


College of Natural and Computational Science

Department of Mathematics

Project on Integer programming for Branch and Bound

Prepared by Weynu Temesgen

Advisor: Mr.Waltengus D (M. SC)

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Department of Mathematics

The undersigned hereby certify that they have read and recommend to the Department of Mathematics for acceptance of a Project on Integer programming of Branch and Bound by Waltenegus D in partial Fulfillment of the requirements for the degree of Bachelor of Science.

Jan, 2021

Advisor name:-……

Examining committee:-……

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# Abstract

*The project is an over view of the integer programming of branch and bounded method. In chapter one defines about integer programming problems and its applications of integer programming problem in chapter two its defines widely about the branch and bounded method with its basic Procedure to solve integer programming problems and also it includes examples that solved by branch and bounded method*

# Chapter One

## Introduction

Integer programming problems is a mathematical optimization or feasibly programs in which some or all the variables are restricted to be integers .many setting the term refers to integer liner programming in which the objective function and the constrains other than the integers are constrains.

Linear programming (LP) is one of the most popular models used in decision making and optimization problems. In numerous and) introduced Integer programming (IP) problems as optimization problems that minimize or maximize the objective function, taking into consideration the limits of constraints and integer variables. More widely, the applications of integer programming can be used to appropriately describe the decision problems concerning the effective use of resources in engineering technology, business management and other numerous fields. An integer programming model is a model that has constraints and that one or more of the decision variables has to take on an integer value in the final solution.

In such cases, the fractional values of variables like 13/3 may be meaningless in the context of the actual decision problem. This is the main reason why integer programming is so important for marginal decisions.

### 1.1. Applications of Integer Programming

Integer programming is applied in business and industry. All assignment and transportation problems are integer programming problems, as in assignment and travelling salesmen problem; all the decision variables are either zero or one.

$i.e\ x_{ij} = 0\ or\ 1$

Other examples are capital budgeting and production scheduling problems. In fact, any situation involving decisions of the type 'either to do a job or not to do' can be viewed as an IPP In all such situations

$x_{ij}\ \ the\ \ j^{th}$ Activity is performed,

$0\ \ if\ \ the\ \ j^{th}$ Activity is not performed.

In addition, allocation problems involving the allocation of men, and machines give rise to IPP, since such communities can be assigned only in integers and not infractions.

integer variable is rounded off, it violates the feasibility and also there is no guarantee that the or If the none undid off solution will be optimal. Due to these difficulties, there is a need for developing a systematic and efficient procedure for obtaining the exact optimal integer solution to such problems. Many decision-making problems involve a process that takes place in such a way that at each stage, the process is dependent on the strategy chosen. Such type of problems is called Dynamic Programming Problem (DPP). Thus, DPP is concerned with the theory of multistage decision process. Mathematically, a DPP is a decision making problem in $n$ variables, the problem being subdivided into $n$ sub-problems (segments), and each sub-problem being a decision-making problem in one variable only. The solution to a DPP is achieved sequentially, starting from one (initial) stage to the next, till the final stage is reached.

# Chapter two

# The Branch-and-Bound Method

## 2.1. Basic Procedure

An integer linear program is a linear program further constrained by the integrality restrictions. Thus, in a maximization problem, the value of the objective function, at the linear-program optimum, will always be an upper bound on the optimal integer-programming objective. In addition, any integer feasible point is always lower bound on the optimal linear-program objective value.

The most effective general purpose optimal algorithm is an LP-based tree search approach called as ***branch and bound*** (B&B).

This is *a way of systematically (implicitly) enumerating* feasible solutions such that the optimal integer solution is found. Where this method differs from the enumeration method is that *not all* the feasible solutions are enumerated but only a part (hopefully a small part) of them. However we can still *guarantee* that we will find the optimal integer solution. By solving a single sub-problem, many possible solutions may be eliminated from consideration. Sub-problems are generated by branching on an appropriately chosen fractional-valued variable. Suppose that in a given sub-problem (call it subp.1), assumes a fractional value between the integers $i \; and \; i+1$ Then the two newly generated sub-problems:

$subp.2 = subp.1 + constraint \; x_i \geq +1"$

$subp.3 = subp.1 + constraint" x_i \; \leq 1"$

If all variables have integer values in the optimal solution to the sub-problem then the solution is a feasible solution for the original IP. If the current feasible solution for the IP has a better $z - value$ than any previously obtained feasible solution, then it becomes a candidate solution, and its $z - value$ becomes the current Lower Bound (LB) on the optimal $z - value$ (for a max problem). If it is unnecessary to branch on a sub-problem, we say that it is fathomed (inactive):

- The sub-problem is infeasible
- The sub-problem yields an optimal solution in which all variables have integer values
- The optimal $z - value$ for the sub-problem does not exceed the current $LB$, so it cannot yield the optimal solution of the IP

## 2.2. Branch and Bound Method

His is applicable of both all integer programming problem as well as a mixed integer programming problem sometimes a few or all the variable of an integer programming problem are constrained by their upper or lower bounds the most general method or the solution of such constrained optimization problem is called Branch and Bound methods his method first divides the feasible region in of smaller subsets and then example each of them successively until a feasible solution give value of objective is obtained.

Let the given integer programming problem be

$$Maximize\ Z = Cx$$
$$subject\ to\ Ax \leq b$$
$$x \geq 0 are\ integer$$

In his method we firs solve the problem by ignoring he integrality condition

- If the solution is in integer the current solution is optimum of the given integer
- If solution is no integer say one the variable $x$ is no integer then $x_r^* \leq x_r \leq x_{r+1}^*$ $where\ x_r^*, x_{r+1}^*$ are consecutive non negative integer

Hence any feasible integer value $x_r$ must satisfy one of the two conditions $x_r \leq x_r\ or\ x_r \geq x_{r+1}^*$

Two conditions are mutually exclusive or both can be true simultaneous by adding two conditions separately the given linear programming problem we form

subproblem1

$$\max \quad z = cx$$
$$s.t \quad Ax \leq b$$
$$x_r \leq x_r^*\ and\ x \geq 0$$

Sub problem2

$$\max \quad z = cx$$
$$s.t \quad Ax \leq b$$
$$x_r \geq x_{r+1}^*\ and\ x \geq 0$$

Thus we have branched or partitioned the original problem in to two sub problem each of these sub problem is then solved separately as linear programming problem

**Example**

Use Brach and Bound technique to solve the following

$$\max \quad z = x_1 + 4x_2$$
$$s.t \quad 2x_1 + 4x_2 \leq 7$$

$$5x_1 + 3x_2 \leq 15$$

$$x_1, x_2 \geq 0 \; and \; are \; integer$$

**Solution**

Ignoring the integrality conditions we solve the linear programming problem

$$\max \quad z = x_1 + 4x_2$$

$$s.t \; to \; 2x_1 + 4x_2 \leq 7$$

$$5x_1 + 3x_2 \leq 15$$

$$x_1, x_2 \geq 0$$

Introducing a slack variable $s_1, s_2 \geq 0$ the standard from of linear programming problem

$$\max \; z = x_1 + 4x_2 + 0s_1 + 0s_2$$

$$s.t \; 2x_1 + 4x_2 + s_1 = 7$$

$$5x_1 + 3x_2 + s_2 = 15$$

| | | $c_j$ | 1 | 4 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| $c_B$ | $B$ | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $minx_B/x_2$ |
| 0 | $s_1$ | 7 | 2 | **4** | 1 | 0 | 7/4 |
| 0 | $s_2$ | 15 | 5 | 3 | 0 | 1 | 15/3=5 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $z_j_c_j$ | | -1 | -4↑ | 0 | 0 | |
| 4 | $x_2$ | 7/4 | 1/2 | 1 | 1/4 | 0 | |
| 0 | $s_2$ | -13/4 | -7/6 | 0 | -1/3 | 1 | |
| | $z_j$ | 7 | 2 | 4 | 1 | 0 | |
| | $z_j_c_j$ | | 1 | 0 | 1 | 0 | |

Since all $z_j_c_j \geq 0$ an optimal solution is obtained

$x_1 = 0, x_2 = \frac{7}{4}$ *and* max $z = 7$

Since $x_2 = 7/4$ this problem should be branched in to two sub problem

$$x_2 = \frac{7}{4} \quad ,1 > x_2 > 2$$

$$= \quad x_2 \leq 1 \quad , x_2 \leq 2$$

Appling these two conditions separately in the give LPP we get two sub problems

**Sub Problem 1**

$$\max\ z = x_1 + 4x_2$$
$$s.t\ 2x_1 + 4x_2 \leq 7$$
$$5x_1 + 3x_2 \leq 15$$
$$x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

**Sub Problem 2**

$$\max\ z = x_1 + 4x_2$$
$$s.t\ 2x_1 + 4x_2 \leq 7$$
$$5x_1 + 3x_2 \leq 15$$
$$x_2 \geq 1$$
$$x_1,\ x_2 \geq 0$$

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|
| $c_B$ | $B$ | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $minx_B/x_2$ |
| 0 | $s_1$ | 7 | 2 | 4 | 1 | 0 | 0 | 7/4 |
| 0 | $s_2$ | 15 | 5 | 3 | 0 | 1 | 0 | 15/3 |
| **0** | $s_3$ | 1 | 0 | **1** | 0 | 0 | 1 | 1/1 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | $minx_B/x_1$ |
| | $z_{j-}c_j$ | | -1 | **-4**↑ | 0 | 0 | 0 | |
| **0** | $s_1$ | 3 | **2** | 0 | 1 | 0 | **-4**↑ | 3/2 |
| 0 | $s_2$ | 12 | 5 | 0 | 0 | 1 | -3 | 12/5 |
| 0 | $x_2$ | 1 | 0 | 1 | 0 | 0 | 1 | |
| | $z_j$ | 4 | 0 | 4 | 0 | 0 | 4 | |
| | $z_{j-}c_j$ | | **-1**↑ | 0 | 0 | 0 | 4 | |

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $c_B$ | B | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ |
| 1 | $x_1$ | 3/2 | 1 | 0 | ½ | 0 | -2 |
| 0 | $s_2$ | 9/2 | 0 | 0 | -5/2 | 1 | 7 |
| 4 | $x_2$ | 1 | 0 | 1 | 0 | 0 | 1 |
| | $z_j$ | 11/2 | 1 | 4 | ½ | 0 | 2 |
| | $z_{j-}c_j$ | | 0 | 0 | 1/2 | 0 | 2 |

Since all $z_j_c_j \geq 0$ the solution is optimum given by

$$x_1 = \frac{3}{2} and\ x_2 = 1 \max\ z = \frac{11}{2}$$

Since $x_1 = 3/2$ is not an integer this sub problem is branched again

Sub Problem 3

$$\max\ z = x_1 + 4x_2$$
$$s.t\ 2x_1 + 4x_2 \leq 7$$
$$5x_1 + 3x_2 \leq 15$$
$$x_1 \geq 2$$
$$x_1, x_2 \geq 0$$

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 | -M | |
|---|---|---|---|---|---|---|---|---|---|
| $c_B$ | $B$ | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $\frac{minx_B}{x_1}$ |
| **0** | $s_1$ | 7 | 2 | **4** | 1 | 0 | 0 | 0 | 7/4 |
| 0 | $s_2$ | 15 | 5 | 3 | 0 | 1 | 0 | 0 | 15/3 |
| -M | $A_1$ | 2 | 0 | 1 | 0 | 0 | -1 | 1 | 2/1 |
| | $z_j$ | -2M | 0 | -M | 0 | 0 | M | -M | |
| | $z_j - c_j$ | | -1 | -M-4 | 0 | 0 | M | 0 | |
| 4 | $x_2$ | 7/4 | ½ | 1 | ¼ | 0 | 0 | 0 | |
| 0 | $s_2$ | 39/4 | 7/2 | 0 | -3/4 | 1 | 0 | 0 | |
| -M | $A_1$ | ¼ | -1/2 | 0 | -1/4 | 1 | -1 | 1 | |
| | $z_j$ | 7-5m/4 | 2+m/2 | 4 | 1+m/4 | 0 | M | -M | |
| | $z_j - c_j$ | | M/2+1 | 0 | M/4+1 | 0 | M | 0 | |

Since all $z_j - c_j \geq 0$.but an artificial $A_1$ is in the basis at positive level there exist no feasible solution hence this sub problem is dropped

In sub problem (1) since $x_1 = 3/2$

$$we\ have\ 1 \leq x_1 \leq 2$$
$$= x_1 \leq 1, x_1 \geq 2$$

Applying these two condition separately in the sub problem (1) we get the sub-problems

**Sub –Problem 3**

max $z = x_1 + 4x_2$

$s.t\ 2x_1 + 4x_2 \leq 7$

$5x_1 + 3x_2 \leq 15$

$x_2 \leq 1$

$x_1 \leq 1$

$x_1, x_2 \geq 0$

**Sub Problem 4**

max $z = x_1 + 4x_2$

$s.t\ 2x_1 + 4x_2 \leq 7$

$x_2 \leq 1$

$x_1 \geq 2$

$x_1, x_2 \geq 0$

Sub Problem 3

Since all $z_j - c_j \geq 0$ an optimum is obtained

It is given by $x_1 = x_2\ and$ max $z = 5$

Since the solution is integer valued this sub-problem cannot be branched further

The lower bound of the objective function

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|---|
| $c_B$ | $^B$ | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $minx_B/x_2$ |
| 0 | $s_1$ | 7 | 2 | 4 | 1 | 0 | 0 | 0 | 7/4 |
| 0 | $s_2$ | 15 | 5 | 3 | 0 | 1 | 0 | 0 | 15/3 |
| **0** | $s_3$ | 1 | 0 | **1** | 0 | 0 | 1 | 0 | 1/1 |
| 0 | $s_4$ | 1 | 1 | 0 | 0 | 0 | 0 | 1 | - |
| | $z_j$ | | 0 | 0 | 0 | 0 | 0 | 0 | $minx_B/x_1$ |
| | $z_{j_}c_j$ | | | -1 | -4↑ | 0 | 0 | 0 | |
| 0 | $s_1$ | 3 | 2 | 0 | 1 | 0 | -4 | 0 | 3/2 |
| 0 | $s_2$ | 12 | 5 | 0 | 0 | 1 | -3 | 0 | 12/5 |
| 4 | $x_2$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 | - |
| **0** | $s_4$ | 1 | **1** | 0 | 0 | 0 | 0 | 1 | 1/1 |
| | $z_j$ | 4 | 0 | 4 | 0 | 0 | 4 | 0 | |
| | $z_{j_}c_j$ | | -1↑ | 0 | 0 | 0 | 4 | 0 | |

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|

| $c_B$ | $^B$ | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
|---|---|---|---|---|---|---|---|---|
| 0 | $s_1$ | 7 | 2 | 4 | 1 | 0 | 0 | 0 |
| 0 | $s_2$ | 15 | 5 | 3 | 0 | 1 | 0 | 0 |
| 4 | $x_2$ | 1 | 0 | **1** | 0 | 0 | 1 | 0 |
| 1 | $x_1$ | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| | $z_j$ | 5 | 1 | 4 | 0 | 0 | 4 | 1 |
| | $z_{j_}c_j$ | | 0 | 0 | 0 | 0 | 4 | 1 |

**Sub –Problems 4**

$$\max \ z = x_1 + 4x_2$$
$$s.t\ 2x_1 + 4x_2 \leq 7$$
$$5x_1 + 3x_2 \leq 15$$
$$x_2 \leq 1$$
$$x_1 \geq 2$$
$$x_1, x_2 \geq 0$$

Since all $z_{j_}c_j \geq 0$ the solution is optimum and give by

$$x_1 = 2, \quad x_2 = \frac{3}{4}$$

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 | 0 | -M | |
|---|---|---|---|---|---|---|---|---|---|---|
| $c_B$ | B | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $A_1$ | $\frac{minx_B}{x_1}$ |
| 0 | $s_1$ | 7 | 2 | 4 | 1 | 0 | 0 | 0 | 0 | 7/2 |
| 0 | $s_2$ | 15 | 5 | 3 | 0 | 1 | 0 | 0 | 0 | 15/5 |
| 0 | $s_3$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | - |
| -M | $A_4$ | 2 | **1** | 0 | 0 | 0 | 0 | -1 | 1 | 2/1 |
| | $z_j$ | -2M | -M | 0 | 0 | 0 | 0 | M | -M | |
| | $z_{j_}c_j$ | - | -M-1 | -4 | 0 | 0 | 0 | M | 0 | |

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 | 0 | -M | |
|---|---|---|---|---|---|---|---|---|---|---|

| $c_B$ | $B$ | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $A_1$ | $minx_B/x_1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **0** | $s_1$ | 3 | 0 | **4** | 1 | 0 | 0 | 2 | -2 | ¾ |
| 0 | $s_2$ | 5 | 0 | 3 | 0 | 1 | 0 | 5 | -5 | 5/3 |
| 0 | $s_3$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1/1 |
| -M | $x_1$ | 2 | 1 | 0 | 0 | 0 | 0 | -1 | 1 | - |
| | $z_j$ | 2 | 1 | 0 | 0 | 0 | 0 | -2 | - | |
| | $z_{j-}c_j$ | | 0 | -4↑ | 0 | 0 | 0 | -2 | - | |
| 4 | $x_2$ | ¾ | 0 | 1 | ¼ | 0 | 0 | ½ | - | |
| 0 | $s_2$ | 11/4 | 0 | 0 | -3/4 | 1 | 0 | 7/2 | - | |
| 0 | $s_3$ | ¼ | 0 | 0 | -1/4 | 0 | 1 | -1/2 | - | |
| | $x_1$ | 2 | 1 | 0 | 0 | 0 | 0 | -1 | - | |
| | $z_j$ | 5 | 1 | 4 | 1 | 0 | 0 | 1 | - | |
| | $z_{j-}c_j$ | | 0 | 0 | 1 | 0 | 0 | 1 | - | |

Since $x_{2=3/4}$ is no integer. This sub problem is branched further

In sub problem (4) since $x_2 = \frac{3}{4}$ $0 \le x_2 \le 1 = x_2 \le 0$ $or$ $x_2 \ge 1$

Appling these two conditions in the sub problem (4) we get two sub problems

**Subproblem6**

**Sub problem 5**

$$\max\ z = x_1 + 4x_2$$
$$s.t\ 2x_1 + 4x_2 \le 7$$
$$5x_1 + 3x_2 \le 15$$
$$x_2 \le 1$$
$$x_1 \ge 2$$
$$x_2 \le 0$$
$$x_1, x_2 \le 0$$

$$maxz = x_1 + 4x_2$$
$$s.t\ 2x_1 + 4x_2 \le 7$$
$$5x_1 + 3x_2 \le 15$$
$$x_1 \ge 2$$
$$x_2 \ge 1$$
$$x_1, x_2 \ge 0$$

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 | 0 | -M | 0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $c_B$ | $B$ | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $A_1$ | $s_5$ | $minx_B/x_1$ |
| 0 | $s_1$ | 7 | 2 | 4 | 1 | 0 | 0 | 0 | 0 | 0 | 7/2 |
| 0 | $s_2$ | 15 | 5 | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 15/2 |
| 0 | $s_3$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | - |
| -M | $x_1$ | 2 | **1** | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 2/1 |
| **0** | $s_5$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | - |
| | $z_j$ | - | -M | 0 | 0 | 0 | 0 | M | -M | 0 | $minx_B/x_2$ |
| | $z_{j_}c_j$ | 2M | -M-1 | -4 | 0 | 0 | 0 | M | 0 | 0 | |
| 0 | $s_1$ | 3 | 0 | 4 | 1 | 0 | 0 | 2 | - | 0 | ¾ |
| **0** | $s_2$ | 5 | 0 | 3 | 0 | 1 | 0 | 5 | - | 0 | 5/3 |
| 0 | $s_3$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 | - | 0 | 1/1 |
| 1 | $x_1$ | 2 | 1 | 0 | 0 | 0 | 0 | -1 | - | 0 | - |
| 0 | $s_5$ | 0 | 0 | **1** | 0 | 0 | 0 | 0 | - | 1 | 0/1 |
| | $z_j$ | 2 | 1 | 0 | 0 | 0 | 0 | -2 | - | 0 | $minx_B/s_4$ |
| | $z_j - c_j$ | | 0 | -4↑ | 0 | 0 | 0 | -2 | - | 0 | |

| | | $c_j$ | 1 | 4 | 0 | 0 | 0 | 0 | -M | 0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{c_B}$ | $B$ | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $A_1$ | $s_5$ | $minx_B/x_1$ |
| 0 | $s_1$ | 3 | 0 | 4 | 1 | 0 | 0 | 2 | - | 0 | 3/2 |
| 0 | $s_2$ | 5 | 0 | 3 | 0 | 1 | 0 | **5** | - | 0 | 1 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $s_3$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 | - | -1 | - |
| 1 | $x_1$ | 2 | 1 | 0 | 0 | 0 | 0 | -1 | - | 0 | - |
| 0 | $x_2$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | - | 1 | - |
| | $z_j$ | 2 | 1 | 4 | 0 | 0 | 0 | -1 | - | 0 | |
| | $z_j _ c_j$ | | 0 | 0 | 0 | 0 | 0 | -1↑ | - | 0 | |
| 0 | $s_1$ | 1 | 0 | 0 | 1 | -2/5 | 0 | 0 | - | 0 | |
| 0 | $s_4$ | 1 | 0 | 0 | 0 | 1/5 | 0 | 1 | - | 0 | |
| 0 | $s_3$ | 1 | 0 | 0 | 0 | 0 | 1 | 0 | - | 0 | |
| 1 | $x_1$ | 3 | 1 | 0 | 0 | 1/5 | **1** | 0 | - | 0 | |
| 4 | $x_2$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | - | 1 | |
| | $z_j$ | 3 | 1 | 4 | 0 | 3/5 | 0 | 0 | - | 4 | |
| | $z_j _ c_j$ | | 0 | 0 | 0 | 3/5 | 0 | 0 | - | 4 | |

Since all$z_j_c_j \geq 0$ the solution is optimum and is given by

$x_1 = 3, x_2 = 0\ and\ maxz = 3$ This problem yields an optimum integers solution .hence this sub problems is dropped

**Sub Problem 6**

$$\max\ z = x_1 + 4x_2$$

$$s.t\ 2x_1 + 4x_2 \leq 7$$

$$5x_1 + 3x_2 \leq 15$$

$$x_2 \geq 1$$

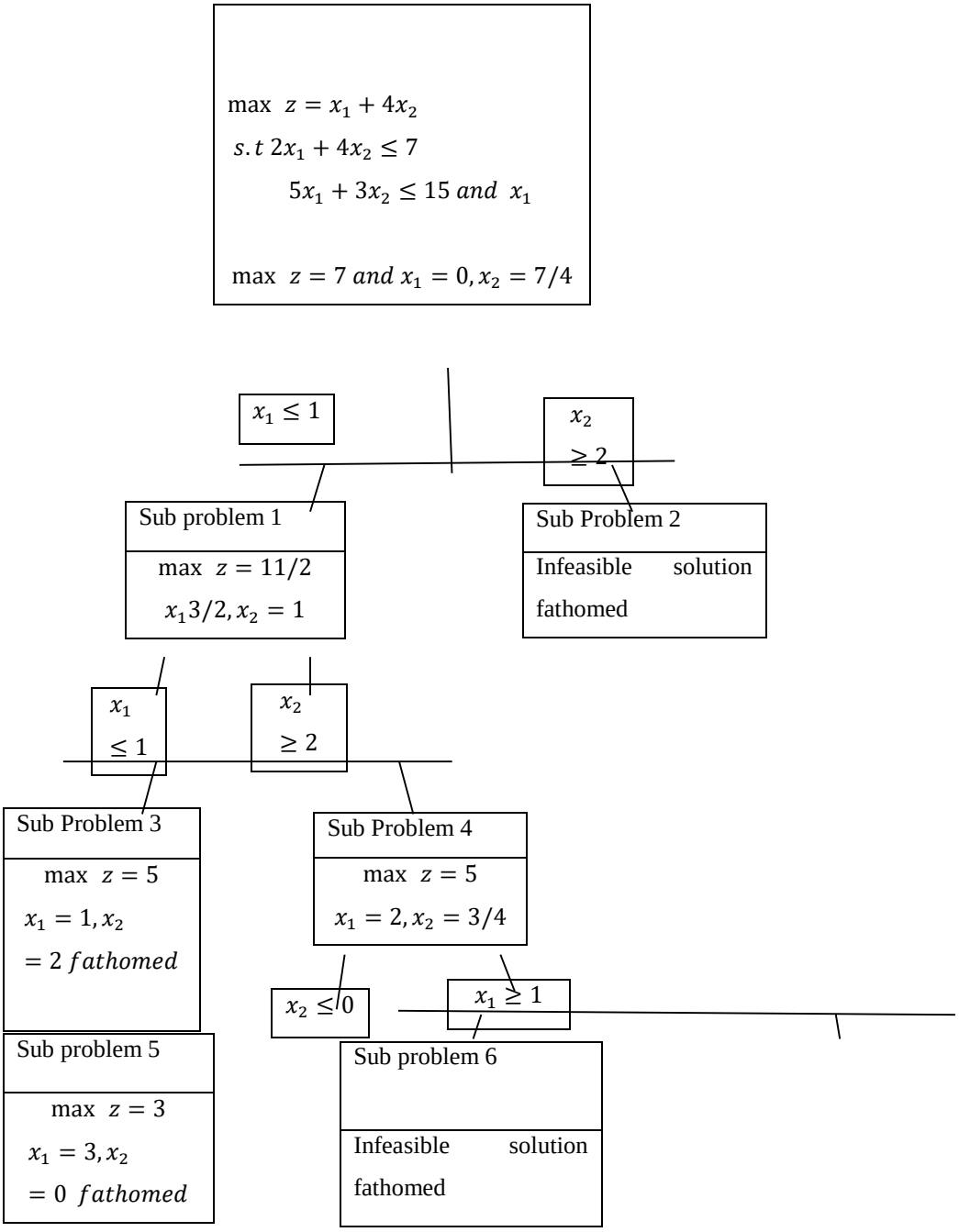
$$x_1 \geq 2$$

$$x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

This sub has no feasible solution hence this sub –problem is also fathomed

**Original problem**




Among the variable integer valued solution, the best integer solution is given by sub problem 3 therefore the optimum integer solution is $maxz = 5\ and\ x_1 = 1, x_2 = 1$ the best variable integer optimal solution is

$maxz = 5\ and\ x_1 = 1, x_2 = 1$

## Conclusion

In this project is some basic concepts of rough intervals are reviewed Then we presented a methodology for solving fully rough integer programming problems and found rough value optimal solutions and decision rough integer variables, where all parameters and decision variables in the constraints and the objective functions are rough intervals. The proposed model depends on sum method, branch and bound method for integer programming, which is a good technique for many LP problems which require that the decision variables are integers. Also, branch and bound algorithm as a method for global optimization for discrete problem the used rough intervals are very important to tackle the uncertainty in decision and imprecise data in decision making problems. Also, a flowchart of the steps to solve the problem is provided for more clarification.

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