WOLKITE UNIVERSITY

College of Engineering and Technology

Department of Electrical and Computer Engineering

DESIGN AND CONTROL OF QUADCOPER USING PID CONTROLLER

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A Final-year Project report submitted for the partial fulfillment of the requirement for Bachlor of Science degree in Electrical and computer Engineering

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Declaration

We here by certify that the work titled by Design and control of a quadcopter using PID controller which submitted to Wolkite University Department of Electrical and Computer Engineering is an authentic record of the work done by Hawi Aboma , Mahlet Alemayehu and Megertu Abebe under the supervision of Mr. Misker Shumeye (Msc). All the sources of the materials used by our work are properly refered.

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Abstract

Quadracopter is a flying object, which flies with a help by four propellers. The objective of this project work is to design, model and control the position and altitude of a quadcopter. To fulfill this objective, a mathematical model of the quadcopter has been developed. The simulation of the model and controller design is developed in MATLAB/Simulink environment. We operates with a simplified mathematical model representation of the quadcopter and modeling of the intended system. The designed a linearized simulation model for a Quadcopter and design a PID controller for the Quadcopter. In order to design the controller, the modeling of the brushless DC motors (BLDCM) which are responsible of the quadcopter motion is done by obtain the relationship between the control input U and the four-motor speed, first through the motor speed and control of four forces between the conversion relationship, a given motor speed corresponding to the control input U_i and obtain that thrust is proportional to the square of angular velocity of the motor.

Linearization refers to finding the linear approximation and stability of an equilibrium point of a system of non linear to a function. The position models is to control the quadcopter around a hovering state, the models can be linearized around the hovering state and assumed that all the linear velocities and all the angular velocities are equal to zero. During control height our system have no maximum overshoot, 3.5se of settling time, rise time is 3se and during control position our stytem performance also with minimum of maximum over shoot (to usually 25%) approximated zero steady state error, 7.5se settling time. The designed controller is setted the value at a specified level and simulation results are discussed.

Keywords: Brushless DC motor, PID Control, Quadrotor

Acknowledgement

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Contents

List of Figures

List of Tables

Abbreviations

AC ……………………………………………………………..Alternating Current

BLDC ………………………………………………………… …Brushless Direct Current

DC ……………………………………………………………..Direct Current

DOF ……………………………………………………………..Degree-Of-Freedom

ECM ……………………………………………………………..Electronically commutated motor

ESC …………………………………………………………… Electronic Speed control

MIMO ……………………………………………………………Multi-Input Multi-Output

ODE ……………………………………………………………Ordinery Diffirential Equation

PID……………………………………………………………..Proportional-Integral-Diffirential

PMSM ……………………………………………………………Permanent Magnet Synchronous Motor

RPM ………………………………………………………………Revolution per minute

UAV ……………………………………………………………Unmanned Aerial Vehicle

Symbols

b ………………………………………………………………..thrust cofficient

d ………………………………………………………………..drag-coefficient

F ……………………………………………………………………… force (N)

g ……………………………………………………………………… acceleration due to Gravity (m·s-2)

I ……………………………………………………………………… Identity-matrix

Ixx ……………………………………………… rotational inertia about the x-axis respectively (N·m·s^2)

Iyy………………………………………………rotational inertia about the y-axis respectively (N·m·s^2)

Izz ………………………………………………rotational inertia about the z-axis respectively (N·m· s^2)

J_r ……………………………………………………………………… motor's moment of inertia (kg·m2)

k_e ………………………………………………………………………motor's back-EMF constant

kd ……………………………………………………………………… Derivative Gain

ki ……………………………………………………………………… Integral Gain

kp ……………………………………………………………………… Proportional Gain.

l ……………………………………………… distance from center of the chassis-frame and the motor (m)

m ………………………………………………………………………mass of quadcopter (kg)

R ……………………………………………………………………… Rotation matrix

Rx ………………………………………………………………………Rotation of φ around the x-axis

Ry ………………………………………………………………………Rotation of θ around the y-axis

Rz ………………………………………………………………………Rotation of ψ around the z-axis

T ………………………………………………………………………Thrust (N)

U ………………………………………………………………………Control input to rotor

V ………………………………………………………………input voltage to quadcopter motor (volts)

X ………………………………………………………………x-axis of the quadcopter

x' ………………………………………………………………velocity of quadcopter along X-axis

x'' ………………………………………………………………acceleration of quadcopter along X-axis

Y………………………………………………………………y-axis of the quadcopter

y' ……………………………………………………………… velocity of quadcopter along Y-axis

y''………………………………………………………………acceleration of quadcopter along Y-axis

z ………………………………………………………………z-axis of the quadcopter

z' ……………………………………………………………… velocity of quadcopter along Z-axis

z''……………………………………………………………… acceleration of quadcopter along Z-axis

τ ………………………………………………………………load-torque (N·m)

ω ………………………………………………………………angular-velocity (rad·s-1)

$\dot{x}$ ……………………………………………………………… rate of change of a particular state

ψ ……………………………………………………………… yaw-angle (rad)

ϕ ………………………………………………………………roll-angle (rad)

θ ……………………………………………………………… pitch-angle (rad

Ω ……………………………………………………………… angular-velocity of motor (rad·s-1)

Chapter one

1 Introduction

1.1 Background

Quadcopter is a device with the ability to fly; this device is equipped with controllers that can change its altitude and direction by taking control to the four rotating blades. Quadracopters are with us almost one century. First experiments on them started at the beginning 20th century. The first functional quadracopter was built in year 1920 by Etienne Oehmichen. At first were created devices that should carry a weight of human body. Actual trend is creation of small unmanned quadcopters. The main reasons for this situation are ability to easy control and maneuverability. Quadcopters are used by enthusiastic modellers, but also find an application in the professional sphere, such as the police or the army. Over the last few years we have seen a massive growth in the manufacture and sales of remote control airborne vehicles known as Quadcopters. These Unmanned Aerial Vehicles have four arms and fixed pitch propellers which are set in an X or + configuration. [1 2]

They are sometimes referred to as Drones, Quadrotors or Quadcopters. Two opposite propellers rotates in one direction, for take-off. First pair opposite propellers rotates in one direction for keeping balance in the X axis. Second pair opposite propellers rotates in opposite direction, for keeping balance in the Y axis. The main reason for opposite rotations of opposite pairs it's the elimination a rotation the quadracopter in the Z axis[2 3]. The rotorcraft UAVs pose a set of advantages compared to the fixed wing UAVs, such as hovering, vertical takeoff and landing and ag- gressive maneuvering. Within the family of the rotor- crafts, Unmanned Quadrotor Helicopters (UQHs) have gained increasing attention among scientists and engi- neers [4].

The Quadcopters is a simple format with very few moving parts and has rapidly becomes a favorite vehicle for remote control enthusiasts and is widely being used as an effective Aerial photographic platform. Hobbyists who understood the simplicity of the vehicle originally built a large majority’s of the Quadcopters. By adding four motors and four propellers to a lightweight frame constructed of lightwood, carbon fiber, or fiberglass then connecting it to a remote control transmitter via a small control board fitted with a gyroscopic stabilization system and connected to a Lipo battery, these craft were relatively simple to construct. The quadracopter needs relatively a high thrust motor. For this purpose, brushless DC motors are good choice. The thrust is continuously controlled with PWM pulses. Generated PWM pulses for motors are from output ports of a kit brought on an ESC (Electronics Speed Control) of each motors.[4 5 7]

1.2 Statement problem

Quadcopter control is a fundamentally difficult and interesting problem. With six degrees of freedom (three translational and three rotational) and only four independent inputs (rotor speeds), quadcopter are severely under actuated. In order to achieve six degrees of freedom, rotational and translational motions are coupled. The resulting dynamics are highly nonlinear, especially after accounting for the complicated aerodynamic effects.

Various control algorithms have already been proposed, however, selection among these algorithms requires accounting for the drawbacks of each control algorithm. In addition, unlike ground vehicles, quadcopters have very little friction to prevent their motion, so they must provide their own damping in order to stop moving and remain stable. Together, these factors create a very interesting control problem. We designed and solve the problem of stability and non lineanerity of quadcopter alttitude and position using PID controller. The PID controller has the advantage and the reasons of this success are mainly three

- simple structure,
- good performance for several processes,
- tunable even without a specific model of the controlled system.

1.2.1 Motivation of the project

The motivation behind choosing quadcopter is that the study of quadcopter model allows the application of many control engineering principles and have several advantages to Quadcopter over comparable- scaled helicopters. However, In Ethiopia the use of quadcopter does not known well and they not applicable in it application area. First, Quadcopter do not require mechanical linkages to vary the rotor blade pitch angle as they spin. This simplifies the design and maintenance of the vehicle. Secondly, the use of four rotors allows each individual rotor to have a smaller diameter than the equivalent helicopter rotor, allowing them to possess less kinetic energy during flight. This reduces the damage caused should the rotors hit anything. For small-scale UAVs this makes the vehicle safer for close interaction. Some small-scale Quadcopter have frames that enclose the rotors, permitting flights through more challenging environments, with lower risk of damaging the vehicle or its surroundings [5]. The technical challenges in UAV modelling and control in various tangled situations and the absence of good arrangements was extremely persuading.

1.2.2 Significance of the project

The design and control quadcopter model will offer a good flying system to the people who use quad in their wanted technology. As the technology has matured and become more mainstream, a number of practical and very interesting uses of Drone technology have emerged. Even though the practical implementation is our future work, Simplifies the design, maintenance of the vehicle, low-cost and small size device which provides simplicity when operating and design simulation first important. In this work the system performance is incease, ease to control reduce risk.

1.3 Objective of project

1.3.1 General objective

- To design and simulate a position and altitude PID controllers for Quadcopter in MATLAB/ Simulink.

1.3.2 Specific objective

- To model a Quadcopter in MATLAB/ Simulink.
- To design an alttitude PID controller for the modelled Quadcopter.
- To design a position PID controller for the modelled Quadcopter.
- To simulate and discuss the results of the designed controllers.

1.4 Methodology

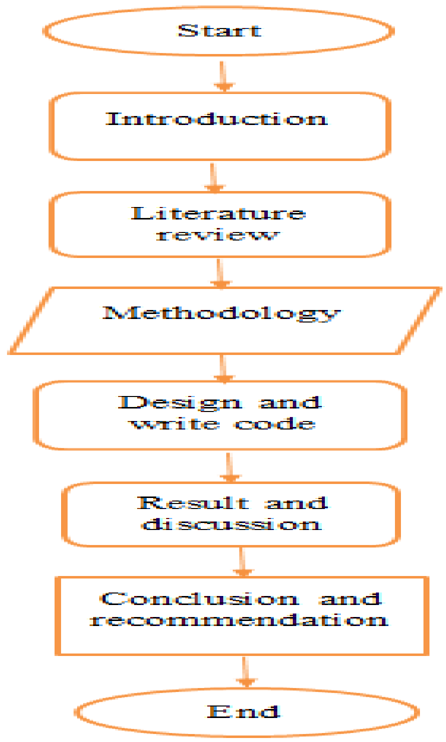

Figure 1-1 method of system

1.5 Scope and Limitations of the project

The scope of this final-year project is to design PID controllers for the control of the altitude and position of a quadcopter during hovering in a MATLAB/ Simulink environment. Practical implementation is not involved in this work but is recommended for the furture work following this. The position and velocity of the quadcopter is estimated in a Cartesian coordinate system. This results in a limited range before the Cartesian estimates starts to deviate from the actual position. Additionally, the area where the quadcopter operates will be flat with little or no inclination. The angle controllers assumed that the quadcopter is close to hovering. This assumption is kept in this project.

1.6 Organization of the report

We organized our project document by starting expain the introduction about the Quadcopter is given followed by the advantages of the Quadcopter over comparable scaled helicopter and the application of Quadcopter in day to day life in chapter 1 and chapter 2, contain about the related project in past as Literature review. In Chapter 3, theoretical and mathematical modeling of the quadcopter is performed. In chapter 4, a brief note on the PID controller is given. The effect of each parameters of the PID controller is defined using a table the simulation of the model for the PID controller is done and the performance of the system for the controller is studied. Chapter 5: this chapter discusses the system simulation results while achieving comparison between linearized and nonlinear system. Furthermore, the real system is tested using experimental testbench and its response is compared to the simulated response. Chapter 6: In this chapter, a summary of the work done in the previous chapter is given as well as explanation of the system limitations and recommendation of some future work. When developing the simulation model, first a detailed theoretical description of the problem is studied followed by a paper calculation for the model, later followed by simulation analysis for designing a PID controller.

Chapter two

2 Literature review

In this chapter, previous projects related to design and control of quadcopter are discussed. Generally quad-rotor carries several uncertainties, such as air disturbances and mainly due to nonlinearity of the system may remarkably disturb the flight and lead to undesired movement. To avoid these types of movements a suitable control algorithm is required for attitude and altitude control for hover. However In the recent years, altitude and attitude controlling of quadrotor has remained an issue due to the constraints and unstable kinematics and dynamics. Some of the techniques already have been developed for these controls mentioned.

Gotbolt *et al*. (2013) used Model based PID control for Autopilot design for Helicopter, this article only focused on software framework optimization. The PD control technique was used for tilt rotor UAV stabilization in year 2012 (Chowdhury *et al*., 2012). In the year 2011 active disturbance rejection controller was proposed by Hua *et al*. (2011) only for attitude controlling of quad-rotor. H-inf control technique was introduced by Jiao *et al*. (2010) for trajectory tracking of quad-rotor.

Robert Mahony presents a comprehensive method for modeling, orientation, and control of a quadcopter with state space methods [17]. Another example of advance control is where researchers at University of Zurich implemented a quadcopter with a model predictive control were able to perform extreme acrobatic maneuvers [18],[19]. Numerous other control techniques have been applied to quadcopters as well such as PID, LQR and L1 [20], [21].

With regards to open source multi-rotor systems, Open Pilot and Clean Flight are perhaps two of the most popular open software ight controller systems [16], These frameworks support a broad range of multi-rotor vehicles from tri-copters to octo-copters. With regards to open source software and hardware systems, the Pixhawk and Sparky systems feature an open source ight controllers [14]. While these systems feature some open hardware and software, they integrate with systems that are proprietary.

Davendra Chaturvedi (2010). use understandable system descriptions and sensor models as a basis to design configurable estimators and controllers, and to build a quadcopter well suited for educational purposes; as well as aiding to more advanced control in the future. The system consists of several components for necessary sensor input, a radio transmitter, Windows user interface and an Arduino microcontroller. All filtering of signals, estimation of system states, calculation of control inputs and communication handling is done on the microcontroller, while the Windows application allows the user to command various actions. [10]

David Riches Emmanuel Akkidas 25 Mar 2016. Quadcopter can achieve vertical flight in a stable manner and be used to monitor or collect data in a specific region such as mapping terrains. Technological advances have reduced the cost and increase the performance of the low power microcontrollers that allowed the general public to develop their own Quadcopter. The goal of this project is to build, modify, and make improvements in Quadcopter design to obtain stable flight, gather and store CO_2 data. The project used a Quadcopter that included a frame, motors, electronic speed controllers, Arduino development board, and sensor boards. Batteries, a transmitter, a receiver, a GPS module, and Sim card were interfaced with the Quadcopters frame. The aim of this project was to build and program a Quadcopter that can be used to collect Co2 information of a surrounding area.[12]

Chapter three

3 Quadcopter Mathematical and Dynamic Model

3.1 Mathematical model

In our project, a quadcopter was first modeled mathematically, after which a simulation was carried out in MATLAB by designing a PID controller, which was applied to the mathematical model. The PID controller parameters were then applied to the designed block on MATLAB simulink. finally, the output of the simulation and the result of simulation is display by using graphical response of system on scope were compared at diffirent value.

In this project, position and altitude dynamics of a quadcopter were considered. Therefore, roll, pitch, yaw, and altitude dynamics were modeled.The mathematical model of the Quadcopter describes its dynamics in a simplified manner. If we model all the effects on the behavior of Quadcopter during the flight, the model would be considerably more complicated resulting in more complex simulation. The basic structure of Quadcopter along with the world coordinate system, the coordinate Quadcopter system, angular velocity directions of each rotor, and the torque and tension forces generated by rotors.

Designing a control system for physical systems is commonly started by building a mathematical model. The model is very important because it gives an explanation of how the system acts to the inputs given to it. In this project, the mathematical model equations of motion are derived using a full quadcopter with body axes.

3.2 Body axes system

The quad copter made of four arms set as its frame. Each of the two are set symmetrically and perpendicularly to each other. On the end of each arm, symmetrically to center point of the device, actuators that provide flying are set. Each actuator consists of a motor and a blade called propeller, where the blade is attached to the rotor’s shaft. Rotation of these blades can lead to quad copter’s movement. In most of papers, the body axes orientation is along the arms of the quadcopter’s are as shown on the following figure. [8]

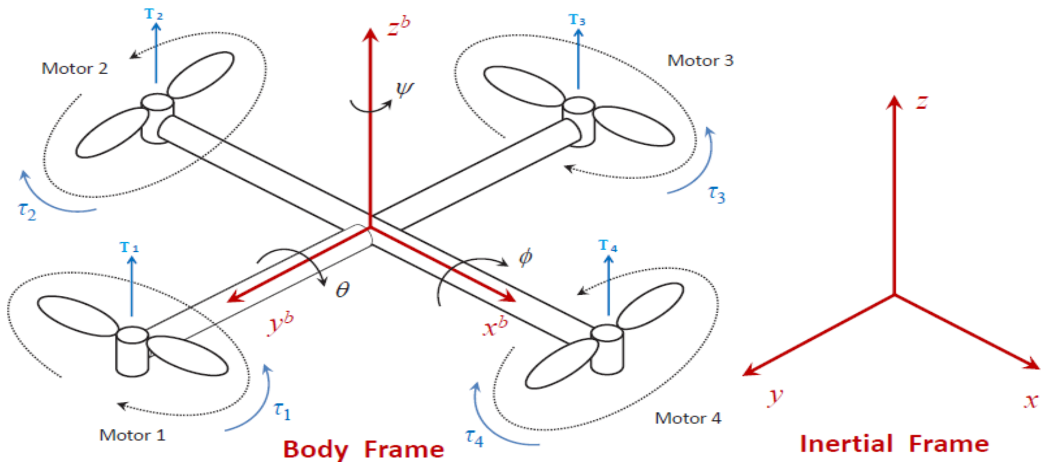

Figure 3-1 quadrotor schematic

However, Mokhtari and Benallegue have tried to model the vehicle with a different axes orientation. [7]

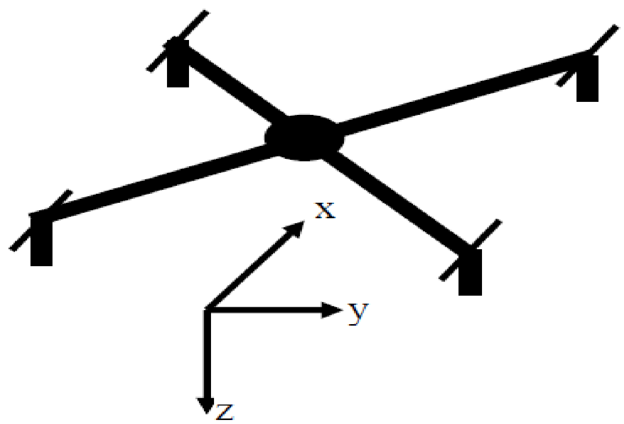

Figure 3-2 alternative orientation for the body axes system

As no comparison has been carried out between the two different axes orientation, we can't say which one is the better one. As this project uses the model realised in [8 9], we are going to work with the more widely used orientation, which means with x and y axes along the arms of the robot. Also, the body axes center is assumed to be at the same position as the center of gravity. Quadcopters are usually defined in spatial-orientation using a two reference frame systems, which are presented and defined as follows:

- Body frame is defined by ground, with gravity pointing in the negative Z-direction. The body-frame vectors describing the helicopter's linear & angular positions are generally represented as: translational velocities $[u, v, w]^T$ and rotational velocities $[p, q, r]^T$.

- Inertia (or Earth) frame is defined by the orientation of the quadcopter with its rotor axes pointing in the positive z-direction and the arms pointing in the X and Y-directions. Within this frame, position $[x, y, z]^T$ and attitude (roll, pitch & yaw) $[\phi, \theta, \psi]^T$ describes its linear & angular positioning.

Combination of these four vectors presented in the body and inertia frame represent the 12-states of the quadcopter. Quadcopters are described dynamically as a highly non-linear helicopter modelled with the following attributes:

- It is a 12-states helicopter (six attitude-states and six position & linear velocity states)
- It possesses 6-DOF (3-translational velocities and 3-rotational velocities)
- It is actuated by four independent rotors.

Inference on this description by various researchers concludes that the resulting helicopter's dynamics is a severely under-actuated and a highly nonlinear helicopter married with erratic aerodynamic uncertainties (because it control-inputs uses four rotors to control its 6-DOF). The four quadrotor targets are thus related to the four basic movements which allow the helicopter to reach a certain height and attitude. It follows the description of these basic movements. While quadcopters are capable of many forms of movements, most literature and publications focus mostly on the following movements:

3.2.1 Throttle (U1)

This command is provided by increasing (or decreasing) all the propeller speeds by the same amount. It leads to a vertical force for body-fixed frame which raises or lowers the quadrotor. If the helicopter is in horizontal position, the vertical direction of the inertial frame and that one of the body-fixed frame coincide. Otherwise the provided thrust generates both vertical and horizontal accelerations in the inertial frame.

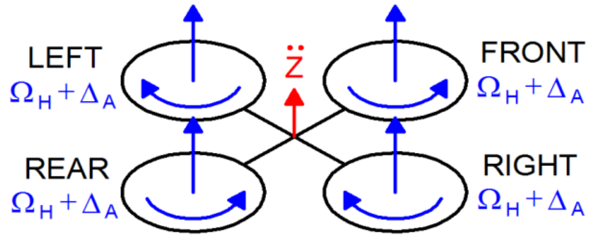

Figure 3-3 Throttle movement

3.2.2 Roll (U2 [N m])

It is provided by concurrently increasing or decreasing the left propellers speed and decreasing or increasing the right propellers speed at the same rate. It generates a torque with respect to the x axis which makes the quadcopter to tilt about the axis, thus creating a roll angle. The total vertical thrust is maintained as in hovering; thus this command leads only to a roll angular acceleration.

- Roll-motion corresponds to quadcopter's rotation about the Xb-axis. It is obtained when $\omega_2=\omega_4=\omega_{hove}$r and ω_1 and ω_3 are changed. For a positive roll, $\omega_1>\omega_{hover}$ and $\omega_3<\omega_{hover}$. A negative rolling action is produced when we set $\omega_1<\omega_{hover}$ and $\omega_3>\omega_{hover}$.

This command is provided by increasing (or decreasing) the left propeller speed and by decreasing (or increasing) the right one. It leads to a torque with respect to the x_B axis which makes the quadrotor turn. The overall vertical thrust is the same as in hovering, hence this command leads only to a roll angle acceleration (in first approximation)

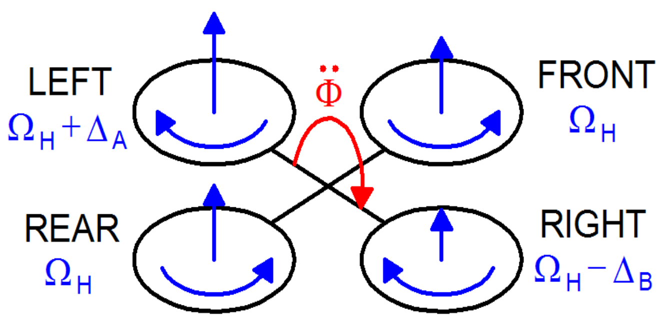

Figure 3-4 Roll movement

3.2.3 Pitch (U3 [N m])

The pitch and roll are very similar. It is provided by concurrently increasing or decreasing the speed of the rear propellers and by decreasing or increasing the speed of the front propellers at the same rate. This generates a torque with respect to the y axis which makes the quad-rotor to tilt about the same axis, thereby creating a pitch.

- Pitch-motion corresponds to the quadcopter's rotation about the Y_b-axis is obtained when $\omega_1=\omega_3=\omega_{hover}$ and ω_2 and ω_4 are altered. For a positive pitch, $\omega_2>\omega_{hover}$ and $\omega_4<\omega_{hover}$ while negative pitch action is obtained when $\omega_2<\omega_{hover}$ and $\omega_4>\omega_{hover}$.

This command is very similar to the roll and is provided by increasing (or decreasing) the rear propeller speed and by decreasing (or increasing) the front one. It leads to a torque with respect to the Y_b axis which makes the quadrotor turn. The overall vertical thrust is the same as in hovering, hence this command leads only to a pitch angle acceleration (in first approximation).

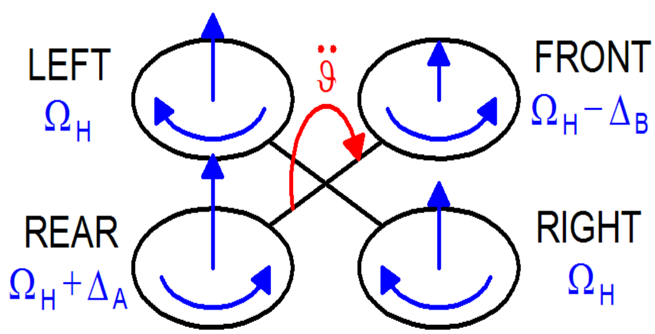

Figure 3-5 Pitch movement

3.2.4 Yaw (U4 [N m])

This command is provided by increasing (or decreasing) the opposite propellers' speed and by decreasing (or increasing) that of the other two propellers. It leads to a torque with respect to the z axis which makes the quadrotor turn clock wise or anti clock wise about the z-axis.

- The yaw motion corresponds to a rotation of the quadcopter about the Zb-axis. It is produced by the difference in the torque developed by each pair of rotors; with a pair creating a clockwise-torque whiles the other pair anticlockwise-torque. By varying the angular speed of one pair over the other, the net torque applied to the helicopter generates the yaw motion. A positive yaw action is obtained by setting (ω_1=ω_3)>ω_{hover} and (ω_2=ω_4) < ω_{hover}. A negative yaw action is achieved when (ω_1=ω_3) <ω_{hover} and (ω_2=ω_4)>ωhover.

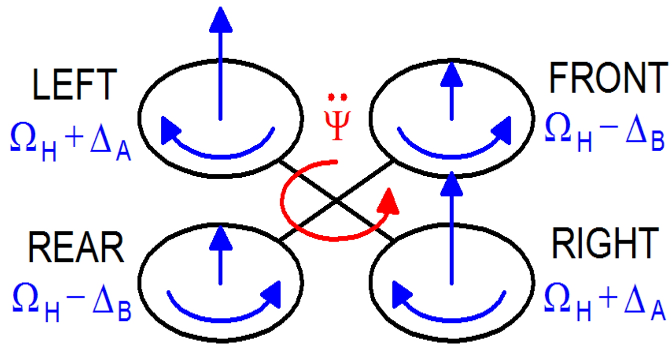

Figure 3-6 Yaw movement

- The vertical take of and landing motions (change in the *z*-frame) is obtained by equally augmenting or diminishing the angular speed of all motors with respect to ω_{hover} and hence the supply voltage. This lead to a vertical upwards (or downwards for decent) force with respect to body-fixed frame which raises or lowers the helicopter.

The modeling approaches of quadcopters are based on the physics of the system with further partitioning of the system into smaller subsystems for easy analysis, design and modeling [10]. Generally in quadcopter modeling, different assumption have been made by simplification of the mathematical equations needed for the helicopter modeling while still establishing a fairly accurate model as possible. The developed model in this work assumes the following:

- The structure is supposed rigid.
- The structure is supposed symmetrical.
- The center of gravity and the body fixed frame origin are assumed to coincide.
- The propellers are supposed rigid.
- Thrust and drag are proportional to the square of propeller's speed.

The way of modelling the quadrotor differs from the one used for fixed wing vehicle in the fact that we are not making the rotational transformations in the same order to go from the earth to body axes. Indeed, the most way is to carry out the final rotation of the earth to body transformation along the thrust direction [8]. Thus, we take for the body to earth transformation, the direction cosine matrix. Translation and rotation matrices are used to transform one coordinate reference frameinto another desired frame of reference.

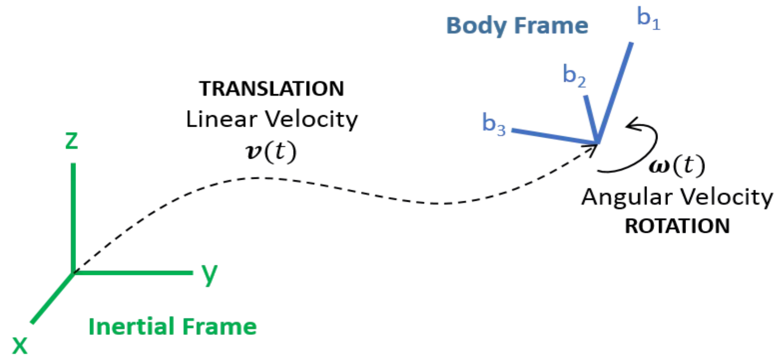

Figure 3-7 Transformation linear to angular

3.2.5 Euler Angles

According to Nelson (1998) it is convenient to define a body-fixed coordinate system [*X Y Z*] in the aircraft. In aeronautic applications, quantities such as acceleration, velocities, and angular rates are often, or at least partially, measured in relation to the aircraft. The body-fixed coordinate system can be used to relate measurements and estimations to the inertial system. In aeronautical applications this is commonly a North-East-Down (ned) coordinate system, with the *X* axis pointing fore, the *Y* axis pointing starboard and the *Z* axis pointing to the keel of the craft.

The rotational velocity of the aircraft is measured by the angular rates *p* around the *X* axis, *q* around the *Y* axis and *r* around the *Z* axis. The aircraft's velocities along the *X*, *Y*, and *Z* axes are denoted *u*, *v* and *w* respectively. To relate the orientation of the local (relating or restricted to a particular area) coordinate system relative to a global (relating to or embracing the whole of coordinate) one, the Euler angles roll (*Φ*), pitch (*θ*), and yaw (*Ψ*) can be used. Nelson (1998) describes the three rotations needed to transform the coordinate system [*x y z*] to the system [*X Y Z*]. Each rotation results in a new coordinate system and the two intermediate coordinate systems are denoted [*x' y' z'*] and [*x'' y'' z''*]. They are related through three successive rotations:

- Roll: Rotation of *Φ* around the x-axis **Rx**;
- Pitch: Rotation of θ around the y-axis **Ry**;
- Yaw: Rotation of *Ψ* around the z-axis **Rz**.

Switch between the coordinates system using rotation matrix **R** which is a combination of rotation about 3 mentioned rotations also Fig**.** shows how rotation is applied

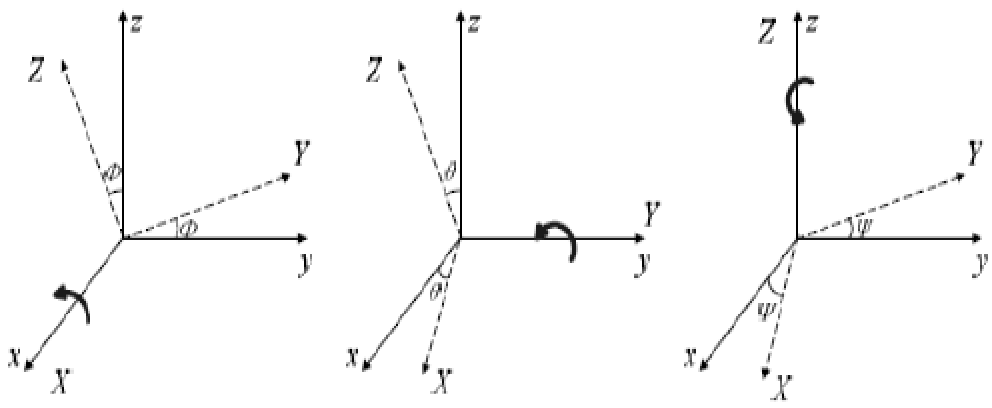

Figure 3-8 Rotations about X, Y, and Z axes.

The rotation matrix *R* is orthogonal thus $R^{-1} = R^T$ which is the rotation matrix from the inertial frame to the body frame.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\Phi & -sin\Phi \\ 0 & sin\Phi & cos\Phi \end{bmatrix} \tag{3.1}$$

$$R_y = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \tag{3.2}$$

$$R_z = \begin{bmatrix} cos\Psi & -sin\Psi & 0 \\ sin\Psi & cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.3}$$

$$\mathrm{R} = \mathrm{Rx.Ry.Rz} = \begin{bmatrix} c_\theta c_\psi & -c_\Phi s_\psi + s_\Phi s_\theta c_\psi & s_\Phi s_\psi + c_\Phi s_\theta c_\psi \\ c_\theta s_\psi & c_\Phi c_\psi + s_\Phi s_\theta s_\psi & -s_\Phi c_\psi + c_\Phi s_\theta s_\psi \\ -s_\theta & s_\Phi c_\theta & c_\Phi c_\theta \end{bmatrix} \tag{3.4}$$

The rotation matrix *R* is orthogonal thus $R^{-1} = R^T$ which is the rotation matrix from the inertial frame to the body frame.

$$R_T = \begin{bmatrix} c_\theta c\Psi & c_\theta s\Psi & -s_\theta \\ -c_\Phi\, s\Psi + s_\Phi s_\theta c\Psi & c_\Phi\, c\Psi + s_\Phi\, s_\theta s\Psi & s_\Phi\, c_\theta \\ s_\Phi\, s\Psi + c_\Phi\, s_\theta c\Psi & -s_\Phi\, c\Psi + c_\Phi\, s_\theta s\Psi & c_\Phi c_\theta \end{bmatrix} \tag{3.5}$$

Where:

Φ, θ, Ψ are the roll, pitch and yaw angle respectively

$s_\Psi = \sin(\Psi)$, $c_\Psi = \cos(\Psi)$, $t_\Phi = \tan(\Phi)$, et.c

Euler angular velocity and the body angular velocity conversion

Before establishing the kinetic model, we first define p, q, r as the angular velocity of the three axes in the body coordinate system; Φ˙, θ ≡, Ψ˙ are the three-axis Euler angular velocity in the ground coordinate system, The relationship between the angular velocity and the body angular velocity is:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \Phi' - \Psi'\sin\theta \\ \theta'\cos\Phi + \Psi\sin\Phi\cos\theta \\ -\theta'\sin\Phi + \Psi'\cos\theta\cos\Phi \end{bmatrix} \quad (3.6)$$

Note that the angular velocity vector ω != θ’. The angular velocity is a vector pointing along the axis of rotation, while θ˙ is just the time derivative of yaw, pitch and roll. To convert these angular velocities into the angular velocity vector, we can use the following relation:

$$\omega = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\Phi & \cos\theta\sin\Phi \\ 0 & -\sin\Phi & \cos\theta\cos\Phi \end{bmatrix} \quad (3.7)$$

where

ω is the angular velocity vector in the body frame

The general form of Newton-Euler equation is expressed as:

$$\begin{bmatrix} mI_{3x3} & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} V' \\ \omega' \end{Bmatrix} + \begin{Bmatrix} \omega * mv \\ w * I\omega \end{Bmatrix} = \begin{Bmatrix} F \\ \tau \end{Bmatrix} \quad (3.8)$$

This eq. is a generic form of equations of motion. It can be applied in any position in the coordinate system. In this case, the main point is the center of mass of the quadcopter. Regarding the main body of the quadcopter and the translational dynamics of the quadcopter in the body frame (B) is defined as:

$$m \begin{Bmatrix} \ddot{X} \\ \tilde{Y} \\ \check{Z} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} KT\omega i\hat{}2 \end{Bmatrix} - Rxyz \begin{Bmatrix} 0 \\ 0 \\ mg \end{Bmatrix} \quad (3.9)$$

$$= \text{Rxyz} \begin{Bmatrix} -\sin\theta\cos\Phi \sum_{i=1}^{4} \text{KT}\omega\text{i}^\wedge 2 \\ \sin\varphi \sum_{i=1}^{4} \text{KT}\omega\text{i}^\wedge 2 \\ -\text{mg} + \cos\theta\cos\Phi \sum_{i=1}^{4} \text{KT}\omega\text{i}^\wedge 2 \end{Bmatrix} \quad (3.10)$$

the main body rotational dynamics can be described in the body frame (B) as

$$\begin{bmatrix} I_{yy} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{Bmatrix} \Phi'' \\ \theta'' \\ \Psi'' \end{Bmatrix} + \begin{Bmatrix} \Phi' \\ \theta' \\ \Psi' \end{Bmatrix} x \begin{bmatrix} I_{yy} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{Bmatrix} \Phi' \\ \theta' \\ \Psi' \end{Bmatrix} = \begin{Bmatrix} \tau x \\ \tau y \\ \tau z \end{Bmatrix} \quad (3.11)$$

$$\begin{Bmatrix} Ixx\Phi'' \\ Iyy\theta'' \\ Izz\Psi'' \end{Bmatrix} + \begin{Bmatrix} (Izz - Iyy)\theta'\Psi' \\ (Ixx - Izz)\Phi'\Psi' \\ (Iyy - Ixx)\Phi'\theta' \end{Bmatrix} = \begin{Bmatrix} \tau x \\ \tau y \\ \tau z \end{Bmatrix} e + \begin{Bmatrix} \tau x \\ \tau y \\ \tau z \end{Bmatrix} r \quad (3.12)$$

Where subscripts and refer to moments due to external forces, which ultimately caused by thrust and drag from rotors, and moments due to rotor gyro effect.

3.3 Rotor Dynamics

The rotor dynamics can be described using by considering coordinate system of each rotor, which is the same plane as the body frame for X and Y axes while Z axis coincides with rotation of the rotor. Considering rotational dynamics of each rotor in the form of Newton-Euler:

$$\begin{bmatrix} I_{yy} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} r,i \begin{Bmatrix} \Phi'' \\ \theta'' \\ \Psi'' \end{Bmatrix} r,i + \begin{Bmatrix} \Phi' \\ \theta' \\ \Psi' \end{Bmatrix} r,i\ x \begin{bmatrix} I_{yy} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} r,i \begin{Bmatrix} \Phi' \\ \theta' \\ \Psi' \end{Bmatrix} r,i = \begin{Bmatrix} \tau x \\ \tau y \\ \tau z \end{Bmatrix} r,i$$

$$\begin{Bmatrix} Ixx\Phi'' \\ Iyy\theta'' \\ Izz\Psi'' \end{Bmatrix} r,i + \begin{Bmatrix} (Izz - Iyy)\theta'\Psi' \\ (Ixx - Izz)\Phi'\Psi' \\ (Iyy - Ixx)\Phi'\theta' \end{Bmatrix} r,i = \begin{Bmatrix} \tau x \\ \tau y \\ \tau z \end{Bmatrix} r,i \quad (3.13)$$

for i= 1,2,3,4 and denotes i^{the} rotor. Since the rotors always rotate about their Z-axes at the rate of Ω with the moment of inertia about Z-axis of J_r and have very low masses I*xx*, and I*yy* can then be omitted and the dynamics of each rotor reduces to:

$$\begin{Bmatrix} 0 \\ 0 \\ Jr\Omega' i \end{Bmatrix} + \begin{Bmatrix} \theta'\omega Jr \\ -\Phi'\omega Jr \\ 0 \end{Bmatrix} = \begin{Bmatrix} \tau x \\ \tau y \\ \tau z \end{Bmatrix} r,i \quad (3.14)$$

Note that Eq. (15) is a function of rotor speed Ω. Since rotor 1 and 3 rotate in the opposite direction of rotor 2 and 4, one can define the total rotor speed as:

$\Omega = \omega 1 - \omega 2 + \omega 3 - \omega 4$ (3.15)

From Eq. (14) and (15), the total moment due to gyro effect from all rotors can be expressed as:

$$\begin{Bmatrix} \theta'\omega Jr \\ -\Phi'\omega Jr \\ 0 \end{Bmatrix} = \begin{Bmatrix} \tau x \\ \tau y \\ \tau z \end{Bmatrix} r \quad (3.16)$$

3.4 Equation of Motion (EoM)

No Now that all necessary dynamics of the entire model has been established, one can write the complete equations of motion of the quadrotor yields:

$$\begin{Bmatrix} translational \begin{Bmatrix} m\ddot{X} = -\sin\theta\cos\Phi \sum_{i=1}^{4} KT\ \omega_i^{\,2} \\ m\tilde{Y} = \sin\varphi \sum_{i=1}^{4} KT\ \omega_i^{\,2} \\ m\check{Z} - mg + \cos\theta\cos\Phi \sum_{i=1}^{4} KT\ \omega_i^{\,2} \end{Bmatrix} \\ Rotational \begin{Bmatrix} Ixx\Phi'' = \Psi'\,\theta'(Izz - Iyy) - \theta'\omega rJr + bl(-\ \omega_2^{\,2} +\ \omega_4^{\,2}) \\ Iyy\theta'' = \Psi'\Phi'(Ixx - Izz) + \Phi'\omega rJr + bl(\ \omega_1^{\,2} - \omega_3^{\,2}) \\ Izz\Psi'' = \Phi'\theta'(Iyy - Ixx) + Jr\omega r + \sum_{i=1}^{4}(-1)^{i} d - \omega_i^{\,2} \end{Bmatrix} \end{Bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (3.17)$$

3.4.1 Rotor of Motors Model

The brushless DC motor is a synchronous electric motor that, from a modelling perspective, looks exactly like a DC motor. These motors are used in a great amount of industrial sectors because their architecture is suitable for any safety critical applications.[7]

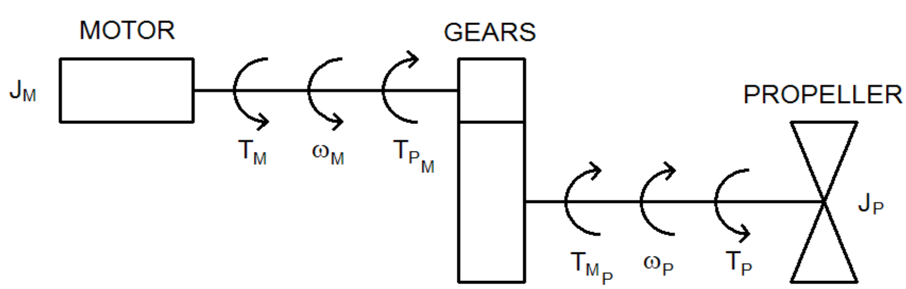

Figure 3-9 Motor system

Another thing to point out is that the price is decreasing due to the advances in permanent magnets, structural design and most especially in its control (Singh and Singh, 2009). They are the new trends as they are compact in housing and have good power density, high efficiency with lesser maintenance, acoustic noise and electromagnetic interference of brushed DC motors. Due to these advantages BLDC motors are the most commonly used motors in UAV technology.

$$\tau = K_t (I - I_0) \tag{3.18}$$

Where τ is the motor torque, I is the input current, I_0 is the current when there is no load on the motor, and Kt is the torque proportionality constant. The voltage across the motor is the sum of the back-EMF and some resistive loss:

$$V = IRm + Kev\,\omega \tag{3.19}$$

Where V is the voltage drop across the motor, Rm is the motor resistance, *w* is the angular velocity of the motor, and Kv is proportionality constant (indicating back-EMF generated per RPM). We can use this description of our motor to calculate the power it consumes. The power is

$$P = IV = \frac{(\tau + Kt\, I0)\,(Kt\, I0Rm + \tau\, Rm + Kt\, Kv\, \omega)}{Kt^2} \tag{3.20}$$

For the purposes of our simple model, we will assume a negligible motor resistance. Then, the power becomes proportional to the angular velocity:

$$P \approx \frac{(Kv)\,\tau\,\omega}{Kt} \tag{3.21}$$

Further simplifying our model, we assume that Kt $I_0 << \tau$. This is not altogether unreasonable, since I_0 is the current when there is no load, and is thus rather small. In practice, this approximation holds well enough. Thus, we obtain our final, simplified equation for power:

$$P \approx \frac{(\tau + Kt\, I0)\, Kv\, \omega}{Kt} \tag{3.22}$$

- Forces

The power is used to keep the quadcopter aloft. By conservation of energy, we know that the energy the motor expends in a given time period is equal to the force generated on the propeller times the distance that the air it displaces moves (Pd t = Fd x). Equivalently, the power is equal to the thrust times the air velocity (P = F$\frac{dx}{dt}$).

$$P = T\ vh \tag{3.23}$$

We assume vehicle speeds are low, so vh is the air velocity when hovering. We also assume that the free stream velocity, is zero (the air in the surrounding environment is stationary relative to the quadcopter). Momentum theory gives us the equation for hover velocity as a function of thrust,

$$Vh = \sqrt{\frac{T}{2\rho A}} \tag{3.24}$$

Where ρ is the density of the surrounding air and A is the area swept out by the rotor using our simplified equation for power, we can then write

$$P = \frac{T^{3/2}}{\sqrt{2\rho A}} \tag{3.25}$$

Note that in the general case, in this case, the torque is proportional to the thrust T by some constant ratio Kt determined by the blade configuration and parameters. Solving for the thrust magnitude T, we obtain that thrust is proportional to the square of angular velocity of the motor:

$$T = \left(\frac{K v K \tau \sqrt{2\rho A}}{Kt}\omega\right)^2 = k\omega^2 \tag{3.26}$$

Where k is some appropriately dimensioned constant

In addition to the thrust force, we will model friction as a force proportional to the linear velocity in each direction. This is a highly simplified view of fluid friction, but will be sufficient for our modeling and simulation. Our drag forces will be modeled by an additional force term.

$$F_D = \begin{bmatrix} -Kd\dot{X} \\ -Kd\dot{Y} \\ -KdZ' \end{bmatrix} \tag{3.27}$$

If additional precision is desired, the constant Kd can be separated into three separate friction constants, one for each direction of motion. If we were to do this, we would want to model friction in the body frame rather than the inertial frame. Brushless motors are used as the actuators of propellers of the quadcopter. Four brushless motors in the system are driven via Electronic Speed Controllers (ESC). ESC's converts PWM signals into a three-phased signal, which rotates the motor continuously. Motor and propeller unit models are identified experimentally. These models are algebraic, steady state ones. Thrust force generated due to the rotation of propellers can be calculated with the equation given below:[9 10]

$$F_i = b\omega^2$$

where b is thrust factor which is determined experimentally.

3.4.2 BLDC motor transfer function

The BLDC motor we have utilized for this project is the Emax Bl4030. It is a 385kv, 11.5 ounce (326g), 1300 watt out runner brushless motor. Contingent upon the propeller and battery utilized, it is generally comparable to 0.60 to0 .90 2 stroke nitro engines. The parameters we used in the modeling are extracted from the datasheet of this motor with corresponding relevant parameters used. Table contains the major extracted parameters used for the modeling task [11].

Table 1: BLDC motor parameter used

Model	EmaxBl4030
Weight	326 g
RPM	6100
Thrust	4200 g
Electric resistance (R)	0.22 ohm
Electric inductance(l)	8.5Mh
Current(I)	55A
Torque constant(kt)	0.44Nm/A
Moment of inertia(J)	0.089kg/m^2

Mechanical constant

$$\tau m = \frac{j \times 0.004 \times R}{ke \times kt}$$

Electrical constant

$$\tau e = \frac{1}{0.004R}$$

Torque constant,

$$kt = \frac{Torque}{Current}$$

= 2.03/55

= 0.04Nm/A

Phase value of the EMF constant

ke=kt×0.0605

Where kt is torque constant, DC current I=0.0605

$$Torque = \frac{Pwx9.554}{n}$$

$$= \frac{1300 \times 9.554}{6100} = 2.03Nm$$

Electrical torque, Ke=0.04×0.0605=0.0024

Mechanical constant,

$$\tau m = \frac{0.089 \times 0.004 \times 0.220}{0.04 \times 0.0024} = 0.8158$$

BLDC Motor Transfer Function

$$G1(s) = \frac{\frac{1}{Ke}}{\tau m \tau e s^2 + \tau m s + 1}$$

=

$$G1(s) = \frac{416.67}{0.38s2 + 0.82s + 1}$$

3.5 Model Parameters

The quadcopter parameters was calculated practically by taking the measurements from the DJI F-450 frame and the applying two tests on a2212 13t 1000kv brushless motor to calculated thrust factor and drag factor, also the total weigh was calculated all the results can be seen in Table 2.

Table 2 Model parameter which is taken from real quadcopter system datasheets.

Symbol	Value	Discription
m	1.4 Kg	Total mass of quadcopter
l	0.56 m	Distance form center of quadcopter to the motor
Ixx	0.05 kg-m^2	Quadcopter moment of inertia around X axes
Iyy	0.05 kg-m^2	Quadcopter moment of inertia around Y axes
Izz	0.24 kg-m^2	Quadcopter moment of inertia around Z axes
Jr	5.225×10-5 kg-m^2	Rotational moment of inertia around the propeller axis
b	7.66×10-5	Thrust coefficient
d	5.63×10-6	Drag coefficient
g	9.86 m/s^2	Gravity

3.6 Rotor Input Modeling

The Quadcopter is a 6-DOF MIMO system defined by four control-inputs to its rotors, responsible for controlling its 12-output-states (six altitude control states and six position & linear velocity states). This control-inputs for controlling the rotor by ways of manipulating - it angular velocities, it torques, force-moment balance and supplied rotor voltages – generally in all reviewed literatures and publications employed the same notion for representing them: U_1 is associated with vertical input movement, U_2 associated with roll input movement, U_3 with pitch input movement and U_4 with yaw input motion. These control inputs falls into one the followings categories:

Two different methods have been investigated to achieve this task. We can either use the Lagrangian equation as in [11 12] or the Newton's law as in the other project. Let's explain the second method which is more comprehensible. The quadrotor is controlled by independently varying the speed of the four rotors. Hence, with the notation of the (u_i and τ_i are respectively the normalized torque and normalized thrust from the i^th rotor), we have the following inputs:

- The total thrust: $u_1 = \tau_1 + \tau_2 + \tau_3 + \tau_4$
- The rolling moment: $u_2 = l\,(\tau_3 - \tau_4)$
- The pitching moment: $u_3 = l\,(\tau_1 - \tau_2)$
- The yawing moment: $u_4 = \tau_1 + \tau_2 - \tau_3 - \tau_4$

A Force-moment Control-Input is presented as:

$$U = \begin{bmatrix} U1 \\ U2 \\ U3 \\ U4 \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\sum_{i=1}^{4} Fi \\ \frac{1}{I_{xx}}(F_1 - F_2 - F_3 + F_4) \\ \frac{1}{I_{YY}}(-F_1 - F_2 + F_{3+} + F_4) \\ \frac{C}{I_{xx}}\sum_{i=1}^{4}(-1)^{l+1}F_I \end{bmatrix} = \begin{bmatrix} b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ b(-\omega_2^2 + \omega_4^2) \\ b(-\omega_1^2 + \omega_3^2) \\ d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{bmatrix} \quad (3.30)$$

3.7 Block diagram of system

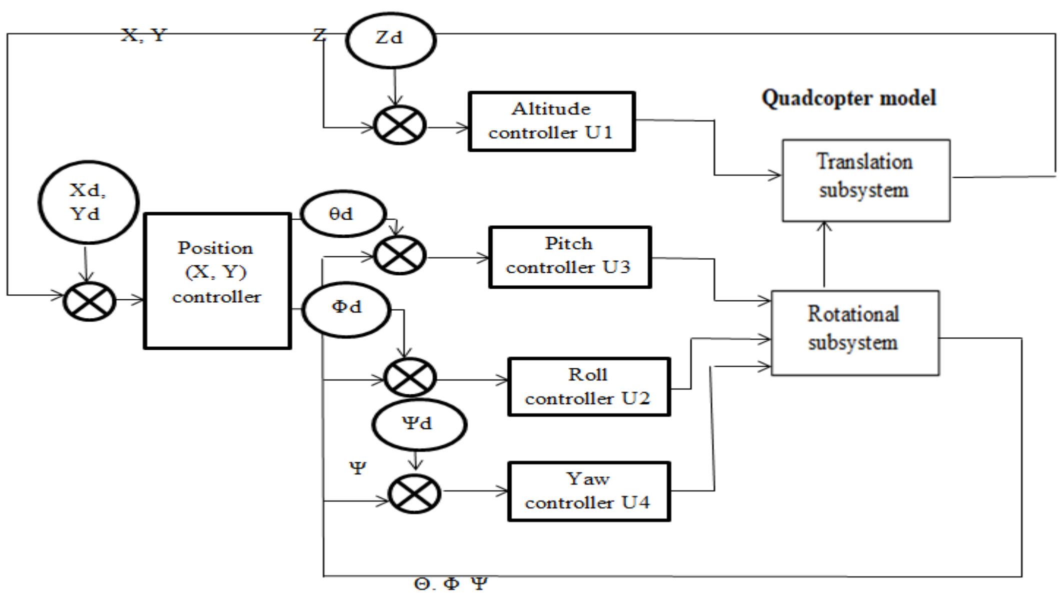

Figure 3-10 overall Block diagram of system

Chapter four

4 Controller Design

4.1 PID Control

PID controllers have use in a broad range of controller applications. It is for-sure the most applied controller in industry. The PID controller shown in Fig has the advantage that parameters (Kp, Ki, Kd) are easy to tune, which is simple to design and has good robustness. However, the quadcopter includes non-linearity in the mathematical model and may include some inaccurate modeling of some of the dynamics, which will case bad performance of the control system, thus it is needed form the designer to be careful when neglecting some effects in the model or simplifying the model. In the industrial area the most used liner regulators are surely the PID.

In robotics, PID technique represents the basics of control. Even though a lot of different algorithms provide better performance than PID, this last structure is often chosen for the reasons expressed above. Quadcopter or multicopters use PID to achieve stability. Tuning of the PID controllers has been attracting interest for six decades. Numerous methods have been suggested so far try to accomplish the task by making use of different representations of the essential aspects of the process behaviour [12 16]. Among the well- known formulas are the Ziegler-Nicolas rule, the Cohen-Coon method and the internal model control.

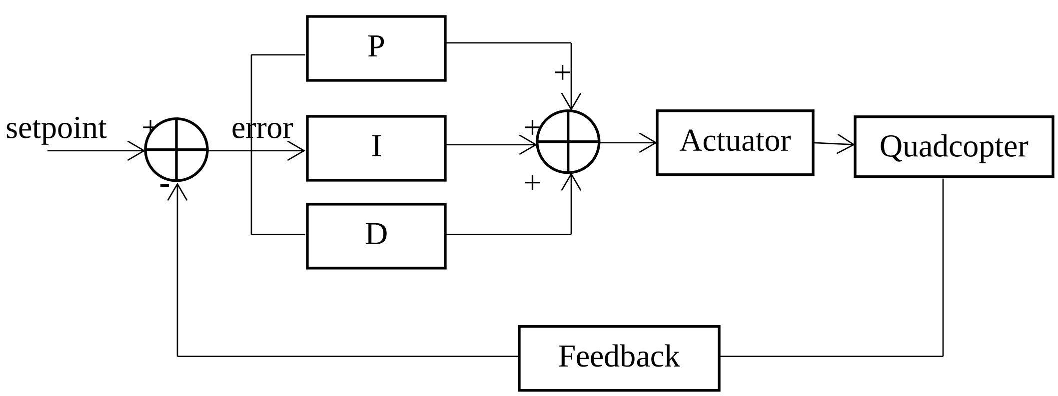

Figure 4-1 General structure of PID controller

Quadcopter is the plant for which the controller has to be designed. But it is need the linearized state space model. The block is the PID controller for the system. The generalized transfer function of the PID controller is given by,

$$PID = Kpe(t) + \frac{de(t)}{dt} + \int e(t)dt \quad (4.1)$$

$$\mathrm{Kp} + \frac{\mathrm{Ki}}{s} + \mathrm{Kds} \quad (4.2)$$

Where Kp = Proportional Gain.

Ki = Integral Gain

Kd = Derivative Gain

A control system is modeled for the quadcopter using four PID controllers to control the Attitude (roll, pitch and yaw) and the Altitude (Z height) to introduce stability these four controllers form the inner loop of control for the quadcopter system. And then two more PID controllers are used to control the position of the quadcopter (X and Y axes) and the output of these two controllers will be input to the roll and pitch controllers these two PID's will form the outer control loop**.**

4.2 Effect of each PID Parameter

The variation of each of these parameters alters the effectiveness of the stabilization. Generally there are 3 PID loops with their own coefficients, one per axis, so you will have to set P, I and D values for each axis. To a Quadcopter, these parameters can cause this behaviour.

- **Proportional Gain coefficient –**Quadcopter can fly relatively stable without other parameters but this one. This coefficient determines which is more important, human control or the values measured by the gyroscopes. The higher the coefficient, the higher the Quadcopter seems more sensitive and reactive to angular change. If it is too low, the Quadcopter will appear sluggish and will be harder to keep steady. You might find the Quadcopter starts to oscillate with a high frequency when P gain is too high.
- **Integral Gain coefficient –** This coefficient can increase the precision of the angular position. For example, when the Quadcopter is disturbed and its angle changes from, in theory it remembers how much the angle has changed and will return . In practise if you make your Quadcopter go forward and the force it to stop, the Quadcopter will continue for

some time to counteract the action. Without this term, the opposition does not last as long. This term is especially useful with irregular wind, and ground effect (turbulence from motors). However, when the I value get too high your Quadcopter might begin to have slow reaction and a decrease effect of the proportional gain as consequence, it will also start to oscillate like having high P gain, but with a lower frequency.

- **Derivative Gain coefficient –** This coefficient allows the Quadcopter to reach more quickly the desired attitude. Some people call it the accelerator parameter because it amplifies the user input. It also decrease control action fast when the error is decreasing fast. In particle it will increase the reaction speed and in certain cases an increase the effect of the P gains.

4.2.1 The characteristics of P, I and D controllers

A proportional controller (kp) will have the effect of reducing the rise time and will reduce but never eliminate the steady-state error. An integral controller (Ki) will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative controller (Kd) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response. Effects of each of controller on a closed-loop system are summarized in the table shown below,

Table 3 Response of the different parameters of the Proportional Integral and Derivative gain

Controller	Rise time	Overshoot	Settling time	Steady-state Error	Stability
Kp	Decreases	Increases	Small Change	Decreases	Degrade
Ki	Decreases	Increases	Increases	Eliminate	Degrade
Kd	Small Change	Decreases	Decreases	No effect in Theory	Improve if Kd is small

The role of control theory is to help us gain insight on how and why feedback control systems work and how to systematically deal with various design and analysis issues. Specifically, the following issues are of both practical importance and theoretical interest:

- Stability and stability margins of closed-loop systems.
- How fast and smooth the error between the output and the set point is driven to zero.

- How well the control system handles unexpected external disturbances, sensor noises, and internal dynamic changes.

4.3 Inner Loop Control

Inner control loop controls quadcopter altitude and attitude. Input variables for inner loop can be divided in two parts, desired and sensor signals. Desired signals are obtained from the control signals coming directly form the pilot or the autopilot program. These signals are the Height (altitude) and pointing (Yaw) of the quadcopter the other two signals desired roll and pitch comes for the output of the outer loop control since they are translated form the desired x and y position in the outer PID's. Figure show the complete control system for the quadcopter including the inner loop and the outer loop.

The quadcopter has six degrees of freedom, which is controlled by the rotation speed of four motors. It belongs to the typical under-actuated nonlinear system. Based on the dynamic model of the four-rotorcraft, the control input and the motor speed and the relationship between the motor speed and the four forces is obtained. Then the closed-loop control loop of the attitude angle is established to obtain the corresponding change of the motor rotation speed when the four-rotor vehicle is turned over, and the simulation is carried out.

By the formula (3.30) to obtain the relationship between the control input U and the four-motor speed, first through the motor speed and control of four forces between the conversion relationship, given a given motor speed corresponding to the U_1, U_2, U_3, U_4. Since there is no sensor element in the simulation environment that can measure the height and position, the feedback value is calculated by calculating the three displacement accelerations (x, y, And acceleration of the three attitude angles ($\ddot{\Phi}$, $\ddot{\theta}$, $\ddot{\psi}$). The acceleration is obtained through two integrals and the displacement is obtained.

- **Altitude control**

Equation for the thrust force control variable U1 is:

$$U_1 = Kpze + Kiz \int ez + Kdz \frac{dez}{dt} \quad (4.3)$$

WhereKpz, Kiz and Kdz are three altitude PID controller parameters. ez is the altitude error, where $ez = Zdes - Zmes$. $Zdes$ Is the desired altitude and $Zmes$ is the measured altitude.

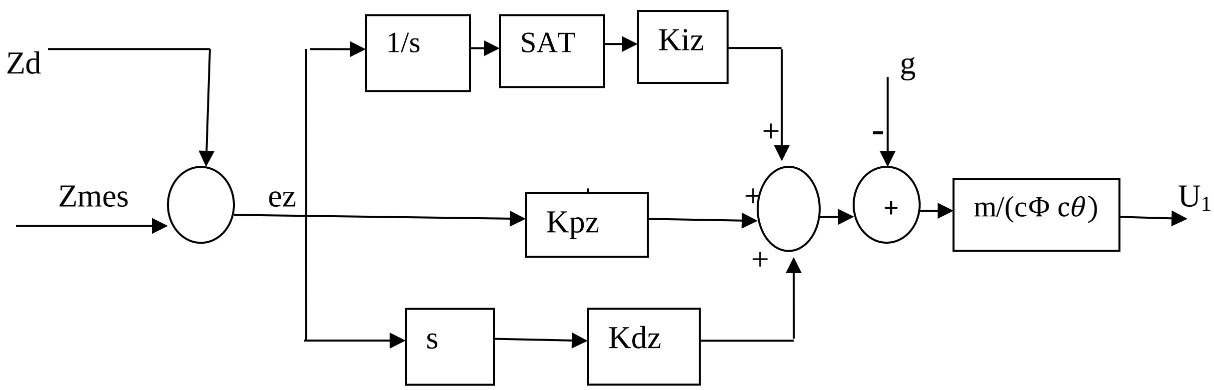

Figure 4-2 Block diagram of the height control

- **Roll control**

Equation for the roll moment control variable U2 is:

$$U_2 = Kp\Phi e + Ki\Phi \int e\Phi \; + Kd\Phi \frac{de\Phi}{dt} \tag{4.4}$$

WhereKpΦ, KiΦ and KdΦ are three roll angle PID controller parameters. eΦ is the roll angle error, where . $e\Phi = \Phi des - \Phi mes$. Φdes is the desired roll angle and Φmes is the measured roll angle.

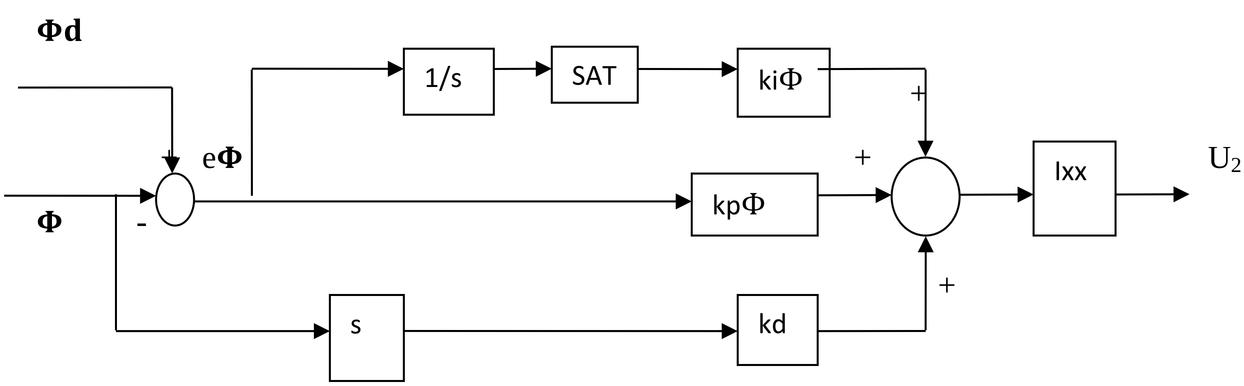

Figure 4-3 Block diagram of the roll control

- **Pitch control**

Equation for the pitch moment control variable U3 is:

$$U_3 = Kp\theta e + Ki\theta \int e\theta \; + Kd\theta \frac{de\theta}{dt} \qquad (4.5)$$

Similar to the roll control, Kpθ , Kiθ and Kdθ are three pitch angle PI-D controller parameters. eθ is the pitch angle error, where $e\theta = \theta des - \theta mes$. θdes is the desired pitch angle and θmes is the measured pitch angle.

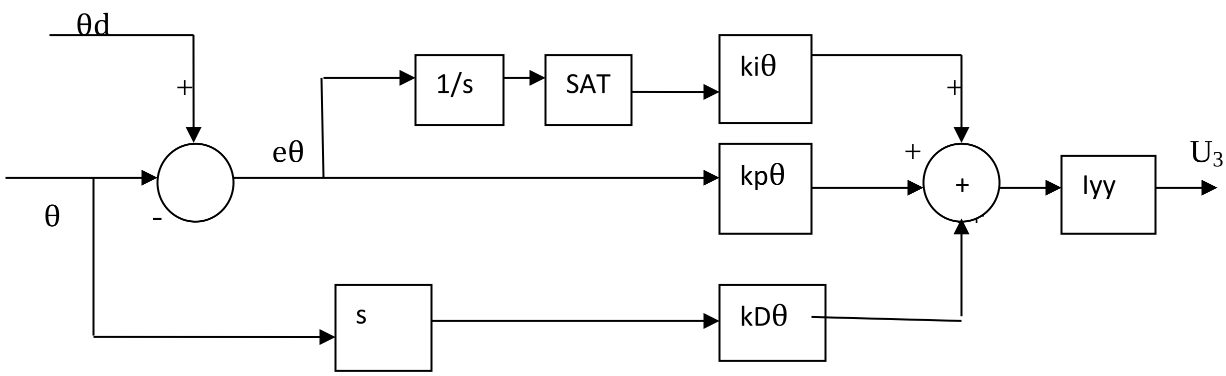

Figure 4-4 Block diagram of the pitch control

- **Yaw control**

Equation for the yaw moment control variable U_4 is:

$$U_4 = Kp\Psi e + Ki\Psi \int e\Psi \; + Kd\Psi \frac{de\Psi}{dt} \qquad (4.6)$$

WhereKpΨ, KiΨ and KdΨ are three yaw angle PI-D controller parameters. eΨ is the yaw angle error, where $e\Psi = \Psi des - \Psi mes$. Ψdesis the desired yaw angle and Ψmes is the measured yaw angle.

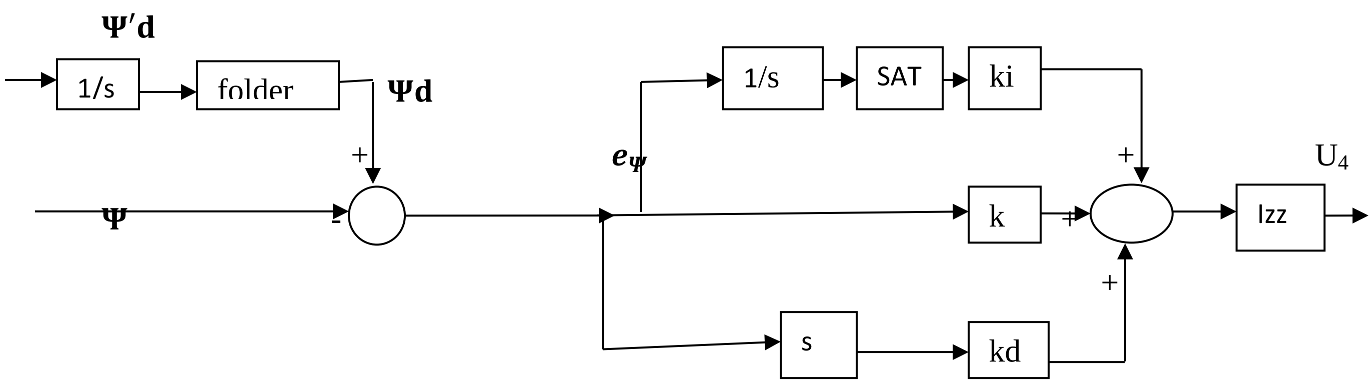

Figure 4-5 Block diagram of the yaw control

4.4 Outer Loop Control

Outer control loop is applied since the quadcopter is under-actuated system and it is not applicable to control all of the quadcopter 6-DOF straightly. As mentioned earlier, the inner loop directly controls 4-DOF (three angles and altitude). To be able to control X and Y position, outer loop is implemented. The outer control loop outputs are desired roll and pitch angles, which they are the inputs to the inner loop for the desired X and Y position. The equations for the quadcopter and linear accelerations:

$$\ddot{X} = \frac{(Cos\Psi \ Sin\theta Cos\ \Phi + Sin\Psi Sin\Phi)U1}{m} \quad (4.7)$$

$$\tilde{Y} = \frac{(Sin\Psi \ Sin\theta Cos\ \Phi - Cos\Psi Sin\Phi)U1}{m} \quad (4.8)$$

Quadcopter dynamics of the X and Y linear accelerations can be simplified in to:

$$\ddot{X} = \frac{(\theta Cos\Psi + \Phi Sin\Psi)U1}{m} \quad (4.9)$$

$$\tilde{Y} = \frac{(\theta Sin\Psi - \Phi Cos\Psi)U1}{m} \quad (4.10)$$

4.5 Design of PID Controllers Using the Ziegler–Nichols Methods

The PID controller has the flexibility of simultaneously tuning three parameters: namely, the parameters Kp, Ti, and Td. This allows a PID controller to satisfy the design requirements in many practical cases, a fact which makes the PID controller the most frequently met controller in practice. The appropriate values of the parameters Kp, Ti, and Td of the PID controller may be chosen by trial and error. This is usually a formidable task, even in cases where the design engineer has great experience on the subject. To facilitate the determination of the appropriate values of the parameters Kp, Ti, and Td, even for cases where a mathematical model for the system under control is not available, Ziegler and Nichols [13] have suggested the following two rather simple and practically useful methods. In this case, the system under control is excited with the unit step function. The shape of the transient response of the open-loop system may have the general form shown in Figure.

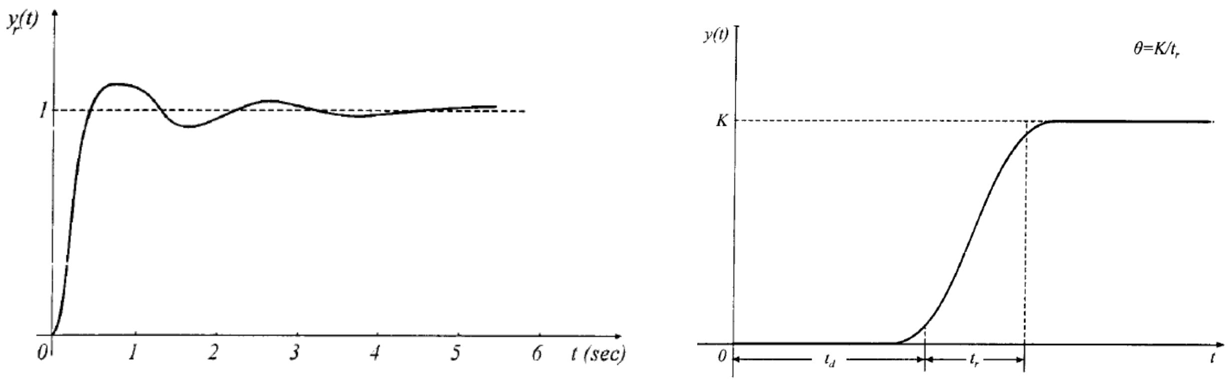

Figure 4-6 Closed-loop response and The transient response method

- The delay (td) time is the time required for the response to reach half the final value the very first time.
- The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value.
- The peak time is the time required for the response to reach the first peak of the overshoot.
- The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. The amount of the maximum (percent) overshoot directly indicates the relative stability of the system.
- The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%). []

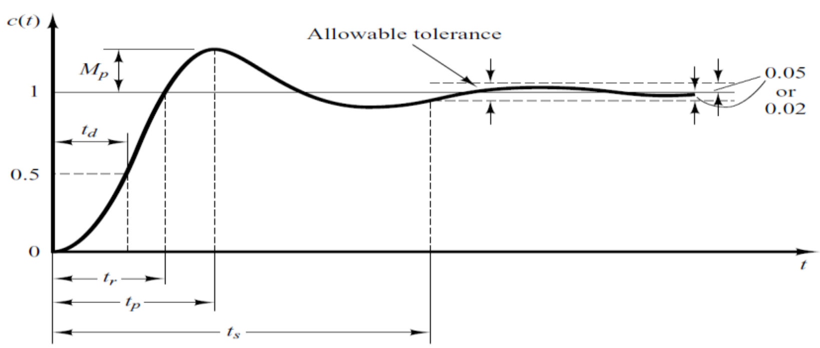

Figure 4-7 charactectic of system responses

In this case, we introduce the parameters td ¼ delay time and tr ¼ rise time. Aiming in achieving a damping ratio ζ of about 0.2 (which corresponds to an overshoot of about 25%), the values of the parameters Kp, Ti, and Td of the PID controller are chosen according to Table. It is useful to mention that here the transfer function of the system under control may be approximated as follows

$$G(s)= k[\frac{e^{-tds}}{1+trs}] \tag{4.11}$$

the PID controller transfer function Gc(S) becomes

$$G(s)= kp[1+\frac{1}{T_i s}+T_d s] = 1.2\frac{t_r}{t_d}[1+\frac{1}{2t_d s}+0.5t_d s] \tag{4.12}$$

$$= 0.6t_r[1+\frac{(s+1/t_d)^2}{s}] \tag{4.13}$$

That is, the PID controller has a pole at the origin and a double zero at s= -1/td.

Table 4: The Values of the Parameters Using the Ziegler–Nichols Transient Response Method

Controller		Kp	Ti	Td
Proportional	P	tr/td	∞	0
Proportional- integral	PI	0.9tr/td	td/0.3	0
Proportional- integral derivative	PID	1.2tr/td	2td	0.5td

4.6 Linearization of the model

In control engineering a normal operation of the system may be around an equilibrium point, and the signals may be considered small signals around the equilibrium. (It should be pointed out that there are many exceptions to such a case.) However, if the system operates around an equilibrium point and if the signals involved are small signals, then it is possible to approximate the nonlinear system by a linear system. Such a linear system is equivalent to the nonlinear system considered within a limited operating range. Such a linearized model (linear, time-invariant model) is very important in control engineering. The linearization procedure to be presented in the following is based on the expansion of nonlinear function into a Taylor series about the operating point and the retention of only the linear term. Because we neglect higher-order terms of the Taylor series expansion, these neglected terms must be small enough; that is, the variables deviate only slightly from the operating condition.[13]

In mathematics, linearization refers to finding the linear approximation to a function at a given point. In the study of dynamical systems, linearization is a method for assessing the local stability of an equilibrium point of a system of nonlinear differential equations or discrete dynamical systems [14]. A set of nonlinear ordiary differential equastion's (forces equations, moments equations) are parameterized by mass characteristics of the system as well as aerodynamics. The nonlinear equations are directly useful for simulations like computer games, flight trainers, flight simulators and validation of control law. They are not directly useable for development of control laws as the design methods rely on the linear systems and control theory. The nonlinear equations come from the geometric transformations and describing functions of aerodynamic coefficient. For this reason, the linearized system and control tools are attractive and viable options for design. In order to design a controller for a Quadcopter it is suggested to have a linearized model of the system to have a precise controller [15]. First-order Taylor-Series approximation was used to linearize the non-linear quadcopter model developed via the mathematical equations. The linearization follows the four point model used by Zambrano [16] with salient aspects of the linearized equation. Linearization at hovering implies that the quadcopter's x-y body-frame is parallel to the x-y inertia-frame; at the same time the vehicle's roll, pitch and yaw angles are all equal to or approximately zero. The derivation for the control voltage used for achieving these conditions via height equation-of-motion is as follows: θ=Φ=Ψ= θ′ =Φ'= Ψ'=0

$$\Phi'' = lb\frac{U3}{Ixx} \quad (4.14)$$

$$\theta'' = lb\frac{U3}{Ixx} \quad (4.15)$$

$$\Psi'' = lb\frac{lbU3}{Ixx} \quad (4.16)$$

$$Z'' = -g + (cos\theta cos\Phi)\frac{U1}{m} \quad (4.17)$$

Chapter Five

5 Simulation Results and Discussion

5.1 Simulation

Now that we have the complete equations of motion describing the dynamics of the system, we can create a simulation environment in which to test and view results of various inputs and controllers. We have used Matlab Simulink and script to create a simulation environment and test the controller for our Quadcopter. The initial simulation, velocity and the angular displacement is defined to zero state. Some disturbances are also defined in the angular velocity and the magnitude of the deviation is in radians per second. The input from the controller is obtained for the time variables. The linear and angular acceleration for the model; is obtained for the input obtained from controllers. The function for the all the physical forces and torques and defined to validate the presented dynamic model and control method, a numerical simulation of the quadcopter was developed using the MATLAB.

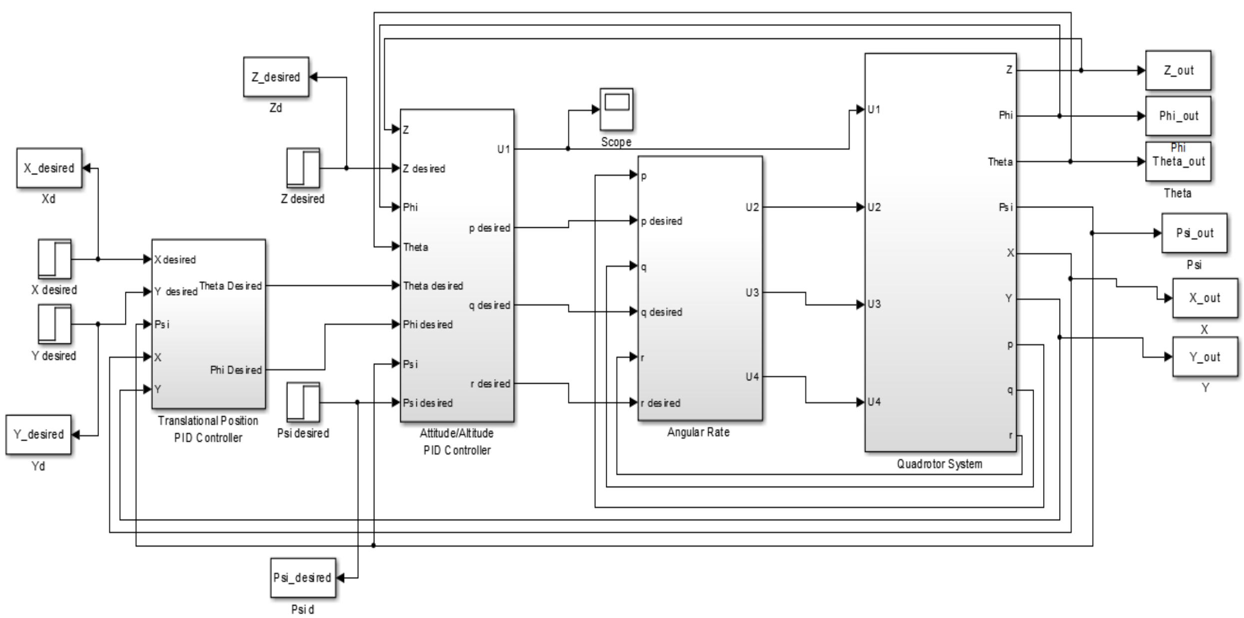

Figure 5-1 Overall Simulink block of quadcopter

Standard Test Signal: - The performance of a system can be evaluated with respect to these test signals. Based on the information obtained the design of control system is carried out. Step input signal (position function), it is the sudden application of the input at a specified time as usual in the figure or instant any us change in the reference input

Example:-

- If the input is an angular position of a mechanical shaft a step input represent the sudden rotation of a shaft.
- Switching on a constant voltage in an electrical circuit.
- Step signal is imitate the sudden change characteristic of actual input signal. the step signal is called unit step signal.

5.2 Control Parameter Optimization

Control system stability and performance of the key lies in the determination of PID control parameters, control parameters to determine the control parameters that are found in the optimal value. The parameters of the position controller are designed first, then the parameters of the height controller are determined, the parameters are determined by trial and error method, then the integral is adjusted first, then the control parameters are obtained through constant trial and error.

Table 5 Table shows the optimal values for all control parameters

Parameter	Value	Parameter	Value	Parameter	Value
Kpx	0.35	Kix	0.25	Kdx	-0.35
Kpy	0.35	Kiy	0.25	Kdy	-0.35
Kpz	5.8823	Kiz	0	Kdz	-5.050
KpΦ	4.5	KiΦ	0	KdΦ	0
Kpθ	4.5	Kiθ	0	Kdθ	0
Kpψ	10	Kiψ	0	Kdψ	0

Zero consideration of Angular velocity and acceleration of our system are achieved in our simulation and the simulation is shown in below figure.

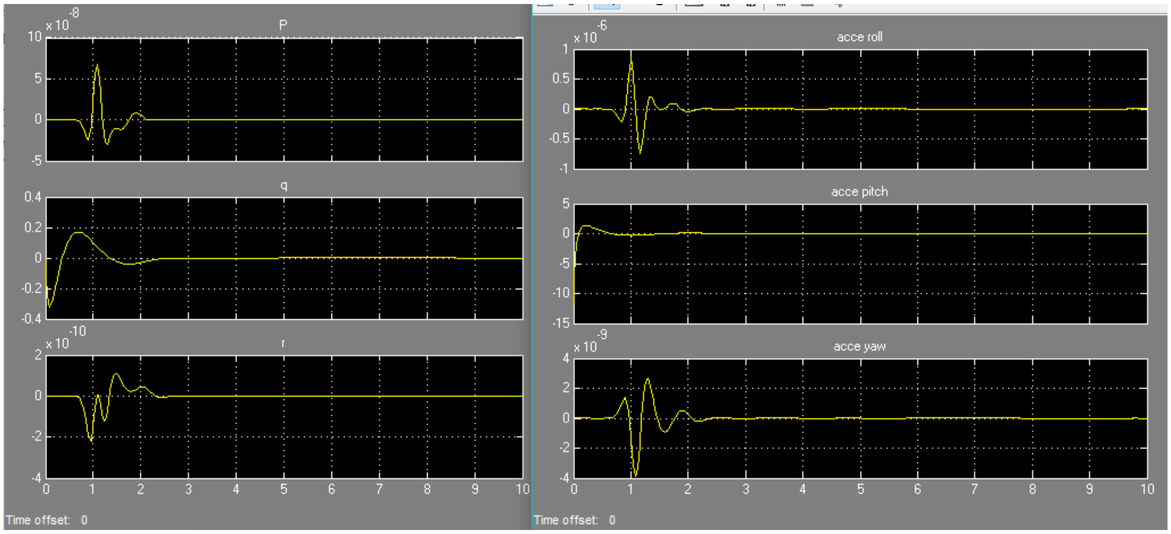

Figure 5-2 Zero consideration of Angular velocity and acceleration

5.3 Position control

- Translational Positions

A control system is modeled for the quadcopter using two more PID controllers are used to control the position of the quadcopter (X and Y axes) and the output of these two controllers will be input to the roll and pitch controllers these two PID's will form the outer control loop**.** It is very important that both roll and pitch errors are kept low to provide stable flight. In the hovering condition, if one of the two angles is different than zero a longitudinal acceleration occurs. This behavior makes difficult to maintain a fixed position without drift. The vehicle's initial position was (0; 0; 0). The final position was set to (0.25:0.05; 0). The desired pitch angle was set and the desired roll angle was set to 00. The acceleration value (x " y ") of the horizontal position is used as the input signal of the inverse resolving module.

- **Case1: change X position, Y position remain constant**

When increase the position of X axis from initial point 0 m to 0.25m and consider the Y position is constant. That mean when quadcopter move to north direction, because of interdependence of both position, it change Y position in small amount of distance.

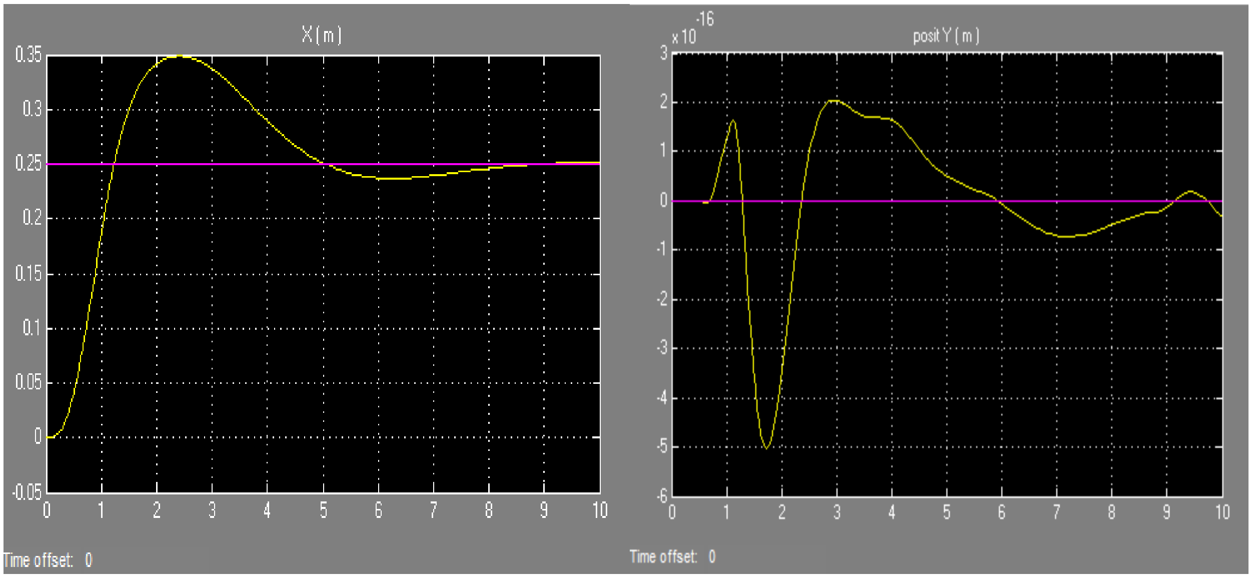

Figure 5-3 Change in position x = 0.25m y = 0 m

- **Case 2: change Y position and X remain constant**

This change in Y or y-axis points towards the east, affects the position controller of the x direction because they are dependent on yaw correction making them interdependent. Overall the closed loop controller seems to be working efficiently in the y direction.

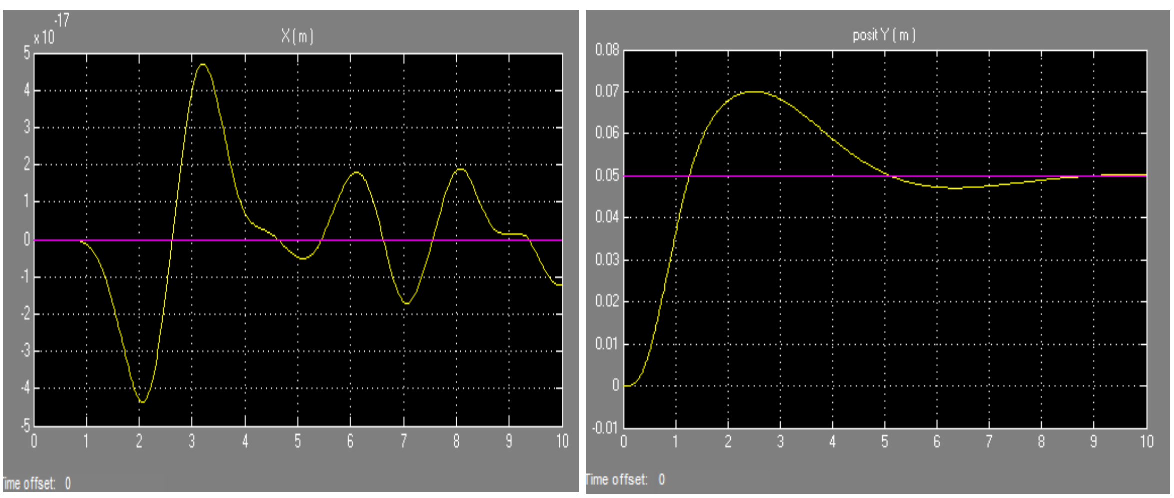

Figure 5-4 Change in position Y=0.05m and X = 0 m

We can see that the quadcopter follows the ground vehicle closely with minimal error in y direction. For all two axes we see that the system takes 8 seconds to settle time, 2% of maximum over shoot, zero steady state error and reach to the position as instructed from the initial position to (0 0.05). From that time on-ward the quadcopter closely follows the y-coordinate as needed.

- **Case 3: when both X and Y position is equal value**

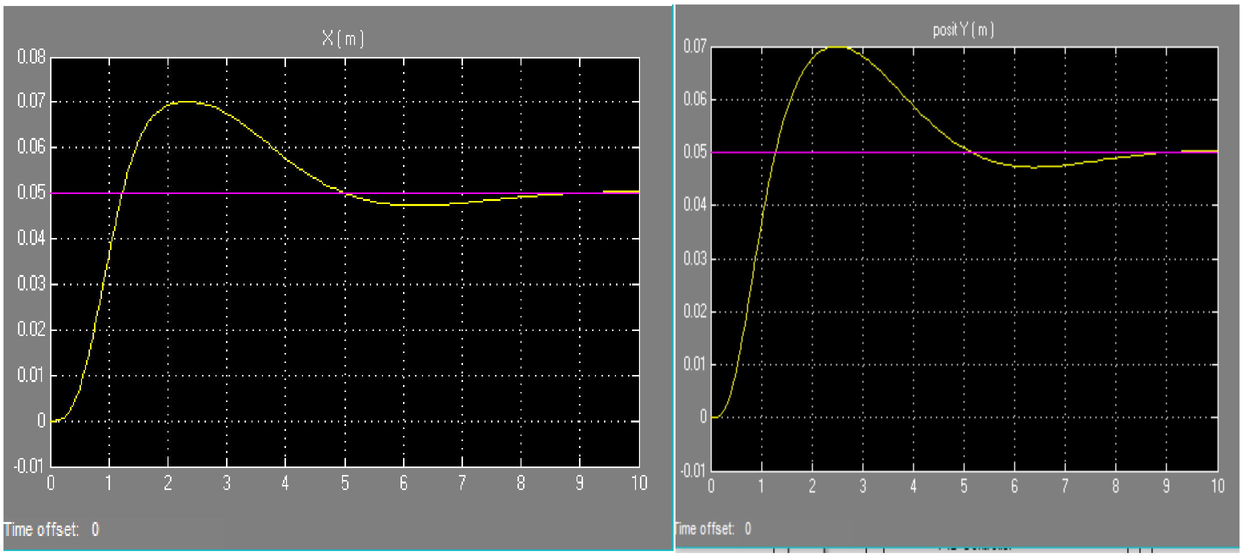

Figure 5-5 when both X and Y position is equal value

The main objective of each simulation was to understand and access the performance of our closed loop feedback architecture for all three axes. The simulations were also helpful in tuning the parameters of the joint centralized controller. This main controller works to minimize the error in the relative position of the quadcopter with respect to the moving ground vehicle.

For first two simulations we keep the quadcopter at a hover position i.e., at a constant z coordinate with respect to the ground vehicle and vary its position in y and x direction one by one to access the performance of the joint centralized controller. In the third set of simulation we vary both x and y coordinates simultaneously to better understand the interdependence**.** The error caused due to interdependence of x and y position controllers on the quadcopter, we test our system by varying the desired position in all three axes. This simulation set shows the overall performance of our system. These simulations validate the controller performance and parameters for varying x, y, and z. Now to better understand the performance of our closed loop feedback system for varying parameters next set of simulations are carried out. For these simulations, we increase and decrease the length and resolution for the feedback model and analyses the system performance.

- Angular Position

We also modeled for the quadcopter using three PID controllers to control the Attitude (roll, pitch and yaw) and to introduce stability these three controllers form the inner loop of control for the quadcopter system the yaw stabilization has lower requirements: an error in the yaw angle does not cause any acceleration in hovering condition. However the dynamic range is much wider than the roll or pitches one. The two (roll and pitch) shows small variation (less than 10 degrees). Therefore, good performance in both dynamic and static tracking is required. Figure shows the yaw stabilization performance. The yaw error in conditions is always lower than two degrees while that one in dynamic tracking is kept under four degrees. For the kinetic model, the first equation is obtained by multiplying sinψ and the second equation multiplying by cosψ,

$$m\ddot{x}\sin\psi - m\ddot{y}\cos\psi = U1\sin\Phi$$

Further,

$$\Phi d = \arcsin\,(mU1\,(m\ddot{x}\sin\psi - m\ddot{y}\cos\psi))$$

From the first equation of the kinetic model,

$$\mathrm{Sin}\theta = (m\ddot{x}U1 - \sin\Phi\sin\psi)/\cos\Phi\cos\psi$$

This is $$\theta d = \arcsin\,((m\ddot{x}U1 - \sin\Phi\sin\psi)\ /\cos\Phi\cos\psi)$$

The input signal Φ_d, θ_d of the inner loop attitude control loop can be obtained. Φd, θd as the attitude control loop closed-loop PID reference value.

- **Case 1: effect of translational position on angular position (when change Y position and x constant)**

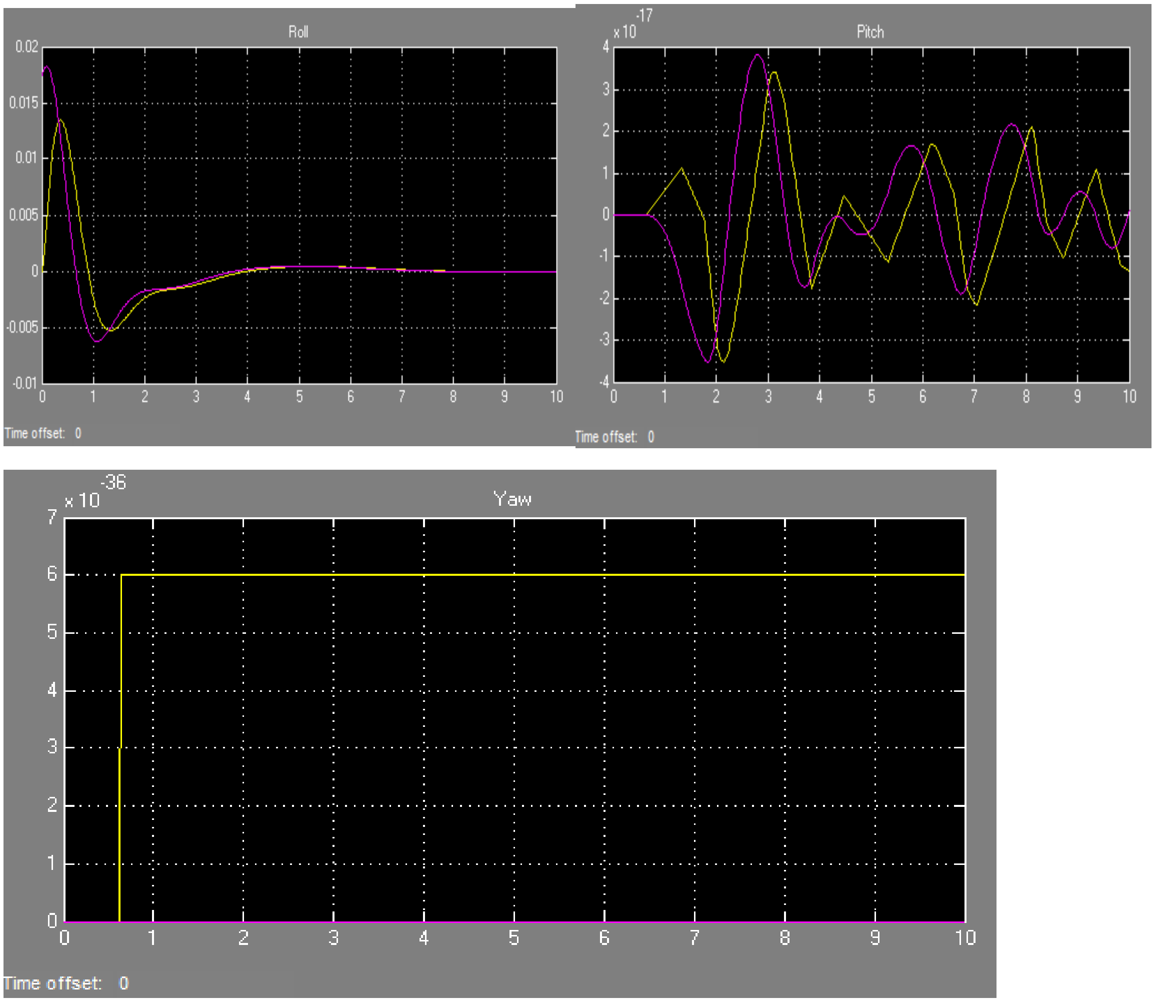

Figure 5-6 Roll, Pitch and Yaw angle when X=0m and Y= 0.05m

- **Case 2: effect of translational position on angular position (when both X and Y are equal)**

When we change the value of X and Y from initial position to same point (x = 0m, y = 0m to x= 0.05m, y = 0.05m) at constant the z value it has effect on the angular displacement or position roll and pitch. The value of angular position interms yaw angle is used in translation motion. Those effects are shown in our simulation and we are minimized the effect.

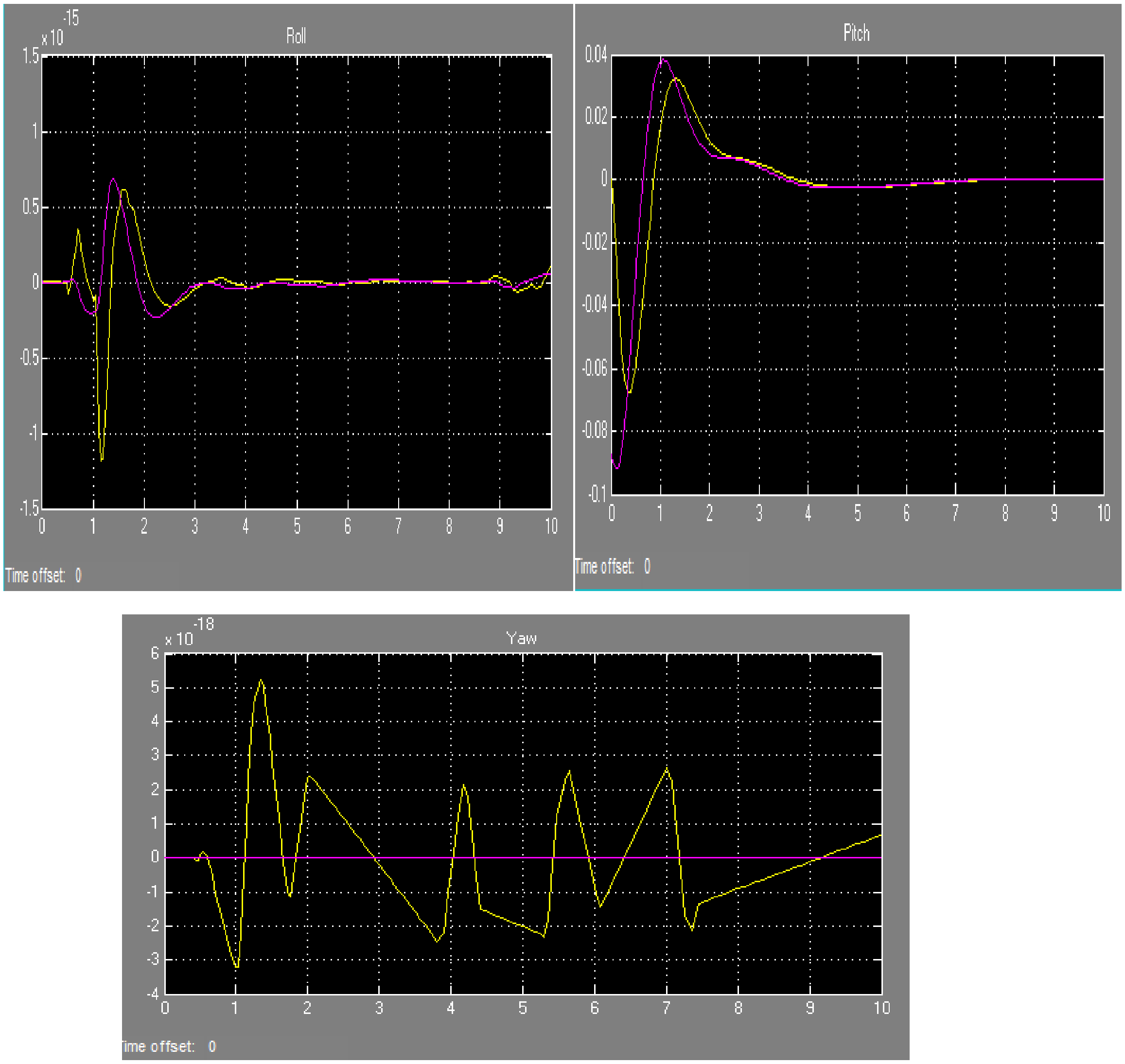

Figure 5-7 Roll, pitch and Yaw position when X and Y equal

Linearization at hovering implies that the quadcopter's x-y body-frame is parallel to the x-y inertia-frame; at the same time the vehicle's roll, pitch and yaw angles are all equal to or approximately zero.

5.4 Height control

We also modeled for the quadcopter using fourth inner loop PID controllers to control the Altitude (Z height). During design our quadcopter we can consider Z axis as altitude of our system. This motion is vertical motion up ward and down along z axis for vertical take-off and landing of quadcopter. We assume that the quadcopter is at hovering state when gravitational force and thrust in terms of U1 is became equal (T = mg). When the value of mass is 1.4 kg, g =

$9.81 m/s^2$.

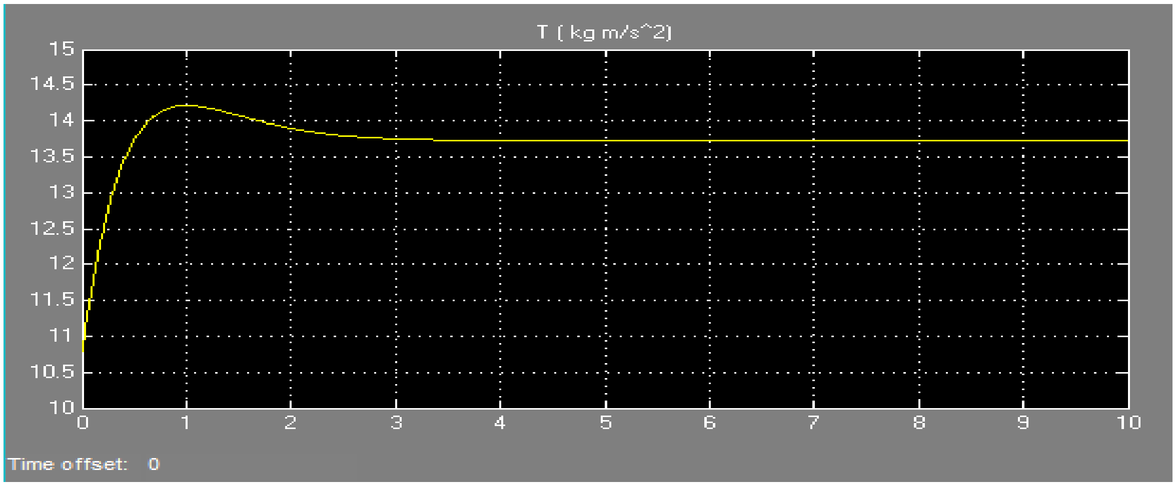

Figure 5-8 Thrust in terms of U1

- **Take off to Hover**

$$Ma = \Sigma F$$

Vertical Takeoff (i.e. stationary to climb velocity $a_z > 0, a_x = a_y = 0$ Sum forces in Body Z axis

$$ma_z = T - mg - D$$

$$T = ma_z + mg + D$$

Thrust required to takeoff = thrust to overcome weight + thrust to overcome inertia + thrust to overcome Drag

- **Hover**

$$az = 0, ax = ay = 0$$

The Sum forces in Body Z axis

$$0 = T - W$$

$$0 = T - mg$$

$$T = mg$$

Thrust required to hover = thrust to overcome weight

- **Case 1: When the position height (Z) is 10m at x and y constant**

The height of our quadcopter is become 10 m with high performance and figure below show out-put of our simulation

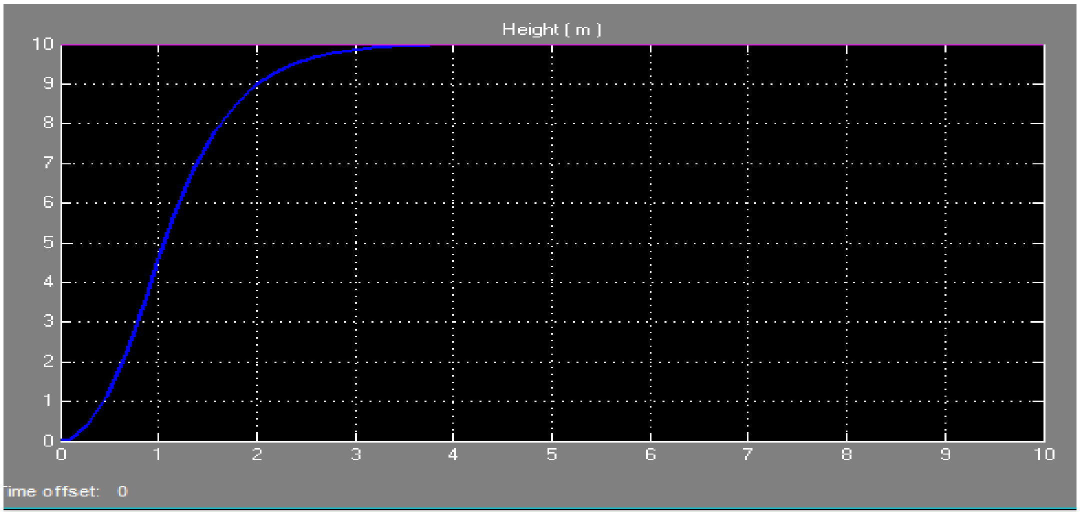

Figure 5-9 Result when height =10 m (Z =10m)

- **Case 2 : when height is increase to 15m at constant X and Y value**

We are increase or decrease the altitude of our system or quadcopter at a given range as we needed. In this thesis one of the objectives is to control altitude of quadcopter, and then it is achieved with high performance. Let as increase the height or our quad to see how the design is done appropriate shown as below figure.

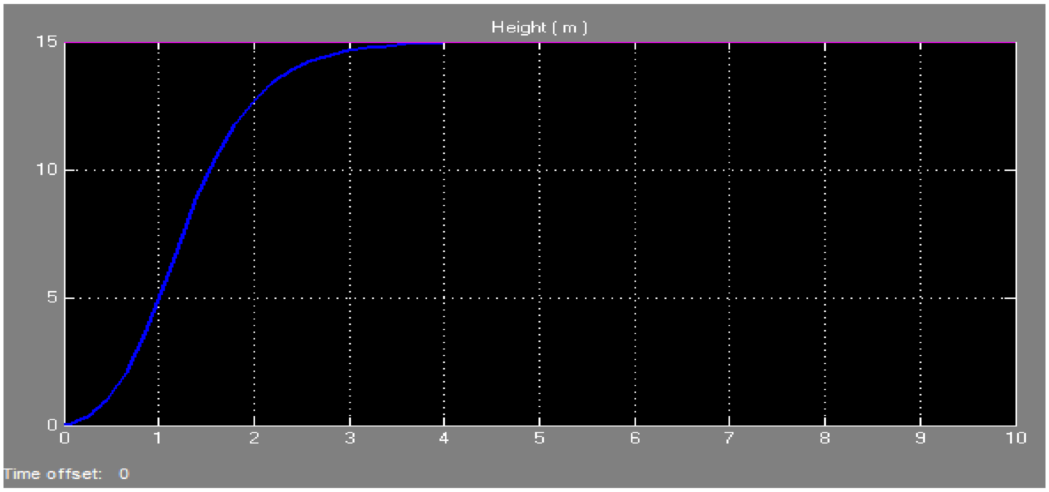

Figure 5-10 Result when height =15m (Z =15m)

Chapter Six

6 Conclusion and Recommendation

6.1 Conclusion

In our project, the introduction about the Quadcopter is given followed by the advantages of the Quadcopter over comparable scaled helicopter and the application of Quadcopter in day to day life. The goals of this thesis work were to model the quadrotor and to test its control algorithm, control attitude and altitude of quadcopter. Furthermore these theoretical considerations were taken into account to develop a real platform. The quadrotor model was presented in chapter 3. In chapter 4 the control algorithm structure was explained. A simulation was adopted to test both dynamics and control, as shown in chapter 5. The dynamics of the Quadcopter is briefly explained theoretically. We derived the equation of motion of the Quadcopter, starting with the Voltage-Torque relation for the brushless motor and working through the Quadcopter kinematics and dynamics. We created the non-linearized model of the Quadcopter is using Matlab Simulink for the equation of motion derived. This simulator was used to test and visualize Quadcopter control mechanics. We also ignored the aerodynamically effects such as blade- flapping and the non-zero free stream velocity. But, the air friction was included as a linear drag forces in all directions. Then, the non-linearized model is linearized for the given operating point using the Taylor-Series approximation method.

The non-linearized and the linearized model are studied for a given set of step input. For stabilization of quadcopter, a detailed explanation about the PID control is given, with the PID plant structure, the transfer function and finally the tuning using a single input single out put tool approach. We are begun with a simple PD controller, even though the response of the model was good there was a significant steady state error. In order to decrease the significant steady state error and introduced the integral term to the controller to create the PID controller. We found out that the steady state error prevented in PID controller was much better that the PD controller when presented with the same disturbances and using the same Proportional and Derivative gains. The height stabilization had an error of just very small in centimeters. This last value was considerably low compared with the resolution of the height sensor in that range.

Generally, according to the goals of our project, we explain detailed in both modelling and simulation. Position and altitude of our system is controlled to desired value.

6.2 Recommendation

When we modeling quadcopter, the each parameter value are depend on another parameter that mean the output of one is used as input for another, the derivative of one is equal to another parameter. Example derivative of displacement is give velocity; its second derivative is equal to acceleration. Because of this interdependent of their parameter design and control quadcopter is difficult. The stabilization and design controller is of system is also complex. The last thing to be noted is that even though the position control is developed and functional it needs more testing on the real system. One has not to forget that the controllers are designed for the hover trim point. When more aggressive or more robust control is desired e.g. recovering from extreme positions, more trim points have to be introduced and some gain scheduling algorithm designed to select or interpolate suitable control law for the given flight envelope.

Future work would involve solving the dynamic model to allow the desired orientation to be achieved in any arbitrary combination of pitch and roll angles. Once the interface for the Gums computers is operational this feature is possible. Then more advanced control algorithms can be as well, such as fuzzy-PID, LQR and $H\infty$ minimization or model predictive control (MPC) algorithm as a higher-level control and planning platform. If This algorithm can use to develop fo outer loops as control interface providing optimal control. Therefore lowering effect of each parametere and improving the performance of system more than PID controller.

References

[1] Chmelař: Building and controlling the Quadracopter, Number 5, Volume VI, December 2011.

[2]. Fundamentals of Small Unmanned Aircraft Flight Jeffrey D. Barton, VOLUME 31, NUMBER 2 (2012)

[3]. L. E. Munoz, O. Santos, and P. Castillo, “Robust nonlinear realtime control strategy to stabilize a pvtol aircraft in crosswind,” in Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on, Oct 2010,.

[4] Beard, R.W., 2008, Quadrotors Dynamics and Control, Report, BYU Scholars Archive, Brigham Young University, Provo.

[5] de Silva, Clarence W. (2009). Modeling and Control of Engineering Systems. CRC Press. pp. 632–633.

[6] Moczala, Helmut (1998). Small Electric Motors. London: Institution of Electrical Engineers. pp. 165–166.

[7] Mokhtari, A. and Benallegue, A. (2004), "Dynamic feedback controller of Euler angles and wind parameters estimation for a quadrotor unmanned aerial vehicle", 2004 IEEE international conference on robotics and automation.

[8]. Cooke, A.K., Cowling, I.D., Erbsloeh, S.D. and Whid borne, J.F., “Low cost system design and development towards an autonomous rotor vehicle”, 22nd International Conference on Unmanned Air Vehicle Systems, April 2007, Bristol,

[9] Kotarski, D., Benić, Z., and Krznar, M., 2016, Control Design For Unmanned Aerial Vehicles With Four Rotors, Interdisciplinary Description of Complex Systems Vol. 14, No. 2, pp. 236-245.

[10] Davendra Chaturvedi (2010). “Modelling and simulation of systems using Matlab and Simulink”. CRC Press, United States.

[11]. Bouabdallah, S., Noth, A. and Siegwart, R. (2004), PID vs LQ control techniques applied to an indoor micro quadrotor, Swiss Federal Institute of Technology.

[12]. Castillo, P., Lozano, R. and Dzul, A., (2005), Stabilization of a Mini Rotorcraft with Four Rotors, IEEE Control Systems Magazine.

[13]. Modern control engineering 5 the edition , chapter 2 page 43

[14] http://www.scholarpedia.org/article/Siegel_disks/Linearization - The Linearization problem in complex dimension one dynamical systems.

[15]. CSAS_flightmechanics1 lecture slides in CVUT by Martin Xhromcik- department of control Engineering.

[16]. Zambrano-robledo P. C. (2013). "Simplifying quadrotor controllers by using simplified design models", pp. 4236–4241.],

[17] R. Mahony, V. Kumar, and P. Corke, \Multirotor aerial vehicles: Modeling, estimation, and control of quadrotor", IEEE Robotics Automation Magazine, vol. 19, no. 3, pp. 20-32, Sept 2012.

[18] M. Mueller and D. Ra_aello, \A model predictive controller for quadrocopter state interception", in Control Conference (ECC), 2013 European. IEEE, 2013.

[19] S. Lupashin, A. Schoellig, M. Sherback, and R. D'Andrea, ''A simple learning strategy for high-speed quadrocopter multi-ips", in Robotics and Automation (ICRA), 2010 IEEE International Conference on, 2010, pp. 1642-1648

[20] L. Argentim, W. Contrimas, P. Santos, and R. Aguiar, \Lqr and lqr-pid on a quadcopter platform", in IEEE Int.Conference on Electronics, Informatics and Vision, 2013.

[21] G. Ra_o, M. Ortega, and F. Rubio, \An integral predictive/nonlinear h infnity control structure for a quadrotor helicopter", in Automatica, 2013, pp. 1-7.