontal plane is controllable. Main focus is to derive the mathematical model and the analysis of its system performance, then design a PID controller in order to get better control and simulate it. This project is based on a linear model of Inverted pendulum. Here, a linear controller is designed based on proportional integral derivative control method. Although another linear controller design method is available like Pole-Placement method. But there are various advantages of PID compared to the other controller: -

Proportional integral controller can stabilize only 1st order unstable process. Changing controller gain K can change closed loop dynamics. A large controller gain will result in control system with:

a)Smaller steady state error, i.e. better reference following from the other controller

b) Faster dynamics, i.e. broader signal frequency band of the closed loop system and larger sensitivity with respect to measuring noise

c) Smaller amplitude and phase margin

d) better tuning than the other

This project is reduced rise time, reduced maximum overshoot and reduced steady state error than the other.

Chapter 3. Theoretical Background

3.1 Theoretical background

The uniqueness and wide application of the technology derived from this unstable system has drawn interest from many researchers. The inverted pendulum are single input and multiple outputs (SIMO), nonlinear and unstable system when open loop system [7]. An inverted pendulum is a pendulum which has its center of mass above its pivot point and the pendulum is free hung in upright condition. Generally, the pivot point mounted at the cart or carriage where the cart can move horizontally. Sensors are attached to the cart and pivot to measure the cart speed and pendulum angle rotation respectively [9].

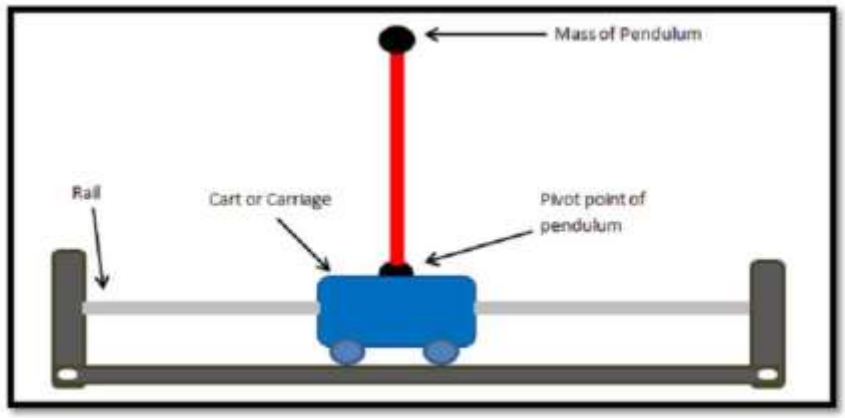


Figure 3. 1 Overview inverted pendulum system

Since the inverted pendulum is in upright situation, is naturally easily to fall downward because of gravity. Thus the inverted pendulum system is inherently unstable. In 2-Dimensional system, to stabilize the system it can be done either vertical or horizontal oscillation with certain frequency. While for 3-Dimensional, a rotational arms or free robot arms are used. An addition, for algorithm controller using feedback system is used to keep the pendulum upright [8].

3.2 Methodology

The methodology in this project includes both qualitative data and system design overview. The data sources are gathered from various literatures, books, websites and other relevant documents.

3.3 Block diagram of the System

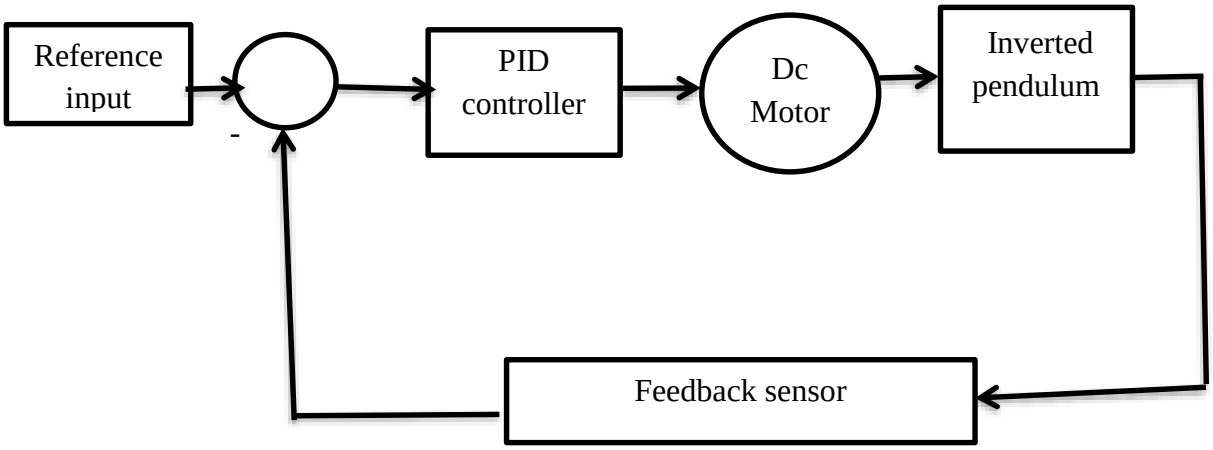


Figure 3. 2 System block diagram

3.3.1 Mechanical System

The design of a mechanical system for this project involves integrating of four main components: (1) the cart, (2) the pendulum, (3) the track, and (4) the mechanism used to move the cart. Each component is quite dependent on the other three.

These components also have to meet some basic requirements such that it is possible to design a controller to balance the pendulum.

These requirements are as follows,

- The cart motion needs to be limited to one degree of freedom which is in the horizontal plane.
- The pendulum motion needs to be limited to two degrees of freedom, one of which is the same as the cart‘s degree of freedom.
- The friction that depends the cart and pendulum motion must be reduced as much as possible.

3.3.2 Track, Cart, and Pendulum Solutions

The track is linear within which the cart would move. Ideally, the cart would fit well in the track and slide smoothly with little friction. Also needed in this design is a mechanism to apply force on the cart in either direction. This could be done in one of two ways. The first way would be to mount a motor at the end of the track with a drive wheel/sprocket attached to the shaft and another wheel/sprocket free-spinning at the opposite end of the track. Around the two wheels would be a pulley or chain with which the cart can be fixed to. The second way would be to have a motor directly drive wheels on the bottom of the cart. A second option for the mechanical design could have the motor as the main component of the cart.

3.3.3 DC-motor

The DC-motor should be in the system as well. This is because the conveyor belt with the cart on it is driven by the DC-motor. Actually, the force acting on the cart is obtained from the torque of the motor. So, a rotational (motor) to linear (conveyor belt) translation takes place. In order to derive the expression for the torque, the electrical circuit of the DC-motor should be evaluated. There are four main types of DC-motors the permanent magnet DC-motor, shunt wound DC-motor, the series wound DC-motor and the compound wound DC-motor [20]. In this project we are use in the permanent magnet DC-motor.

Mechanical system

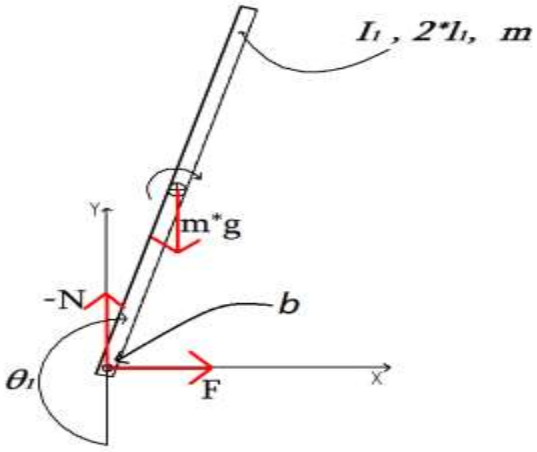


Figure 3. 3 Figure of the pendulum.

I2Ö2 = Mmotor – Fl (1)

F = m ẍ+ml1ö1 cos θ1 - *m l*1$\dot{\theta}^2$1 sin θ1 (2)

Equation 2 is the force equation of the pendulum in horizontal direction seen in Figure 3.3, ***m***, ***θ1***, ***l1*** is the mass, angle and half the length of the pendulum shown in the above figure. From equation (1) and (2) the following expression is obtained.

I2Ö2-Mmotor-lm(ẍ+l1Ö1cosθ1-l1$\dot{\theta}^2$1*sin* θ1)

With substitution as in equation(3) the following equation for the torque of the motor is derived**.**

ẍ=Ö2l (3)

Substitution of the acceleration to polar coordinates.

Mmotor= *Ö2(I2+l^2m)+lm(ẍ*+ l1ö1 *cos* θ1−*l1*$\dot{\theta}^2$1*sin* θ1 *)*

Due to that the pendulum will be balanced at an angle $\pi + \theta$ where θ is small.

linearization with small angle approximations are made.

If θ approximately zero sin (π + θ) = - θ

Cos (π + θ) = -1

$\dot{\theta}^2 = 0$

These approximations result in the following equations for the mechanical system

Mmotor= *Ö2(I2+l^2m)-lm l1ö*1 (4)

Electrical system

This is the derivation how the voltage to the motor transitions into mechanical torque.

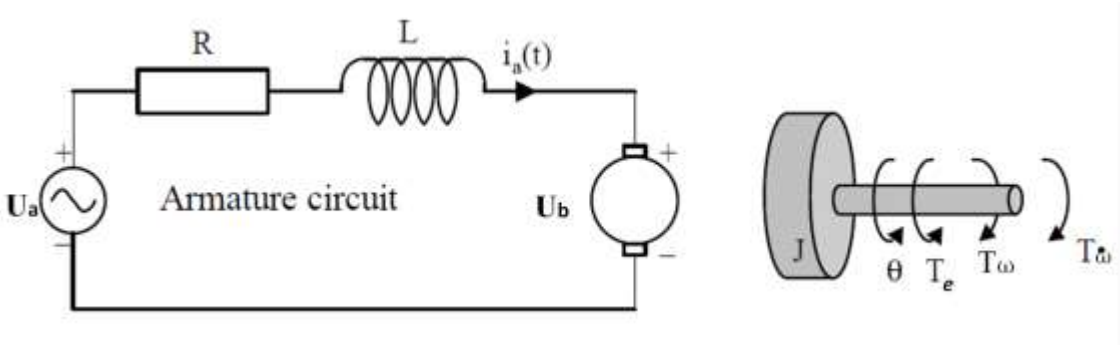


Figure 3. 4 Electrical circuit of DC motor

Using Kirchoff´s voltage law on the circuit above following equations can be derived

Ua-URa-ULa-Ub=0 (5)

where Ua is a voltage source, i.e the voltage across the coil of the armature, URa is the voltage across the resistor R**,** ULa is the voltage across the inductor *L* and *U.*

is the back emf voltage. With Ohm's law URa can be derived as

URa=IaRa (6)

where *Ra* is the resistance of Ra and *Ia* is the armature current. The voltage across the inductor is

ULa=IaLa (7)

where La is the inductance of the armature coil. The back emf *Ub* can be written as

Ub=Keθa (8)

where *Ke* is the velocity constant and *θa* is the angular velocity of the armature, i.e the angular velocity of the motor axis. Equations 6, 7 and 8 inserted in 5 gives the following expression.

$$U_a - I_aR_a - L_aI'_a - K_e\theta_a = 0 \quad (9)$$

To get the sum of the torques an energy balance is done. And the following expression is derived

T***e*** **-** Mmotor - $T\omega\dot{}$ - $T\omega = 0$

where *Te* is the electromagnetic torque, $T\omega\dot{}$ is the torque due to acceleration of the rotor, $T\omega$ is the torque due to the velocity of the rotor.

Te=KtIa (10)

where *Kt* is the motor torque constant. $T\omega\dot{}$

$T\omega\dot{}$=JaÖa (11)

where *Ja* is the inertia of the rotor and $\theta\ddot{}\,a$ is the angular acceleration of the rotor.

Tω=Bθ˙*a* *(12)*

where *B* is the motor viscous friction constant and $\theta\dot{}a$ is the angular velocity of the rotor.

equation 10,11 and 14 in equation 13 gives the following expression

ktIa-Mmotor- JaÖa-Bθa=0 (13)

substituting Ia from equation 13 gives the following expression

$$Ia = \frac{Ua - La\dot{I}a - ke\dot{\theta}a}{Ra} \quad (14)$$

substitution of Mmotor and equation 15 inserted in 14

$$Mmotor = kt\left(\frac{Ua - La\dot{I}a - ke\dot{\theta}a}{Ra}\right) - JaÖa - B\theta\dot{}a \quad \textbf{(15)}$$

Combined systems in Laplace domain

where we define the transfer function *Gi***(***s***)**

$$Gi(s) = \frac{s^2\left(\frac{I1}{l1} - ml1\right) + \frac{bs}{l1} + mg}{mls^2}$$

$$G(s) = \frac{kt}{Ra\left(s^2 lml1 - Gi(s)\left(I2 + l^2 m + \frac{Ja}{14}\right)\right) - \frac{sGi(s)ktke}{14Ra}}$$

Table 3. 1 DC motor parameter used

Model	EmaxBl4030
Mass	230 g
RPM	6100
Length	0.4m
Electric resistance (R)	2.2 ohm
Electric inductance(l)	8.5Mh
Current(I)	22A
Torque constant(kt)	0.44Nm/A
Moment of inertia(Ja)	0.089kg/m^2

$$G(S)=\frac{3.9865}{S^2-2.86S}$$

3.3.4. Feedback sensor

Designing an accurate feedback sensor is essential to stabilizing the system. Thus the sensors need to relatively noiseless and have a fast response such that the information retrieved from the sensors accurately reflects the state of the system. Determining the variables of the system to measure can be difficult. In this case there are four parameters that govern the inverted pendulum system (which will be derived in a later section). They are (1) the angle, (2) the angle‘s velocity, (3) the displacement of the cart, and (4) the velocity of the cart. Thus there are four measurable parameters that could be used for feedback, which would determine the control necessary to stabilize the system. Most conventional approaches to this problem only measure the angle and velocity and derive the other two parameters from these. This project follows in suit since those two parameters are the easiest to measure and give the most information about the system.

3.4 velocity Sensors

One way to measure velocity would be to attach a potentiometer to either the drive sprocket or the free-spinning one. It is then possible to determine the velocity of the cart by using the diameter of the sprocket, the measured voltage, and the number of turns of the potentiometer is capable.

3.5 Angle sensor

One of the easiest solutions would be to mount the pendulum on a circular potentiometer. Ideally, the potentiometer would have little friction. Though potentiometers have some friction, which could influence the dynamics of the pendulum falling. More friction would slow down the reaction of the pendulum to any of the forces exerted on it, making it easier to balance.

3.6 Angle Controllability Range

The maximum control angle of inverted pendulum can be calculated from stability of rigid body. From basic physics a rigid body is stable if the perpendicular line passing through the center of gravity of a rigid body passes through the base of the rigid body. If the base has length of p distance and the center of gravity is located at (x_g, y_g) above the base, the maximum angle of tilt of the rigid body from vertical is computed as follows.

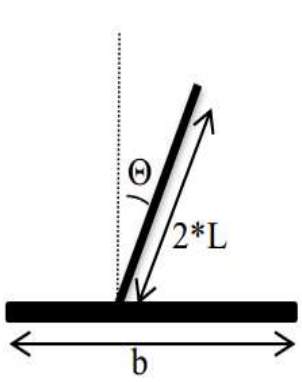


Figure 3. 5 stability Consideration Diagram

$y_g = 0.5*b$ (1.26)

but $y_g = L*\sin\theta$ (1.27)

$L*\sin\theta = 0.5*b$

$\theta max = \sin^{-1} 0.5 * \frac{b}{l}$ = approximately (0 degree to 20 degree)

3.7 Control methods

The following control methods are presented here to control the nonlinear inverted pendulum-cart dynamic system.

3.7.1 PID control

PID (Proportional-Integral-Derivative) control is the widest type of automatic control used in industry. to stabilize the inverted pendulum in upright position and to control the cart at desired speed using PID control approach two PID controllers:-angle PID controller, and cart PID controller have been designed.

PID Controller is achieved as product of PI controller and PD controller. One method of designing a control algorithm that uses the rotation of the angle of the pendulum and the speed of the cart as input measurement is by using the PID-controller. This controller maintains the output of the system to ensure zero error between the desired output and the process variables in closed loop operations. The PID-controller consists of a proportional controller, integral controller and a derivative controller. First, a proportional controller gives output, which is proportional to the current error. The I-controller integrates the error over a period of time until the value of the error becomes zero. Lastly, the derivative controller anticipates future behavior of the error. In figure 3.6 a schematic overview of the working of the PID is given.

Proportional controller (*Kp*) will have the effect of reducing the rise time and steady state error, but never eliminate it. Integral controller (*Ki*) will have the effect of eliminating the steady-state error, but it may make the transient response worse. Derivative control (*Kd*) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

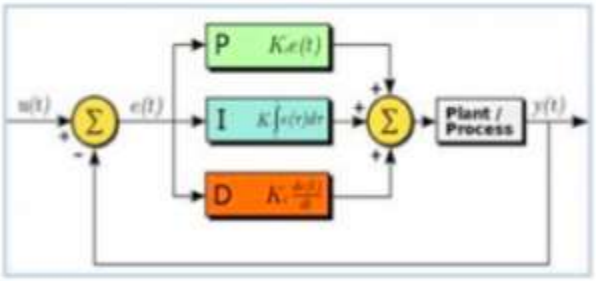


Figure 3. 6 Overview PID controller [6]

Table 3. 2 PID characteristics

Controller	**Rise Time**	**Overshoot**	**Settling Time**	**SS Error**
Kp	Decrease	Increase	Small change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small change	Decrease	Decrease	Small change

3.8 Mathematical modeling

Mathematical modeling is a significant tool to gain a clear understanding of the system. First,throughout the project the pendulum will be analyzed in its vertical downward position, the normal pendulum. This is because this helps validating the parameters of the models for a stable system with similar characteristics. Thereafter, the pendulum when in its vertical upwards position, the inverted pendulum, can be tested by filling in another initial condition (π) or by adjusting the potentio-meter differently by which the zero point becomes the point when the pendulum is straight in its upwards vertical position. To understand the dynamics of the system first mathematical modeling has to be applied. In figure 3.7 a schematic of the cart and the pendulum is given with the applicable forces. Below a list of the relevant variables and its meaning is given:

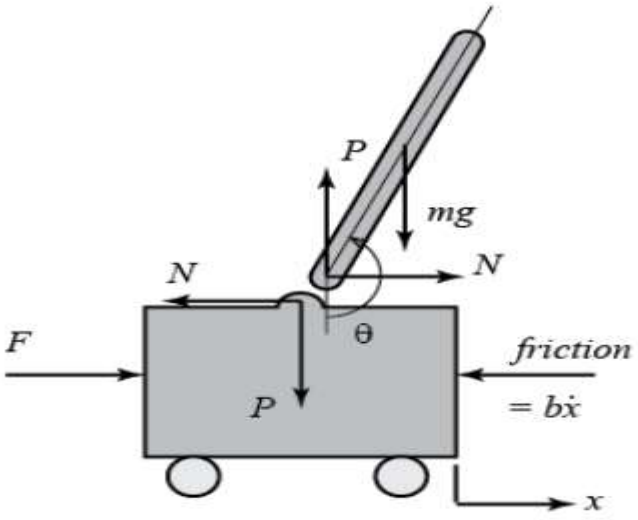


Figure 3. 7 Schematic of pendulum [10]

x = cart direction coordinate

m = mass of the pendulum

M = mass of the cart

b = coefficient of friction of the cart

I = mass moment of inertia of the pendulum

l = length of the pendulum

F = force applied to the mass

θ = the angle that the pendulum makes

The force is the input of the system, the speed of the inverted pendulum and the angle of the pendulum are the outputs of the system. The variable b is defined as the coefficient of friction of the cart. Actually, there is also a coefficient of friction for the pendulum, but compared to the friction on the cart the friction on the pendulum is considerably small, which is the friction of the pendulum is ignored.

3.9 Equations of motion

To get an overview of the mechanics of the system, the equations of motion have to be calculated. Moreover, in order to model the pendulum in Simulink, the equations of motion corresponding to figure 3.7 are significant. Below an overview of the equations is given. Equation 1 belongs to the cart and equation 2 belongs to the angle of the pendulum. In order to calculate the equations for the forces P and N, both the vertical and horizontal position of the pendulum have to be taken into account.

$\ddot{x} = \frac{1}{m}(F - N - b\dot{x})$…….(1)

$\ddot{\theta} = \frac{1}{I}(-Nl\cos(\theta) - Pl\sin(\theta))$……...(2)

Vertical position of the pendulum

$m\ddot{y}_p = P - mg$

Then P becomes:

$P = m\ddot{y}_p + mg = m(\ddot{y}_p + g)$…….….. (3)

Where:

yp =-l cos(ɵ)…….... (3.1)

ẏp = l sin(ɵ)ò……. (3.2)

ÿp = l cos(ɵ)ò ˙2+l sin(ɵ)ë ……..….. (3.3)

Horizontal position of pendulum

x p=N……..*(4)*

Where:

Xp = x + l sin(ɵ)…….…. (4.1)

ẋp = ẋ + l cos(ɵ)ò …… (4.2)

ẍp = ẍ+l cos(ɵ)ë-l sin(ɵ)ò2 ……..... (4.3)

N and P then become:

N = m(ẍ-lò2 sin(ɵ)+lëcos(ɵ)) …… (5)

P = m(lò2 cos(ɵ)+lësin(ɵ)+g) …… (6)

When filling in equation 5 and 6 in equation 1 and 2 the following equation can be obtained

(M+m)ẍ+mlcos(ɵ)ë-mlsin(ɵ)ò2=F-bẋ …… (7)

$(ml^2+I)ë+mlcosɵẍ+mglsinɵ=0$……. (8)

3.9.1 Linearization

Before a model can be obtained in Simulink and the transfer functions of both the angle of the pendulum and the speed of the cart can be determined from equations 7 and 8, these equations first have to be linearized. Since first the normal pendulum is considered, the pendulum starts in its downward vertical position, and presuming that the pendulum swings through small angles. Therefore, equations have to be linearized around this point.

Ɵ= 0

Cos(ɵ)~ 1

Sin(ɵ)~ ɵ

ò^2~ 0

This results in equations of motion expressed by equation 9&10

(M+m) ẍ+mlë=F-bẋ (9)

$(ml^2+I)ë+mlẍ+mglɵ=0$ (10)

3.9.2 Laplace transform

The last step before the transfer function can be obtained is taking the Laplace transform of the linearized equations of motion (9 and 10). Then the following equations can be obtained:

$(M+m)\ s^2 x(s) + mls^2 \theta(s) = F(s) - bsx(s)$ (11)

$(ml^2+I)\ s^2 \theta(s) + mls^2 x(s) + mgl\theta(s) = 0$ (12)

3.9.3 Transfer function

Transfer function of angle of the pendulum

From equation 12 can be obtained

$$X(s) = \frac{-(ml^2+I)s^2 - mgl}{mls\hat{}2}\theta(s) \quad (13)$$

$$X(s) = \left(\left(\frac{ml^2+I}{ml}\right) + \left(\frac{g}{s^2}\right)\right)\theta(s)$$

When substituting equation 13 in equation 11 the final equation is obtained:

$$(M+m)\left(\left(\frac{ml^2+I}{ml}\right) + \left(\frac{g}{s^2}\right)\right)\theta(s)\ s^2 + b\left(\left(\left(\frac{ml^2+I}{ml}\right) + \left(\frac{g}{s^2}\right)\right)\theta(s)\ s + mls^2\theta(s) = F(s) \quad (14)$$

The transfer function of the pendulum can be obtained by equation 14 & then becomes:

$$T_{pend} = \frac{\theta(s)}{F(s)} = \frac{\frac{ml}{q}s^2}{s^4 - \frac{b(ml^2+I)}{q}s^3 - \frac{(M+m)gml}{q}s^2 - \frac{bmgl}{q}s}$$

where, $q = [(M+m)(I+ml^2) - (ml)^2]$ (15)

Table 3. 3 Estimated parameters of the components

Parameters	Value
Mc	2.4kg
Mp	0.23kg
Lp	0.4m
The moment Inertia of pole	0.099kg. m^2
b cart friction coefficient	0.05 Ns/m
gravity	9.81 m/s^2
M body	85 kg

$$\text{Tpend} = \frac{\theta(s)}{F(s)} = \frac{\frac{ml}{q}s^2}{s^4 - \frac{b(ml^2+I)}{q}s^3 - \frac{(M+m)gml}{q}s^2 - \frac{bmgl}{q}s}$$

Transfer function of the cart

$$\theta(s) = -bs - \left(\frac{(M+m)s^2}{mls\text{^}2}\right)x(s) + \frac{1}{mls^2}F(s)$$

Substitute in equation 12:

$$\frac{(ml^2+I)(-bs-(M+m)s^2)+(mls^2)^2+mgl(-bs-(M+m)s^2)}{s^2}X(s) = \frac{-(1+mgl)}{mls^2}F(s)$$

The transfer function of the cart then turns into:

$$\text{Tcart} = \frac{X(s)}{F(s)} = \frac{\frac{(ml^2+I)s^2-bgml}{q}}{s^4+\frac{b(ml+I)}{q}s^3+\frac{(M+m)gml}{q}s^2+\frac{bmgl}{q}s} \qquad (16)$$

Calculating unknown variables

The unknown variables are the length of the pendulum (l), the mass of the pendulum (m), the mass of the cart (M), the mass moment of inertia (I) and the coefficient of friction of the cart (b).

inverted pendulum model

Figure 3.8 shows the cart-pendulum system. It consists of a cart with a mass *M*. An inverted pendulum with a mass *m*, and a moment of inertia *I*, is attached to the cart. The center of mass of the pendulum is located at a distance *l* from the base. A force *F* is applied to the cart.

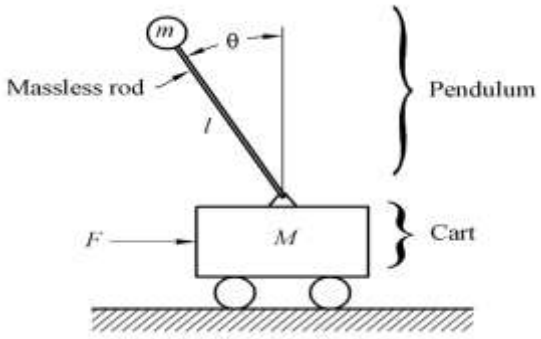


Figure 3. 8 Cart-pendulum system

3.10 Two-wheeled inverted-pendulum

Two-wheeled inverted pendulum human transporter (TWIPHT) is developed in this section. Attached to the base of the pendulum are motors that are used to drive each of the wheels.

3.11 Assumption of wheels

This section first shows to design two-wheeled inverted pendulum. These relations were introduced into a commercial multi body dynamics package to obtain the motion of the system.

It is assumed that:

- Both wheels have same radius.
- Both wheels have the same mass.
- The wheels are not deformable.
- The wheels roll on the ground without slipping.
- The ground is at surface with a certain slope.
- The handlebar of the vehicle cannot be tilted sideways.
- The wheels always make contact with the ground so the vehicle is restricted from tilting in the roll direction.
- The vehicle can rotate at any desired yaw rate independently of the speed.

3.12 Assumption of Human body

In this section, a simple human body considering is described.

- There is no relative motion between the person and the transporter.
- The human can actively command the movements of the transporter.
- The connections between body segments are rigid, so there is no movement between body parts.
- The human assumes a single static posture.
- There is no flexibility between the body and the transporter. This means that the arms and legs are assumed to be rigid solid bodies.

Chapter 4. Controller Design

4.1 Introduction

PID (Proportional, Integral and Differential) controller is the most common form of feedback. In PID controller the basic idea is the examination of signals from sensors placed in the system, called feedback signals.

Let's consider the following unity feedback system

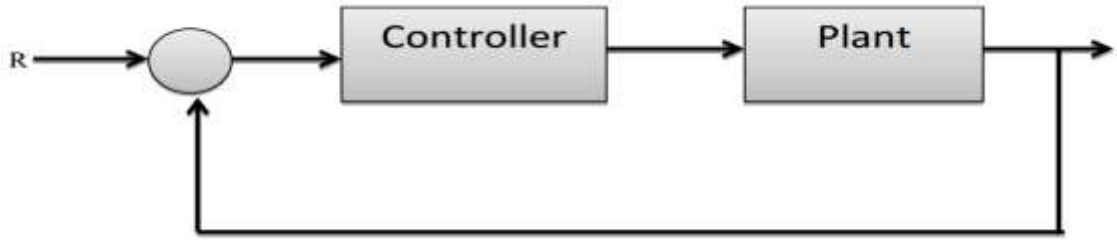


Figure 4. 1 Schematic diagram of the a feedback control system [11]

Plant: A system is to be controlled.

Controller: Designed to control the overall system behavior as per requirement.

The three-term controller

the transfer function of the PID controller is like,

$$kp + \frac{ki}{s} + kds = \frac{kps+ki+kds\text{^}2}{s}$$

kp =Proportional gain

ki=Integral gain

kd=Differential gain

The error signal is sent to the PID controller, and the controller computes both the derivative and integral of the error signal. The signal (u) just past the controller is now equal to the proportional gain (kp) times the magnitude of the error plus the integral gain (ki) times the integral of the error plus the derivative gain (kd) times the derivative of the error. The controller takes the new error signal and computes it derivative and integral again.

The characteristics of P, I, and D controllers

A proportional controller (kp) will have effect of reducing the rise time, but never eliminates the steady-state error. An integral controller (ki) will reduce the steady-state error but may make the transient response worse. A derivative controller (kd) will have an effect on stability of the system, it reduces the overshoot, and improving the transient response. Effects of these three controllers can be summarized as shown in the table 3.1.

PID Control Method

$$\frac{\theta(s)}{F(s)} = \frac{\frac{-mls}{q}}{s^3 + \frac{b(ml^2+I)}{q}s^2 + \frac{(M+m)gml}{q}s + \frac{bmgl}{q}} \qquad (41)$$

If the frictional force is neglected here, then the equation can be written as,

$$\frac{\theta(s)}{F(s)} = \frac{\frac{-mls}{q}}{s^3 + \frac{(M+m)gml}{q}s}$$

Now, cancelling the common poles i.e. s=0, we have,

$$\frac{\theta(s)}{F(s)} = \frac{\frac{-ml}{q}}{s^2 + \frac{(M+m)gml}{q}} = \frac{b1}{s^2 - a^2}$$

Where $b1=\frac{ml}{q}$ and $a = \frac{\sqrt{(M+m)mgl}}{q^2}$

a=6.807

b1=3.96

$$\frac{\theta(s)}{F(s)} = \frac{3.96}{s^2 - 6.807}$$

$$\frac{X(s)}{F(s)} = \frac{\frac{(ml^2+I)s^2 - bgml}{q}}{s^4 + \frac{b(ml+I)}{q}s^3 + b\frac{(M+m)gml}{q}s^2 + \frac{bmgl}{q}s} \qquad (42)$$

If we neglect the frictional force and consider the mass of the pendulum (m) is negligible compared to the mass of the cart (M), then the above transfer function can be reduced to:-

$$\frac{X(s)}{F(s)} = \frac{\frac{1}{M+m}}{s^2} = \frac{b2}{s^2}$$

Where: $b2 = \frac{1}{(M+m)}$ *(43)*

b2=5.841

$$\frac{X(s)}{F(s)} = \frac{5.841}{s^2}$$

With this transfer functions obtained in equation 42 and 43 Here the initial condition for the pendulum's angle ɵ(s)=0.4rad The motor which helps to move the cart along the horizontal direction has a gain of 15.

$$u = -u1 + u2$$

$$u = -15c1\,ɵ - 15c2x$$

$$u = -u15c1\frac{b1}{s^2-a^2} - u15c2\ \frac{b2}{s^2}$$

$$u(1 + 15c1\frac{b1}{s^2-a^2} + 15c2\ \frac{b2}{s^2}) = 0 \qquad (44)$$

Equation 44 is known as the closed loop equation, where,

$$c1=kp1 + \frac{ki1}{s} + kd1s$$

$$c2 = kp2 + \frac{ki2}{s} + kd2s$$

Solving the closed loop equation,

$$s^5 + 15s^4(b1kd1 + b2kd2) + s^3(15b1kp1 - 15b2kp2 - a^2) + 15s^2(b1kI1 + b2kI2 - b2kd2a^2) - 15b2kp2a^2s - 15b2kI2a^2 = 0 \qquad (45)$$

The next step is to determine the PID gains for both pendulum's angle and speed of inverted pendulum. There are several methods for determining the PID gains. We will adopt here the Pole Placement method. There are several PID tuning methods available in literature like Ziegler-Nichols method, relay method for non-linear systems; here a pole placement method is presented.

Pole Placement Method

The pole-placement method is somewhat similar to the root-locus method. The basic difference is that in the root-locus design we place only dominant closed loop poles (Poles that are close to the imaginary axis) at the desired locations, while in the pole-placement design we place all closed-loop poles at desired locations. Here we assume that all state variables are measurable and are available for feedback. The necessary and sufficient condition for pole-placement is that the system should be completely state controllable, such that the poles of the closed-loop system may be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix The present design technique begins with the determination of closed-loop poles based on the required transient response. From this closed-loop poles PID gains are obtained.

$\boldsymbol{\gamma}1$=-2.5191+1.05091

$\boldsymbol{\gamma}2$=-2.5191-1.05091

$\boldsymbol{\gamma}3$=-0.4359+0.53971

$\boldsymbol{\gamma}4$=-4359-0.53971

Form these four closed-loop poles it is clear that and are dominant poles. the system should have five poles, let's assume another pole in such a way that it will not affect the system response and the best way is to choose $\boldsymbol{\gamma}5$ as 6 times the real part of the dominant pole, i.e $\boldsymbol{\gamma}5$=-16.2

So, the characteristic equation becomes

(s- $\boldsymbol{\gamma}1$)(s- $\boldsymbol{\gamma}2$) (s- $\boldsymbol{\gamma}3$) (s- $\boldsymbol{\gamma}4$) (s- $\boldsymbol{\gamma}5$)=0

Putting all the values of λ in the above equation and solving it,

It can be seen that the value of K_{d2} can be chosen arbitrarily. Choose K_{d2}=10 . We have chosen the desired pole polynomial as

$$s^5 + 26.4s^4 + 218.6s^3 + 871.3s^2 + 1721.4s + 1343.7 = 0 \quad (46)$$

The gains of the PID controller are obtained from as follows $Kp1 = 43.3$, Ki1=33.796, $K\,d1 =$ 2.254, kp2=120.9, ki2=247.3 and kd2=10.

4.2 Flow chart of the system

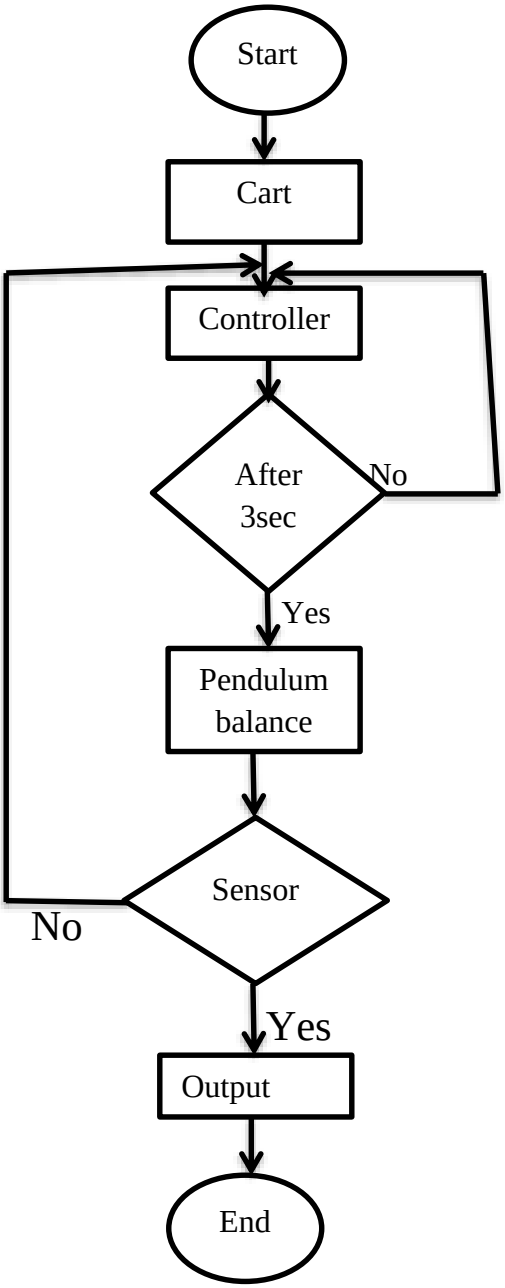


Figure 4. 2 flow chart

4.3 PID controller for the motor speed control of the Inverted pendulum

The building blocks of the system, namely how to drive the motor, how to measure its speed, and how to model the dynamics. The next steps involve putting the building blocks together to move up the control hierarchy, first to achieve motor speed control, then cart control, and finally the balancing pendulum. These final steps are most closely related to the theoretical content of a linear

control systems course. A brief overview of the modeling and control design is given here. Figure 4.3 is a basic feedback control system. The plant represents the system that is to be controlled, e.g. the motor, cart, or inverted pendulum. The output of this system is measured and subtracted from the desired reference input. This error signal *e*(*t*) is fed to the controller which then determines how to drive the plant so that the output matches the reference input. The input to the plant is *u*(*t*).

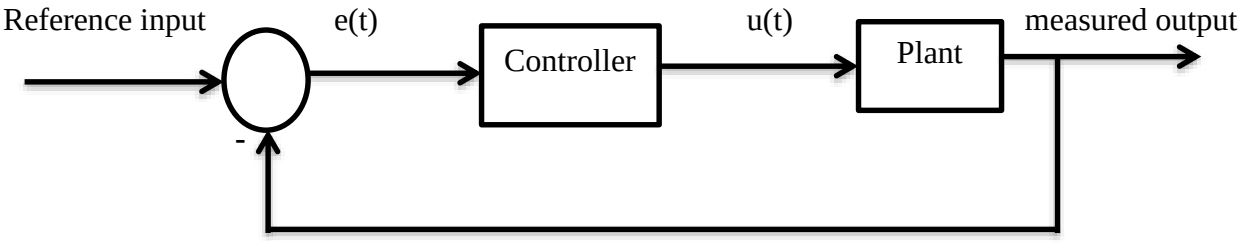


Figure 4. 3 Block diagram of basic control system

Figure 4.4 is a common closed loop system for motor speed control. In the figure, the plant is given by GM(s), represented in (8), which is derived from the first order dynamics of the motor as developed in the previous sections. The controller is represented by GC(s) and one possible form is that of a PID (proportional-integral-derivative) controller (9) whose time domain counterpart is (10).

$$Gm(s) = \frac{\omega(s)}{Va(s)} = \frac{\frac{Kt}{JmRa}}{S+\frac{bm}{Jm}+\frac{KtKm}{JmRa}} \qquad (8)$$

$$Gc(s) = \frac{KI+Kps+KdS^2}{S} \qquad (9)$$

$$U(t) = Kd\ \frac{de(t)}{dt} + Kpe(t) + KI\int e(t)dt \qquad (10)$$

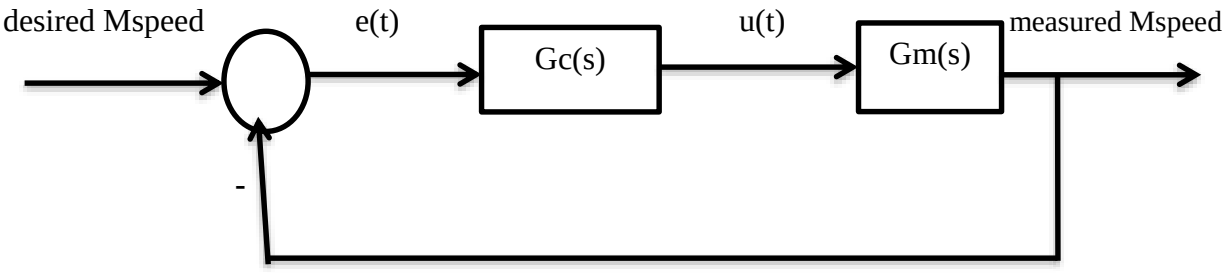


Figure 4. 4 Block diagram of the motor feedback system

In the above model, the input to the plant is V_a, a D.C. voltage and the output is the rotational velocity of the motor shaft. With the closed loop motor controller designed, the next step up in the hierarchy is cart control.

4.4 PID controller for the speed control of the Inverted pendulum

The physical representation of the cart control system is in Figure 4..4. The input to the plant is the force applied to the car F_c and the output is the car's speed, v_c.

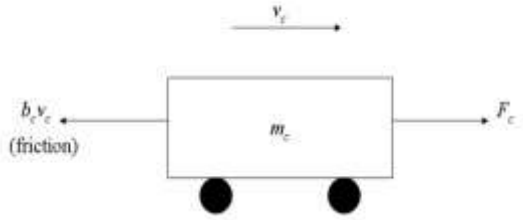


Figure 4. 4Physical representation of the cart control system

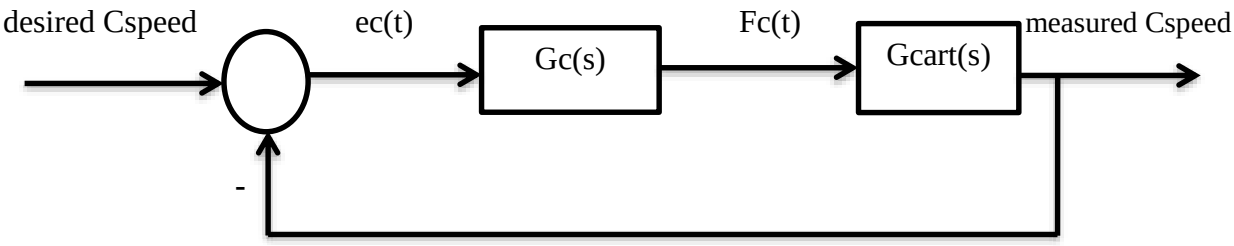


Figure 4. 5 Signals block diagram of the cart control system

The dynamics of the plant are (11), the PID controller is (12), with the time domain counterpart (13). Notice that the controller (13) takes the same form as (10). At the abstract level the cart control design is the same as for the motor speed control.

$$Gcart(s) = \frac{\frac{(ml^2+I)s^2 - bgml}{q}}{s^4 + \frac{b(ml+I)}{q}s^3 + b\frac{(M+m)gml}{q}s^2 + \frac{bmgl}{q}s} \quad (11)$$

$$\text{Gc(s)} = \frac{KIc + Kpcs + KdcS^2}{S} \quad (12)$$

$$\text{Fc(t)} = \text{Kdc}\ \frac{\text{dec(t)}}{\text{dt}} + \text{Kpcec(t)} + \text{Kic}\int \text{ec(t)dt} \quad (13)$$

4.5 PID controller for the angle control of the Inverted pendulum

Finally, consider the inverted pendulum. As before, conversions must be applied in order to get the signals into the correct representations for this model. The control signal F_c must be converted to motor speed to move down in the hierarchy and act as the desired reference input for the motor controller.

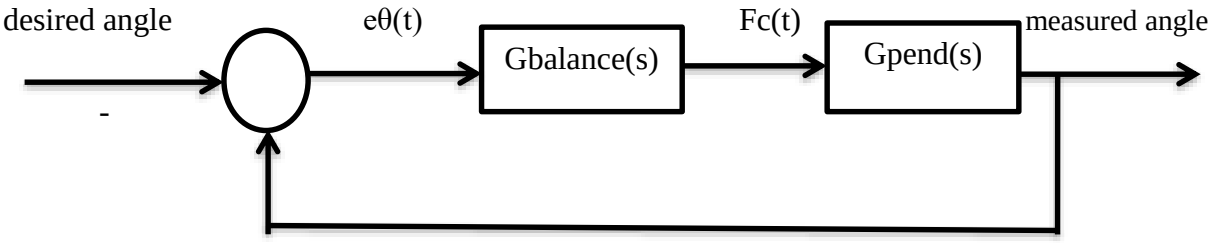


Figure 4. 6 block diagram of angle inverted pendulum system

The plant input is the force applied to the car *Fc* and the output is the pendulum angle θ*p*. The plant model is equation (14). Control design for this system can be treated in exactly the same way as the motor speed control and cruise control. The error signal is used to determine the plant input.

$$\text{Gpend(s)} = \frac{\theta(s)}{F(s)} = \frac{\frac{ml}{q}s^2}{s^4 - \frac{b(ml^2+I)}{q}s^3 - \frac{(M+m)gml}{q}s^2 - \frac{bmgl}{q}s} \quad (14)$$

4.7 Circuit diagram of cart speed control

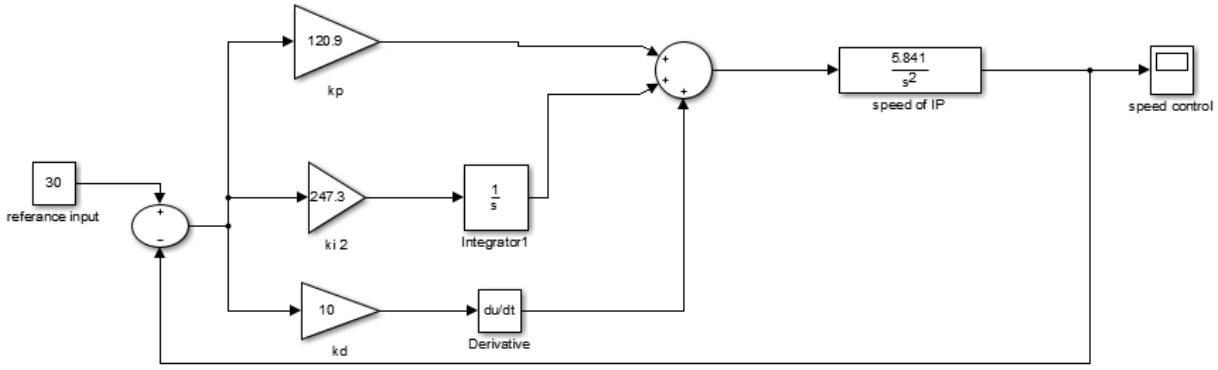


Figure 4. 7 Circuit diagram of speed control

4.8 Circuit diagram of control pendulum angle

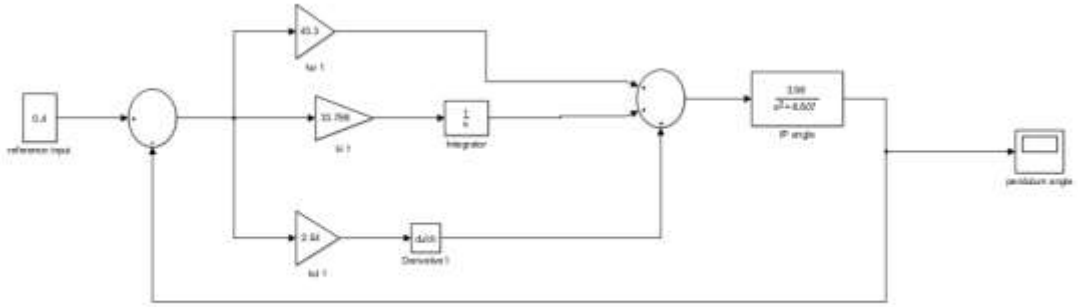


Figure 4. 8 Circuit diagram of pendulum angle

4.9 Circuit diagram of motor speed control

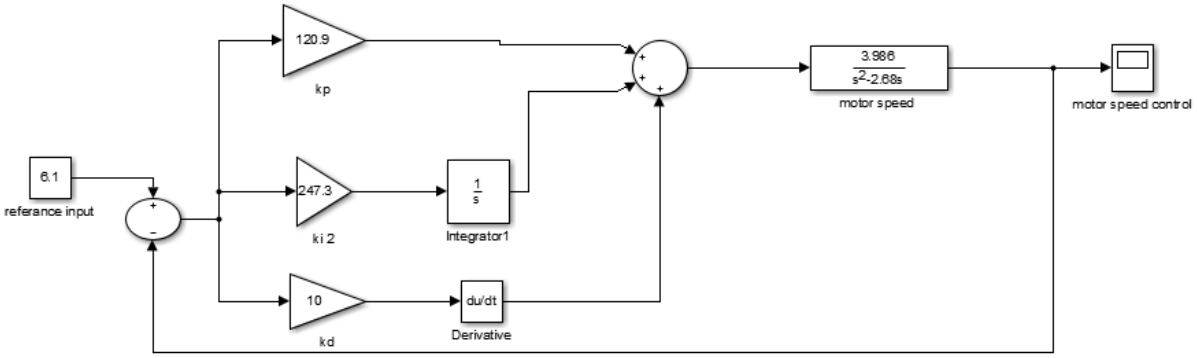


Figure 4. 9 Circuit diagram of motor speed control

Chapter 5. Result and Discussion

5.1 PID characteristics of speed controller

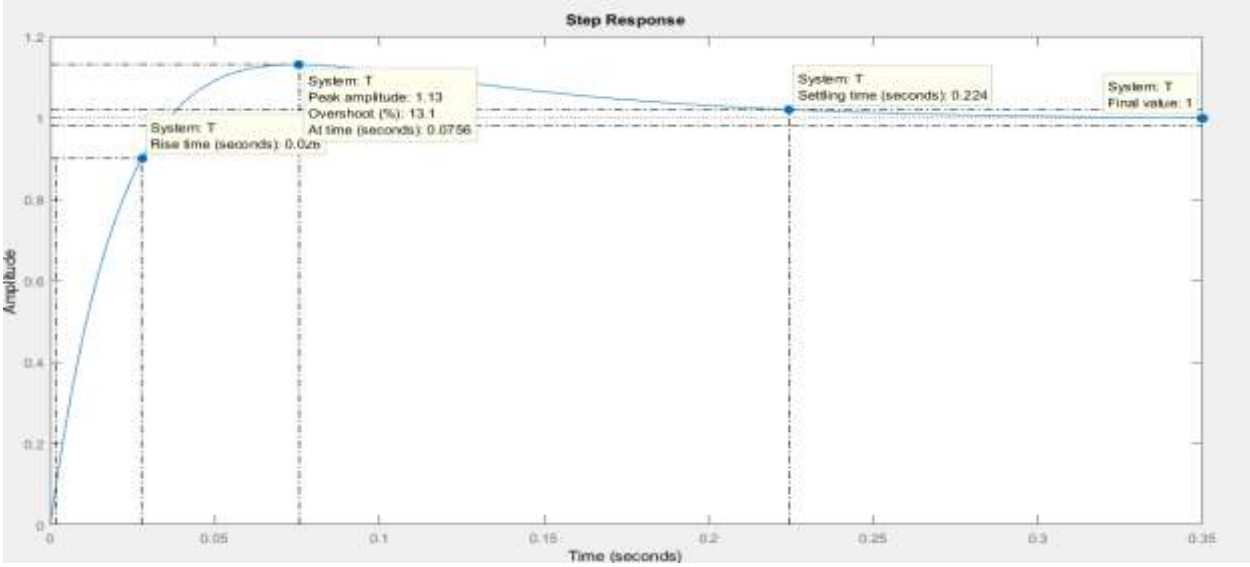


Figure 5. 1 PID characteristics of speed controller

5.2 PID characteristics of pendulum angle

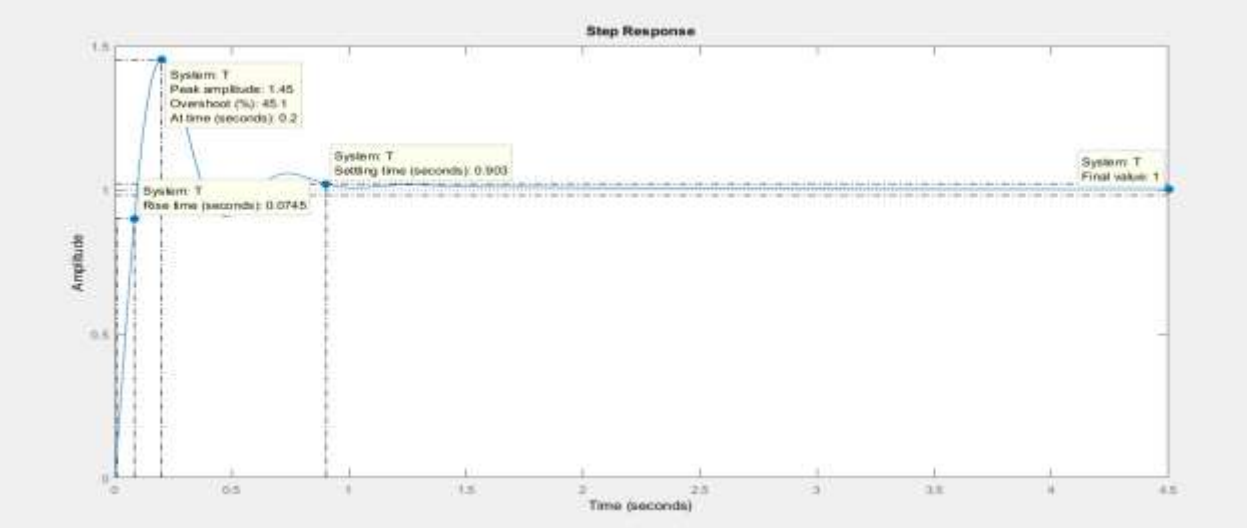


Figure 5. 2 PID characteristics of angle

5.3 Motor Speed control of the inverted pendulum

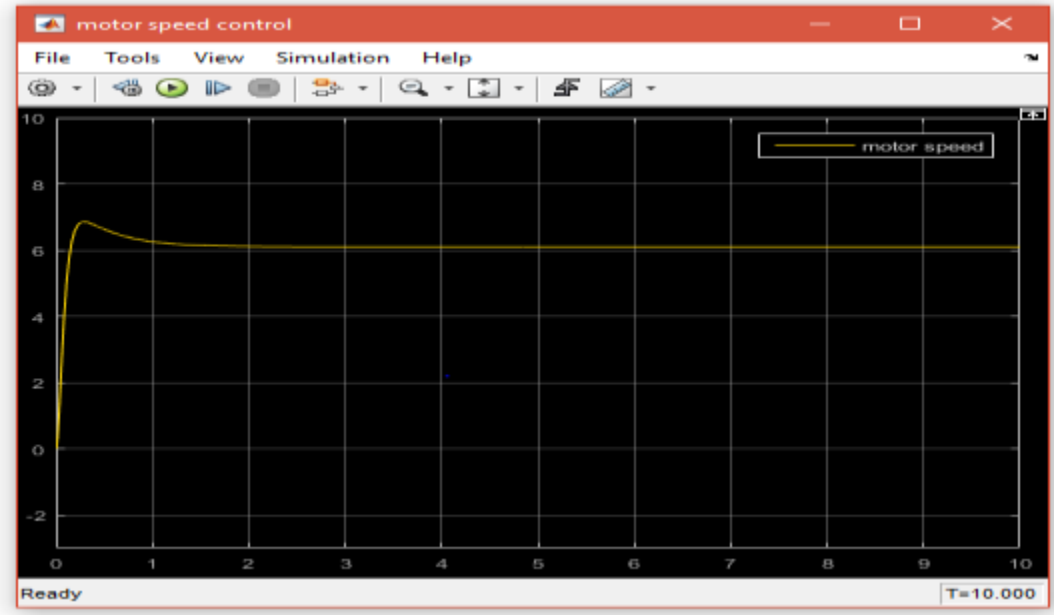


Figure 5. 3 motor speed control

5.4 Speed control of the inverted pendulum

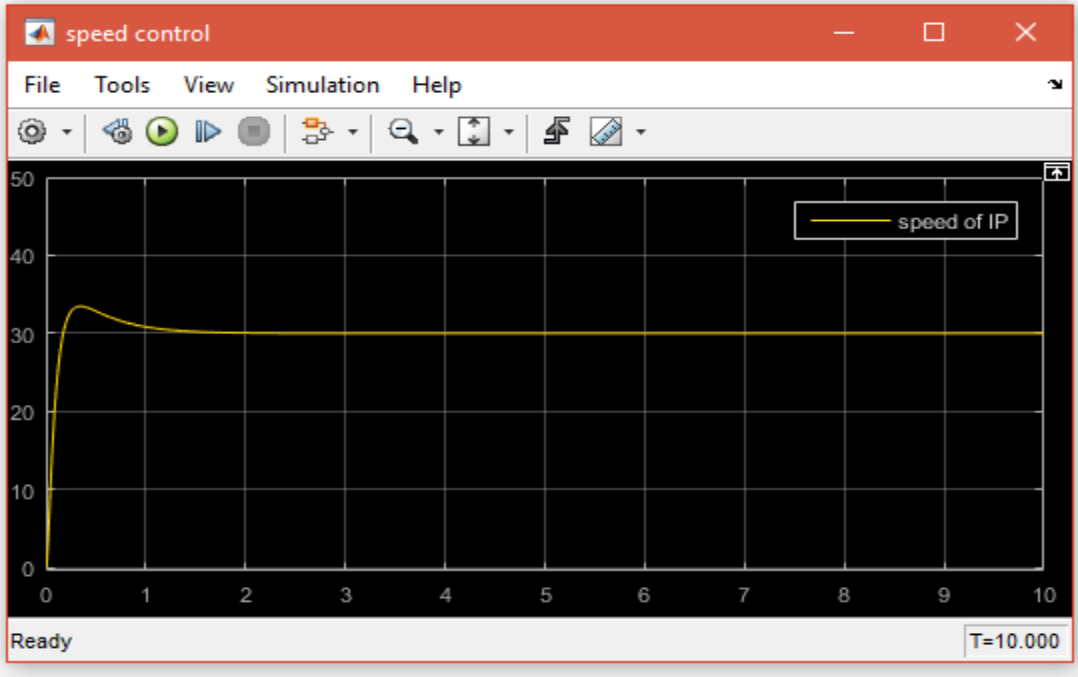


Figure 5. 4 simulation of speed controller

5.5 Angle control of the inverted pendulum

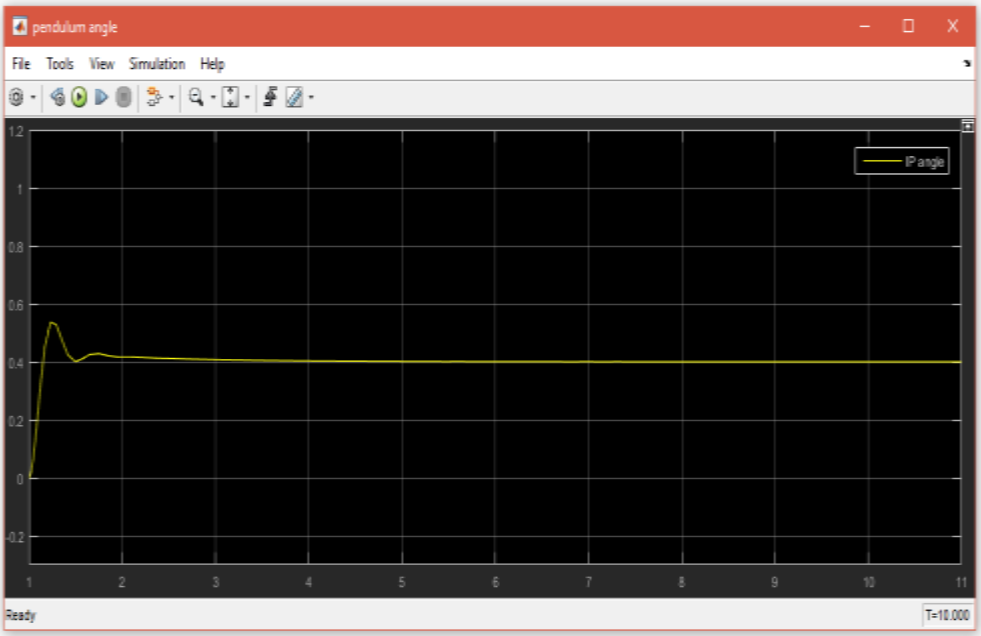


Figure 5. 5 simulation of pendulum angle

5.6 Discussion

Fig.5.4 gives simulation result (initial angle of 0.4 rad), Figure 5.3 gives the simulation result for the cart-inverted pendulum system and fig 5.2 gives motor speed control system. The requirements to keep the inverted pendulum firstly control motor speed, control speed of inverted pendulum and lastly balance pendulum angle. Only an unexpected disturbance occurred, the resonance in the inverted pendulum. Since it cannot easily be concluded where this resonance in the system is coming from, a plastic pendulum with a certain weight on the system. With this pendulum the weight is on the end of the pendulum, while the weight of the pendulum is considered to be around the center of the pendulum. Therefore, actually for the plastic pendulum different models have to be obtained. This way the values for the PID controller corresponding to this inverted pendulum can be obtained

Furthermore, further research to the DC-motor used in this project could be done. It is complex to conclude what type of DC-motor is used; it could become more clear what type of DC-motor exactly is used. Besides this, also the motor constant and the ohmic resistance can be obtained. In this report just the general values are used, since these values are of minor influence compared to the overall system and the mechanical part of the system in specific.

Lastly, besides a PID-controller also a model-based controller can be designed. The advantage of a model-based controller compared to a PID-controller is that when designing a model-based controller probably both the angle of the pendulum and the speed of inverted pendulum are controlled using one controller, where with the PID-controller two separate controllers have to be used.

Chapter 6. Conclusion and Recommendations

6.1 Conclusion

Two-wheeled inverted pendulums are systems that can be stabilized by simple controllers, such as a PID feedback controller. This approach is satisfactory in conditions where the disturbance forces are negligible and the system parameters are fixed. The human transporter is a two-wheeled inverted pendulum that has been commercialized for its use in public spaces. In these spaces, there are numerous factors that can interfere with the performance of the balancing controller and can cause safety hazards to its user and the surrounding environment. Furthermore, operators with variable inertia parameters and time-varying actions make the system deviate considerably from simple linear dynamical behavior. In this project, a dynamic model of a two-wheeled inverted pendulum carrying a human rider was developed. The literature review showed that many control techniques have been applied to the stabilization problem of two-wheeled inverted pendulums.

The mass properties of two-wheeled inverted pendulum were used to produce the dynamic equations of motion of the system. Following, a rigid-body model of a human rider was defined and added to the model to produce the equations of motion of a two-wheeled inverted-pendulum human transporter.

6.2 Recommendation

The work presented here can be extended in several different directions. The dynamic model presented here could be extended in order to account for an active human rider. experimental testing can be carried out to track the human motion on different roles such as accelerating, decelerating, and turning. Markers can be placed on different segments of the person operating the vehicle. A more complex human model, in which each body segment is modeled as one rigid body could be developed. Then, the motion tracking data can be used to command the movements of the model person in the simulation.

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Appendix

1. Matlab code for PID characteristics of speed

```
>> clc
syms s
%G=5.84/s^2
num=(5.84);
den=sym2poly(s^2);
G=tf(num,den);
H=1;
kp=120.9;
ki=247.3;
kd=10;
C=pid(kp,ki,kd);
T=feedback(C*G,H);
step(T)
```

apendex2

2.Matlab code for PID characteristics of angle

```
>> clc
syms s
%G=3.96/s^2-6.807
num=(3.96);
den=sym2poly(s^2-6.807);
G=tf(num,den);
H=0.4;
kp=43.3;
```

```
ki=33.796;

kd=2.54;

C=pid(kp,ki,kd);

T=feedback(C*G,H);

step(T)
```